

$$\begin{aligned}
 \mathbf{x} \cdot \mathbf{y} &= x_1 y_1 + x_2 y_2 + \dots + x_n y_n \\
 (\mathbf{x} + \mathbf{x}') \cdot \mathbf{y} &= (x_1, \dots, x_n) + (x'_1, \dots, x'_n) \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \\
 &= (x_1 + x'_1) y_1 + \dots + (x_n + x'_n) y_n \\
 &= \underbrace{x_1 y_1}_{\mathbf{x} \cdot \mathbf{y}} + \underbrace{x'_1 y_1}_{\mathbf{x}' \cdot \mathbf{y}} + \dots + \underbrace{x_n y_n}_{\mathbf{x} \cdot \mathbf{y}} + \underbrace{x'_n y_n}_{\mathbf{x}' \cdot \mathbf{y}}
 \end{aligned}$$



$$\Lambda_i(A \cdot B) = (\Lambda_i(A) \cdot D_i(B), \dots, \Lambda_i(A) D_i(B))$$


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$$\begin{array}{cc} A & B \\ m \times n & n \times m \end{array}$$

$$A \cdot B = (\Lambda_i(A) D_i(A))$$

$$\Lambda_i(A) \cdot B = (\Lambda_i(A) \cdot D_i(A))_{i=1}^n$$

$$\begin{aligned} A \cdot (B + B') &= ( \wedge_i (A) \text{ } \text{OR} (B + B') ) \\ &= ( \wedge_i (A) \cdot ( \text{OR} (B) + \text{OR} (B') ) ) \\ &= ( \wedge_i (A) \text{ } \text{OR} (B) + \wedge_i (A) \text{ } \text{OR} (B') ) \\ &= ( \wedge_i (A) \text{ } \text{OR} (B) ) + ( \wedge_i (A) \text{ } \text{OR} (B') ) \\ &= A \cdot B + A \cdot B' \end{aligned}$$

$$I_m A = A$$

$m \times m$

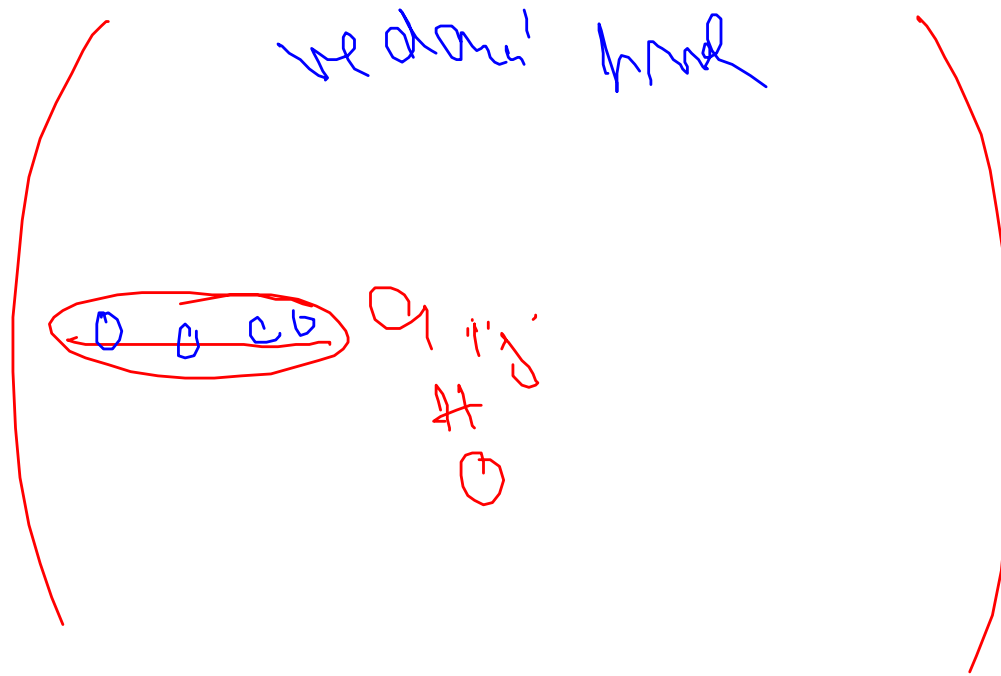
$$I_m A = \left( \begin{matrix} I_i(I_m) \\ \vdots \\ I_m(I_m) \end{matrix} \right) \left( \begin{matrix} R_1(A) \\ \vdots \\ R_m(A) \end{matrix} \right) =$$

