$$F(v_1, \dots, v_i, \dots, v_j, \dots, v_n) = 0$$

$$F(v_i, v_i) \stackrel{?}{=} 0$$

$$F(-v_i, v_i) = -F(v_i, v_i)$$

$$F(v_i, v_i) = -F(v_i, v_i)$$

$$\boxed{2}\, F(v_i, v_i) = 0$$

$\frac{1}{2}$

$\mathbb{K}$

σ = transpozice

$$F(u_1, \ldots, u_j, \ldots, u_i, \ldots, u_n) =$$

$$= (-1)^1 F(u_1, \ldots, u_i, \ldots, u_j, \ldots, u_n)$$

$$F(u_{\sigma 1}, \ldots, u_{\sigma n})$$

(2) $\exists$ i, j $\sigma(i) = \sigma(j)$
$i \neq j$

$$F(u_{\sigma(1)}, \ldots u_{\sigma(n)}) = 0$$

$$\det A^T = \det A = \sum_{\sigma \in S_n} (-1)^{|\sigma|} a_{\sigma(1)1} \dots a_{\sigma(n)n}$$

$$A^T = (b_{ij}) \quad , \quad b_{ij} = a_{ji}$$

$$\det A^T = \sum_{\beta \in S_n} (-1)^{|\beta|} \cdot b_{\beta(1)1} \dots b_{\beta(n)n}$$

$\beta(1) = j \qquad \beta^{-1}(j) = 1$

$$= \sum_{\beta \in S_n} (-1)^{|\beta|} \cdot a_{1\beta(1)} \dots a_{n\beta(n)} =$$

ij

$a_{\beta^{-1}(1)\,1} \dots a_{\beta^{-1}(n)\,n}$

$$= \sum_{\beta^{-1} \in S_n} (-1)^{|\beta^{-1}|} a_{\beta^{-1}(1)\,1} \cdots a_{\beta^{-1}(n)\,n}$$

$$= \sum_{\sigma \in S_n} (-1)^{|\sigma|} a_{\sigma(1)\,1} \cdots a_{\sigma(n)\,n}$$

$$= \det A$$

$$\begin{array}{c|cc} & 1 \dots m & m+1 \dots n \\ \hline \begin{array}{c}1\\ \vdots \\ m \\ m+1 \\ \vdots \\ n\end{array} & A = \begin{pmatrix} B & C \\ 0 & D \end{pmatrix} & \end{array}$$

$$(-1)^{|\sigma|} \cdot a_{\sigma(1),1} \dots a_{\sigma(n),n}$$

$$\text{if } \sigma\{1, \dots, m\} \Longrightarrow \underline{\underline{\sigma(j) > m}}$$

$$j \in \{1, \dots, m\} \Rightarrow \sigma(j) \in \{1, \dots, m\}$$

$$j \in \{m+1, \dots, n\} \Rightarrow \sigma(j) \in \{m+1, \dots, n\}$$

$$\sigma = (\sigma_1, \sigma_2) \qquad |\sigma| = |\sigma_1| + |\sigma_2|$$

$$\sigma_1 : \{1, \dots, m\} \to \{1, \dots, m\}$$

$$\sigma_2 : \{m+1, \dots, n\} \to \{m+1, \dots, n\}$$

$$\sum_{\sigma\in S_n} (-1)^{|\sigma|}\, a_{\sigma(1)1} \cdots a_{\sigma(m)m}\; a_{\sigma(m+1)\,m+1} \cdots a_{\sigma(n)\,n}$$

// def B

$$= \sum_{\sigma_1\in S_m} (-1)^{|\sigma_1|}\, a_{\sigma_1(1)1} \cdots a_{\sigma_1(m)m}$$

def D

$$\sum_{\text{"}\sigma_2\in S_{n-m}\text{"}} (-1)^{|\sigma_2|}\, a_{\sigma_2(1)1} \cdots a_{\sigma_2(n-m)\,n-m}$$

