$$c^T G c > 0 \quad \forall c \neq 0$$

$$\sum c_i u_i = 0$$ ma c' nelle ipotesi

$$\langle \underbrace{\sum c_i u_i}_{\neq 0}, \sum c_i u_i \rangle = 0$$

Ass.

$$0 \le \langle u - \lambda v, u - \lambda v \rangle = \cdots$$

$c \qquad b = -2\langle u, v\rangle \qquad a$

$$= \langle u, u\rangle - 2\lambda\langle u, v\rangle + \lambda^2 \langle v, v\rangle$$

$$D = b^2 - 4ac$$

$$D = 0$$

$$|\langle u, v\rangle| = \|u\|\,\|v\|$$

$$D \le 0$$

$$0 \ge 4\langle u, v\rangle^2 - 4\langle u, u\rangle\langle v, v\rangle$$

$$|\langle u, v\rangle| \le \|u\|\,\|v\|$$

$$\sqrt{\langle cx, cx\rangle} = \sqrt{c^2 \langle x, x\rangle}$$

$$\|cx\| = |c|\,\|x\|$$

$$\|x+y\|^2 \overset{?}{\le} \left(\|x\| + \|y\|\right)^2$$

$$\langle x+y, x+y\rangle \overset{?}{\le} \|x\|^2 + 2\|x\|\,\|y\| + \|y\|^2$$

$$\cancel{\langle x,x\rangle} + \cancel{2}\underline{\langle x,y\rangle} + \cancel{\langle y,y\rangle} \overset{?}{\le} \cancel{\|x\|^2} + \cancel{2}\underline{\|x\|\,\|y\|} + \cancel{\|y\|^2}$$

CBS

$$\|x \pm y\|^2 = \underbrace{\langle x, x\rangle}_{\|x\|^2} \pm 2\langle x, y\rangle + \underbrace{\langle y, y\rangle}_{\|y\|^2} =$$

$$= \|x\|^2 + \|y\|^2 \pm 2\,\|x\|\,\|y\| \cos \measuredangle(x, y)$$

$$x \perp y \iff \langle x, y\rangle = 0$$

$$\|x+y\|^2 = \langle x, x\rangle + 2\langle x, y\rangle + \langle y, y\rangle$$

$$+\ \|x-y\|^2 = \|x\|^2 - 2\langle x\ y\rangle + \|y\|^2$$

$$\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + X + 2\|y\|^2$$

$$\|x\| = \|y\| \qquad x+y \overset{?}{\perp} x-y$$

$$\langle x+y, x-y \rangle = \langle x, x\rangle + \cancel{\langle y, x\rangle} - \cancel{\langle x, y\rangle} - \langle y, y\rangle$$

$$= 0$$

$\forall i, \langle u, u_i \rangle = 0$

$\sum_i \langle u, \alpha_i u_i \rangle = 0$

$\langle u, \sum \alpha_i u_i \rangle = 0$

$u \perp u_i$?

$u \neq \sum_j \alpha_j u_j$

$\alpha_j = 0 \quad i \neq j$

$\alpha_j = 1$

$u_1 \ldots u_k$ OG / u_j

$\sum_{i=1}^{k} c_i u_i = 0$

$\sum_{i=1}^{k} c_i \langle u_i, u_j \rangle = 0 = \langle 0, u_j \rangle$

$c_j \langle u_j, u_j \rangle = 0$

> 0

$\Downarrow$

$c_j = 0$

u_1 $\qquad e_1 = u_1$ $\qquad e_1, e_2 \in \langle u_1, u_2 \rangle$

$e_2 = \mu_1 e_1 + u_2$ $\qquad / \qquad e_1$ $\quad \langle e_1, e_2 \rangle \subseteq \langle u_1, u_2 \rangle$

$u_1, u_2 \in \langle e_1, e_2 \rangle$

$0 = \mu_1 \langle e_1, e_1 \rangle + \langle u_2, e_1 \rangle$

$\langle u_1, u_2 \rangle \subseteq \langle e_1, e_2 \rangle$

$e_1 \neq 0 \Rightarrow \mu_1 = - \dfrac{\langle u_2, e_1 \rangle}{\langle e_1, e_1 \rangle}$

$$\left\langle \sum_i \lambda_i a_i, \sum_j c_j b_j \right\rangle \qquad a_i \in A, \quad b_j \in B$$

$$= \sum_{i,j} \lambda_i c_j \underbrace{\langle a_i, b_j \rangle}_{=0} = 0$$

$x \perp R$

$y \perp R \quad \Rightarrow \quad x+R \perp R$

$cx \perp R$

$W \subseteq V$

$u = x + x'$, $x \in W$, $x' \in W^{\perp}$!

$W \oplus W^{\perp} = V$

$\langle e_1, \ldots, e_k \rangle = W$ $e_1, \ldots, e_k, u_{k+1}, \ldots, u_n$

OG

$W \cap W^{\perp} = \{0\}$ $e_1, \ldots, e_k, e_{k+1}, \ldots, e_n$

$e_{k+1}, \ldots, e_n \in W^{\perp}$