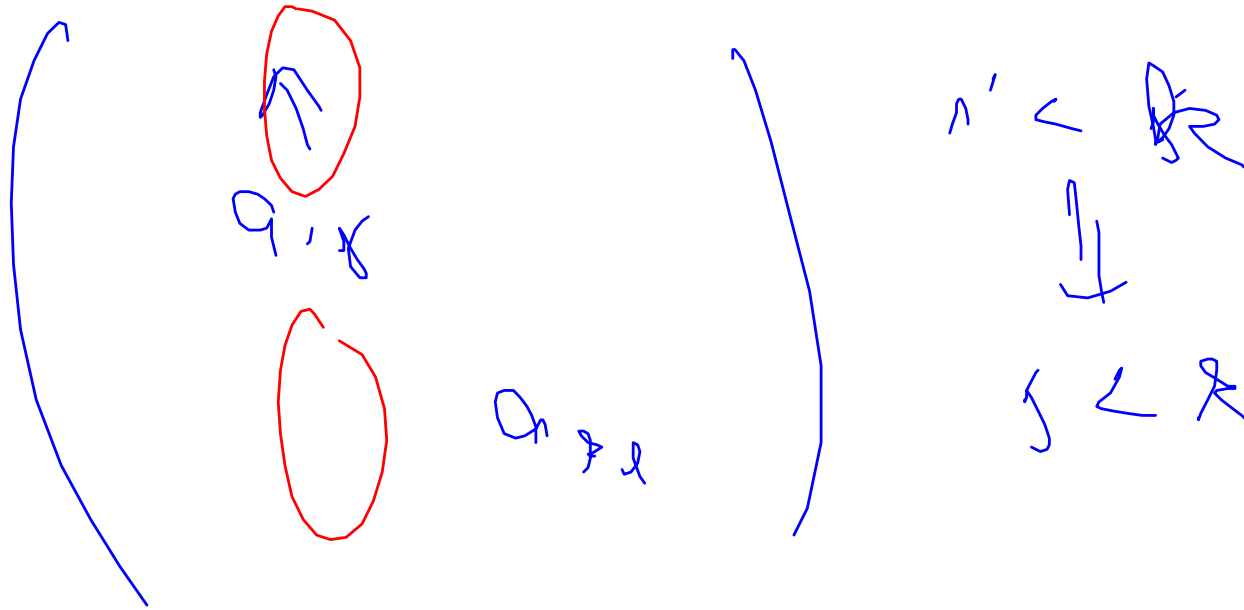
$$\left( \begin{matrix} (0 \dots 1 \dots 0) \\ \vdots \\ (0 \dots 1 \dots 0) \end{matrix} \right) \cdot \left( \begin{matrix} R_1(A) \\ \vdots \\ R_m(A) \end{matrix} \right) = (1 \cdot R_i(A))$$

