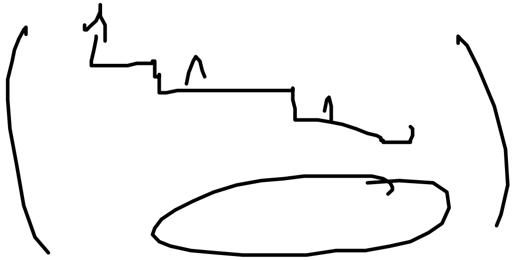
$\varphi : V \to U$

$r(\varphi) = r(A) = k \qquad A = (\varphi)_{\alpha', \beta'}$

$A \sim$  k red. r..

α', β'

$\downarrow$

α, β

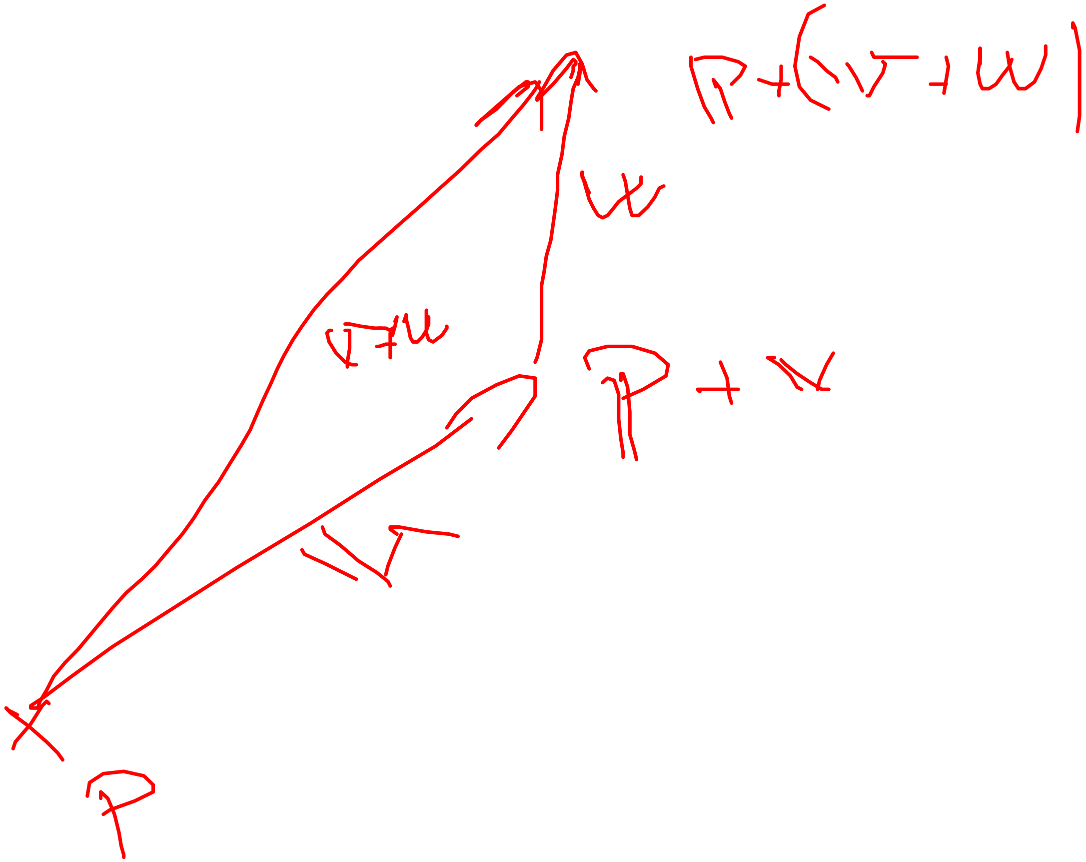
$B \sim$ 

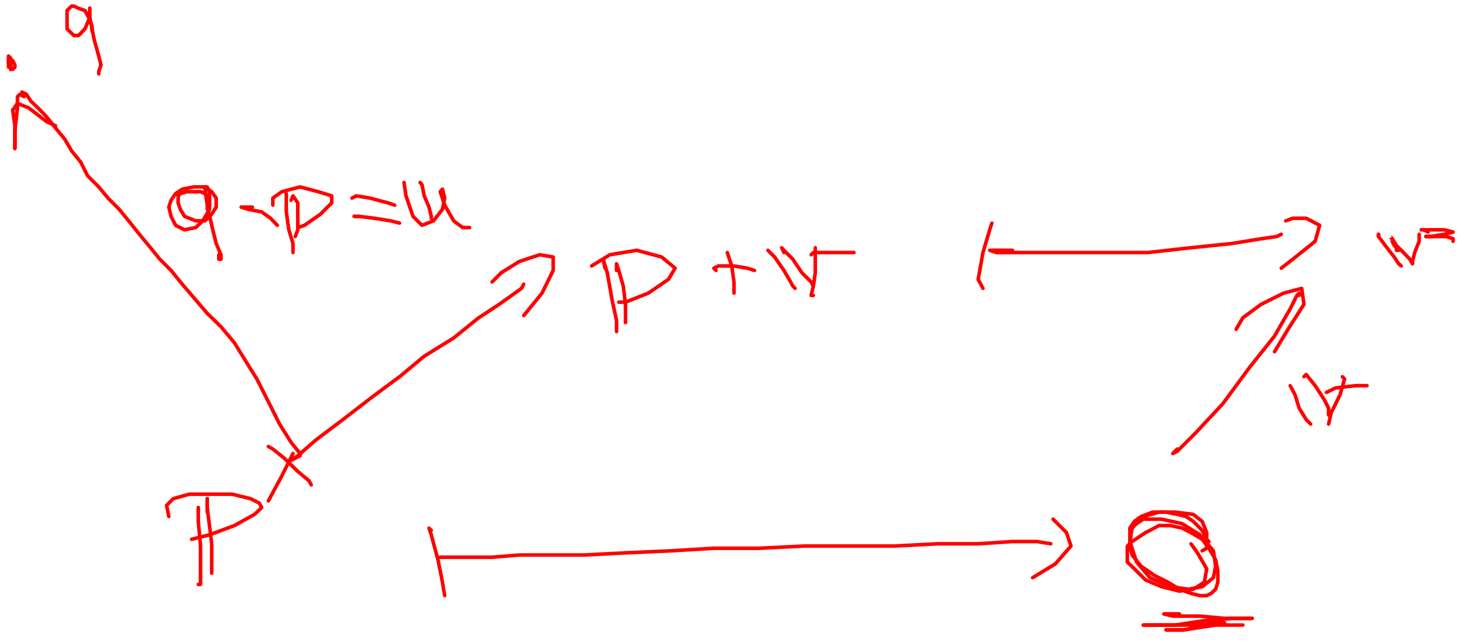
$$\underline{1+1+1+\ldots+1} = n \cdot 1 \neq 0$$

$\mathbb{Z}_3$

$$2 = 2 \cdot 1 \neq 0$$

$$\underline{\underline{3 \cdot 1 = 0}}$$


P+(V+W)
w
V+W
P+V
V
P


q
q - p = u
p + v
v
v
p
Q

(I) $\Leftarrow$ (II) $\qquad \{pn + (1-p)q : n \in K\}$

$= l(p, q)$

(III) $\Rightarrow$ (II) $\qquad n = 1$

(II) $\Rightarrow$ (III)

$n = 0 \qquad I_0 = 1 \qquad 1 \cdot p_0 = p_0 \in M$

$n \Rightarrow n+1$

$\exists\, k \quad 0 \le k \le n+1 \quad b_k = 0$

$\forall\, k, \quad 0 \le k \le n+1 \quad b_k \ne 0$

$$\sum_{k=0}^{n+1} b_k = 1 \qquad b_{n+1} \ne 1$$

$$\alpha = \sum_{k=0}^{n} b_k \ne 0 \qquad \alpha + b_{n+1} = 1$$

$\cup M$

$$\sum_{k=0}^{m+1} h_k p_k = \sum_{k=0}^{m} h_k p_k + (1-\alpha) p_{m+1} =$$

$$= \alpha \left(\sum_{k=0}^{m} \frac{h_k}{\alpha} p_k \right) + (1-\alpha) p_{m+1}$$