$$= (R_i(A)) = A$$

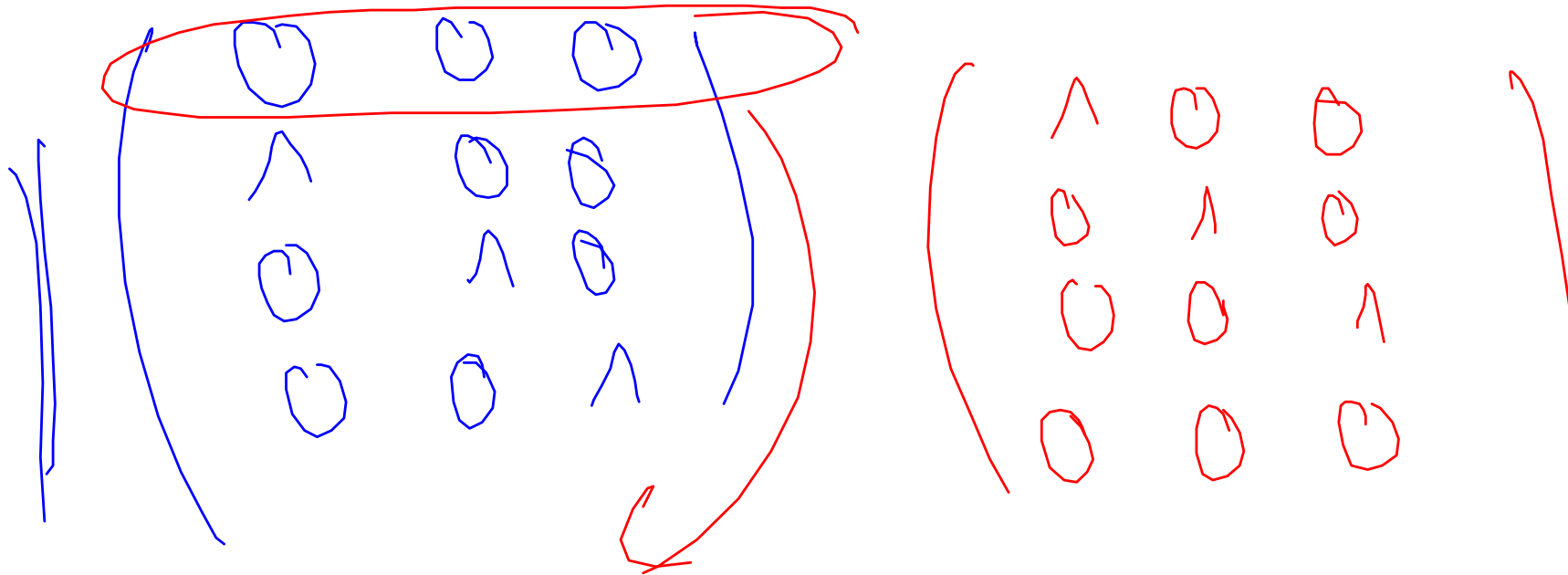


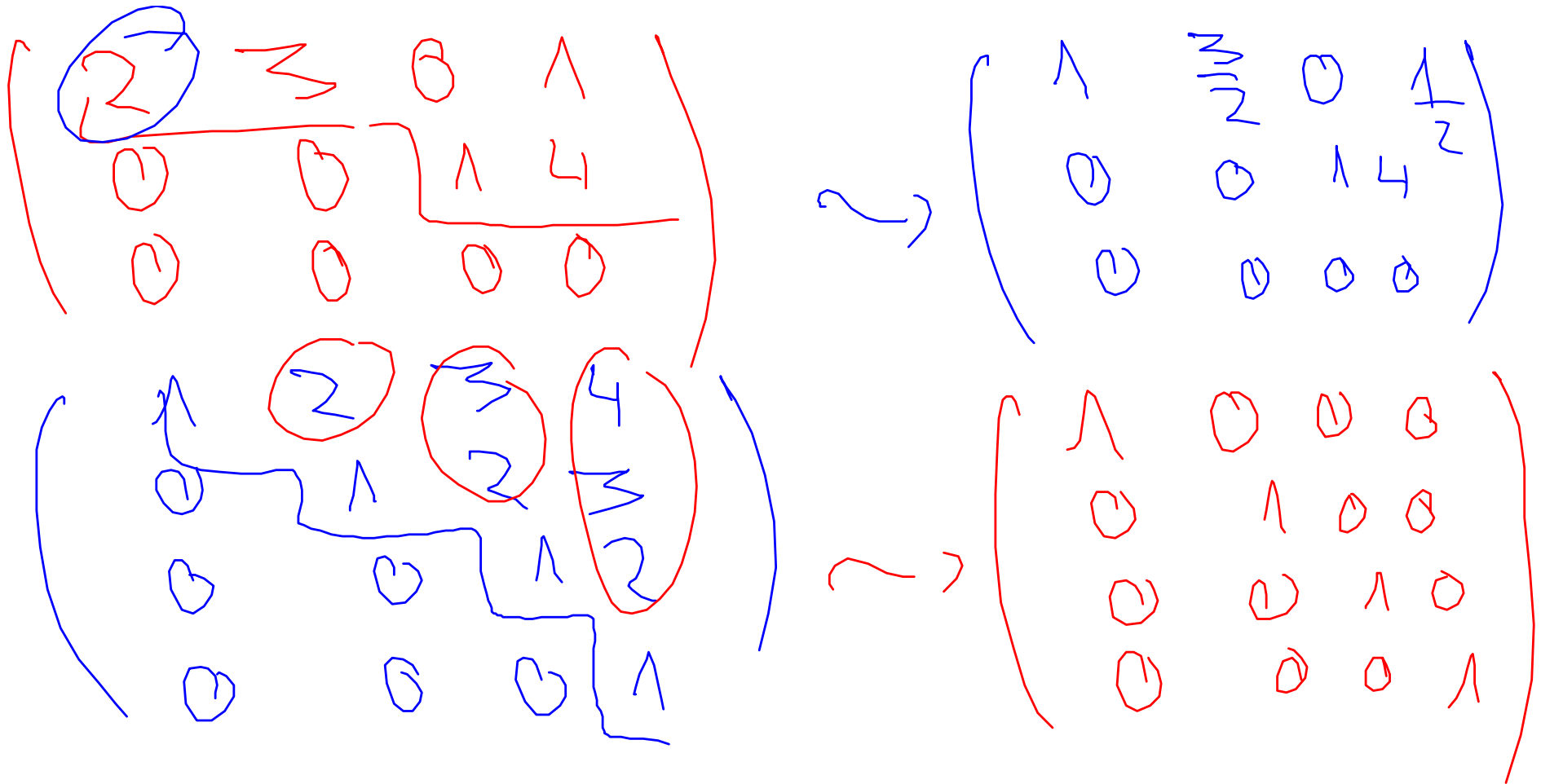
$$\begin{aligned}
 (A \cdot B)^T &= (P_i(A) \cdot P_r(B))^T \\
 &\stackrel{R \rightarrow}{=} \left( \underline{P_i(A) \cdot P_r(B)} \right) = \left( \underline{P_r^T(B)} \cdot \underline{P_i^T(A)} \right) \\
 &\stackrel{I}{=} \left( P_r(B^T) \cdot P_i(A^T) \right) = \\
 &\stackrel{I}{=} B^T \cdot A^T
 \end{aligned}$$





$$\begin{pmatrix} 0 & 2 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$





$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\left( \begin{array}{cccccc|c} 1 & 0 & -2 & 0 & 0 & 3 \\ 0 & 1 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ \hline x_1 & x_2 & x_3 & x_4 & x_5 & \end{array} \right)$$

$0, k \in \mathbb{R}$

$$x_4 = 1$$

$$\begin{aligned} x_3 &= k \\ x_5 &= 0 \end{aligned}$$

$$\begin{aligned} x_2 &= 0 - 6x_3 \\ x_1 &= 3 + 2x_3 \end{aligned}$$

$$(3 + 2k, -6k, k, 1, 0)$$



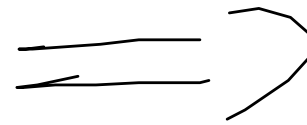
$(0, \dots, 0 \mid 1)$  ←  
 $(B|c)$  mai răsun' a B ordoniz

$$0x_1 + \dots + 0x_n = 1$$



$$0 = 1$$

$$0 \neq 1$$

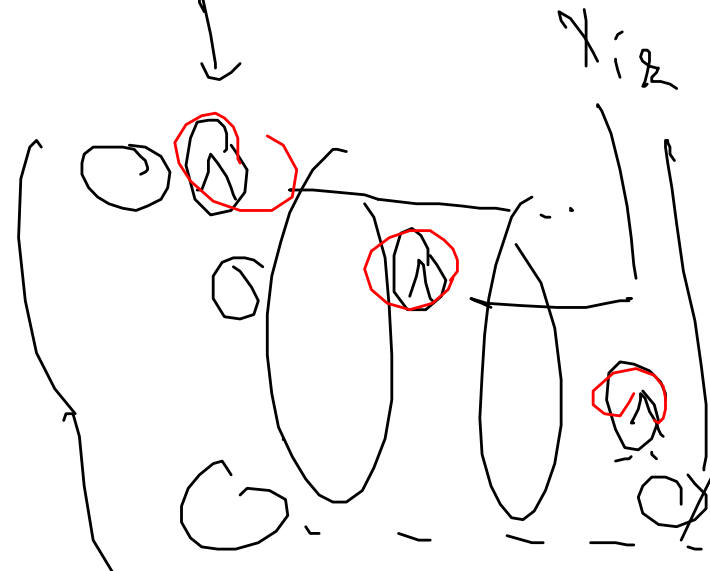


← para se  $\mathbb{B}/\mathbb{C}$  (memória)

$$(\mathbb{C}, \dots, \mathbb{C} | \mathbb{1}) \xrightarrow{2} \mathbb{R} \leq \mathbb{M}$$

$$\mathbb{B} \times \mathbb{C} = \mathbb{C} \quad \text{mó} \quad \text{no} \quad \mathbb{X}_i$$

me mó se den



$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 4 & 4 & 4 \\ 4 & 4 & 4 \\ 4 & 4 & 4 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ 5 & 2 & 2 \end{pmatrix}$	$\begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix}$
$x_1$	$x_2$	$x_3$	$D$
$f_3 =$	$4$	$D$	$+$
$f_2 =$	$1$	$4$	$D$
$f_1 =$	$1$	$4$	$D$

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$m < n$       $A \neq \mathbb{0}$       $m \times n$       $n \times n$

$A \neq R \neq T$

$\exists$  closed  $\rightarrow$  flexion      $m \times n$       $n \times n$       $n \times n$

$\mathbb{0}$

$X \neq$       $\mathbb{0}$       $n \times n$       $n \times n$

$X \neq \mathbb{0}$

$R \neq \mathbb{0}$ ,  $X \neq \mathbb{0}$  ;

$\Rightarrow$

$R$  red.  $n \times n$

$\begin{matrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{matrix} \Bigg|$

$$A = \begin{pmatrix} D_1(A) & \dots & \dots \\ & & \downarrow \\ & & D_m(A) \end{pmatrix}$$

$$Ax = 0$$

$\left\{ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{matrix} \right\}$

$$J = \{ j_1, \dots, j_r \}$$

$$\begin{pmatrix} D_{j_1}(A) & \dots & \dots \\ & & \dots \\ & & D_{j_r}(A) \end{pmatrix} \begin{pmatrix} x_{j_1} \\ \vdots \\ x_{j_r} \end{pmatrix} = \underline{D_{j_i}(A) x_{j_i}}$$