$\in M$ $\qquad\qquad$ $\in M$

$$\sum \frac{h_k}{\alpha} = \frac{\sum h_k}{\alpha} = \frac{\alpha}{\alpha} = 1$$

M AP

$M \neq \emptyset$

$\forall p, q \in M \Rightarrow \ell(p,q) \subseteq M$

$\exists p \in M$

$S = M - p = \{ q - p : q \in M \}$

$\in V$

$0 \in S \subseteq V$

$\|$

$p - p$

$u, v \in S, \quad c, d \in k$

$\Rightarrow cu + dv \in S$

$cu + dv \;\overset{2}{\in}\; S$

$= cq_1 + dq_2 + (1-c-d)P$

$u = q_1 - P \;,\; v = q_2 - P \;\in\; \Rightarrow P \in S$

$c(q_1 - P) + d(q_2 - P) =$

$= cq_1 - cP + dq_2 - dP =$

$= cq_1 + dq_2 - (c+d)P$

$-(c+d)P$

$= cq_1 + dq_2 - (1-c-d)P + (1-c-d)P$

$M = P + S \overset{?}{\Rightarrow} M\ AP$

$O \in S \Rightarrow P \subseteq M \neq \emptyset$

$q_1, q_2 \in M$, $u, v \in S$

$q_1 = p + u, \quad q_2 = p + v$

$\lambda q_1 + (1-\lambda) q_2 = \lambda p + \lambda u + (1-\lambda)p +$

$+ (1-\lambda) v = p + \underbrace{\lambda u + (1-\lambda) v}_{\in S}$

$$X \subseteq V \mapsto \ell(X)$$

erz. AP des $\underline{X}$

$$\ell(p_0, \dots, p_n) = p_0 + \underbrace{[p_1 - p_0, \dots, p_n - p_0]}_{\dim(\ell(\dots))}$$

$$\ell(p_0, \dots, p_n) = \{ \sum_{i=0}^{n} \lambda_i p_i :$$

$$n \in \mathbb{N}, \sum_{i=0}^{n} \lambda_i = 1 \} \qquad \overset{?}{\equiv} p_0$$

$$+ [\underbrace{p_1 - p_0}_{\in \operatorname{Dim} \ell(\dots)}, \dots, \underbrace{p_n - p_0}_{\in \operatorname{Dim} \ell(\dots)}]$$

$p_0, \dots, p_n$

$$[p_1 - p_1, \dots] \leq \operatorname{Dim} \ell(p_0, \dots, p_n)$$

$$q = \sum_{l=0}^{m} \lambda_l p_l \in Q(p_0, \ldots, p_n)$$

$$= \underline{p_0} + \sum_{l=0}^{m} \lambda_l p_l - \left(\sum_{l=0}^{m} \lambda_l\right) \underline{p_0}$$

$$= p_0 + \sum_{l=0}^{m} \lambda_l (p_l - p_0) =$$

$$= p_0 + \sum_{l=1}^{m} \lambda_l \underline{(p_l - p_0)} \in p_0 + [p_1 - p_0, \ldots]$$

$\Rightarrow$ $M \cap N$ ist AP

$\neq \emptyset$

$\mathrm{Dim}(M \cap N) \leq \mathrm{Dim}\,M \cap \mathrm{Dim}(N)$

$\Leftarrow$ $M \cap N \neq \emptyset$

$p, q \in M \cap N$

$\ell(p,q) \subseteq M$, $\ell(p,q) \subseteq N$

$\ell(p,q) \subseteq M \cap N$

$\mathrm{Div}(M \cap N) \subseteq \mathrm{Div}(M)$

$p \in M \cap N$

$S = \{q - p;\ q \in M \cap N\}$

$T = \{r - p:\ r \in M\}$

$S \subseteq T$

$\mathrm{Div}(M \cap N) \quad \mathrm{Div}(M)$

$u \in D_{ii}(M) \cap D_{ii}(N) \Rightarrow$

$u \in D_{ii}(M \cap N)$

$p^\perp$

$p \in M \cap N \subseteq M$

$\subseteq N$

$u = q_1 - p = q_2 - p$

$q_1 \in M \qquad q_2 \in N$

$q_1 = q_2 \in M \cap N \qquad u \in D_{ii}(M \cap N)$

$\mathrm{Dim}\, M + \mathrm{Dim}\, N = V$

$\forall v \in V \;\; \exists\, u \in \mathrm{Dim}\, M,\; w \in \mathrm{Dim}\, N$

$u + w = v$

$M = P + \mathrm{Dim}\, M$

$N = Q + \mathrm{Dim}\, N$

$Q - P \in V$

$\parallel$

$u + w$

$\in \mathrm{Dim}\, M \quad \in \mathrm{Dim}\, N$

$$q - p = u + w$$

$$q - w = p + u \in M \cap N$$

$$\underbrace{\in N \quad \in D \cap N}_{\in N} \qquad \underbrace{\in M \quad \in D \cap M}_{\in M}$$

$$M \sqcup N = Q(M \cup N)$$

$$M = P + \mathrm{Di}\, M \qquad \sum_{i=0}^{n} b_i = 1$$

$$N = Q + \mathrm{Di}\, N$$

$$M \cup N = M + ([Q - P] + \mathrm{Di}\, N)$$

$$x \in M \cup N = Q(M \cup N)$$

$$x = \sum_{i=0}^{k} b_i p_i + \sum_{i=k+1}^{n} b_i q_i$$

$$\in M \qquad \in N,$$

$$x = \sum_{i=0}^{k} l_i p_i + \sum_{i=k+1}^{m} l_i p - \sum_{i=k+1}^{m} l_i p$$

$$+ \sum_{i=k+1}^{m} l_i q_i + \sum_{i=0}^{m} l_i q - \sum_{i=0}^{m} l_i q$$

$\in M$

$$= \tilde{p} + \tilde{q} - \sum_{i=k+1}^{m} l_i p - \sum_{i=0}^{m} l_i q$$

$$= \tilde{p} + (\tilde{q} - q) + q - \Sigma - \Sigma$$

$$= \tilde{p} + (\tilde{q} - q) + \sum_{i=k+1}^{m} \lambda_i (q - p)$$

$\in M \qquad \in D_{in} N \qquad \in [q - p]$

$$M \cup N \subseteq M + ([q-p] + D_{in} N)$$

$$x = p + u + \alpha (q - p) + v$$

$\in D_i (M \cup N)$

$u \in D_{in} M \subseteq D_i (M \cup N)$

$v \in D_{in} N$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 2 & 0 \\ 2 & -3 & 1 & -2 & 6 & 0 \\ 3 & 1 & 0 & 4 & 2 & 1 \\ 4 & -1 & 0 & 3 & 5 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 2 & 0 \\ 0 & -3 & -1 & -2 & 2 & 0 \\ 0 & 1 & -3 & 4 & -4 & 1 \\ 0 & -1 & -4 & 3 & -3 & 1 \end{array}\right) \checkmark$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 2 & 0 \\ 0 & 1 & -3 & 4 & -4 & 1 \\ 0 & 0 & -10 & 10 & -10 & 3 \\ 0 & 0 & -7 & 7 & -7 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 2 & 0 \\ 0 & 1 & -3 & 4 & -4 & 1 \\ 0 & 0 & 1 & -1 & 1 & -\frac{3}{10} \\ 0 & 0 & -1 & 1 & -1 & \frac{2}{7} \\ 0 & 0 & 0 & 0 & 0 & \neq 0 \end{array}\right)$$

$$\dim(S+T) = 4 = \dim \mathbb{R}^4$$

$$M \cap N \neq \emptyset$$

$$\dim S = \dim T = 3$$

$$\dim(S+T) = 4$$

$$\dim(S \cap T) = 3+3-4 = \underline{\underline{2}}$$

$$\dim(M \sqcup N) = 4$$

$$\dim(M \cap N) = \underline{\underline{2}}$$

$\#$

$\emptyset$