$$\det(A\cdot B) = \det A \cdot \det B$$

$$F(A) = \det(A\cdot B)$$

$$F: V^n \to K$$

$u, D_2(A), \ldots, D_n(A)$

$v, D_2(A), \ldots, D_n(A)$

$cu+dv, D_2(A), \ldots, D_n(A)$

$$\det((cu+dv, D_2(A), \ldots)\cdot B)$$

$$= \det(cuB + dvB, D_2(A)B, \ldots, D_n(A)\cdot B)$$

$$= c\det(uB, D_2(A), \ldots) + d\det(vB, D_2(A), \ldots)$$

$$= c\det A_u + d\det A_v$$

$$F(A) = \det(A \cdot B)$$

$$\| $$

$$F(I_n) \det A$$

$$\|$$

$$\det(I_n B) \cdot \det A = \det A \det B$$

$$A A^{-1} = I_n$$

$$\det(A A^{-1}) = \det(A) \cdot \det(A^{-1}) = \underbrace{\det(I_n)}_{} = 1$$

$$\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$$

A reg $\Rightarrow$ $\det A \neq 0$

$$\det A^{-1} = \frac{1}{\det A}$$

$A \in K^{m \times n}$ $\quad$ $i < j$

$D_i(A) = D_j(A)$ $\quad$ τ $\quad$ $i \leftrightarrow j$

$a_{kl} = a_{\tau(k)l}$

A_σ $\quad$ $\tau \circ \sigma$

m_{kl}' $\quad$ $k_i \; k_i'$

$$S_n = \bigcup_{i=1}^{n} S_n(i,j) \qquad (\tilde{a}_{1j} \ldots \tilde{a}_{nj}) \begin{pmatrix} a_{1j} \\ a_{nj} \end{pmatrix}$$

$$\tilde{a}_{ij} = \sum_{\sigma \in S_n(i,j)} (-1)^{|\sigma|} a_{\sigma(1)1} \ldots a_{\sigma(j-1)\,j-1}\, a_{\sigma(j+1)\,j+1} \ldots a_{\sigma(n)n}$$

$$\det A = \sum_{i=1}^{n} a_{ij}\tilde{a}_{ij} \qquad a_{ij} = \sum_{\sigma \in S_n} (-1)^{|\sigma|} \ldots a_{ij} \ldots$$

$A \qquad b_{ij}(A)$

$B \qquad b_{ij}(B) = e_{ij}$

$b_{i}(B) = b_{i}(A) \qquad i \neq j$

$(-1)^{i+j}$

$\det B = 0$

A_{ij}

$= C$

$\delta_{ij} = a_{ij}$

i

(-1)

$\delta_{ij} = \det B = \det C \cdot (-1)^{i+j}$

$$\begin{vmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & & \vdots \\ \vdots & & \vdots & & \\ 1 & x_n & x_n^2 & & x_n^{n-1} \end{vmatrix} = \begin{vmatrix} \boxed{1} & 0 & 0 & & \\ \vdots & x_2 - x_1 & x_2^2 - x_1 x_2 & \cdots & \\ & & & & \\ 1 & x_n - x_1 & x_n^2 - x_1 x_n & \cdots & \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x_2 & x_2^2 & \cdots & x_2^{n-2} \\ 1 & x_3 & x_3^2 & \cdots & \\ & & & & \\ 1 & x_n & x_n^2 & \cdots & 1 \end{vmatrix} (x_2 - x_1)(x_3 - x_1) \cdots (x_n - x_1)$$

$$VD_n(x_1, \ldots, x_n) = (x_2 - x_1) \ldots (x_n - x_{n-1}) \cdot$$

$$\cdot VD_{n-1}(x_2, \ldots, x_n) = \ldots$$

$$\ldots = V_2(x_{n-1}, x_n) \cdot \prod_{i<j} (x_j - x_i)$$

$$= \begin{vmatrix} 1 & x_{n-1} \\ 1 & x_n \end{vmatrix} \cdot \prod_{j \neq x_{n-1}} =$$

$$= (x_n - x_{n-1}) \cdot \prod = \prod_{i<r} (x_j - x_i)$$

$$A x = b \qquad A^{-1} = \frac{1}{|A|} (\tilde{A})^T$$

$$x = \frac{1}{|A|} \cdot \tilde{A}^T \cdot b$$

$$x_j = \frac{1}{|A|} \cdot e_j (\tilde{A}^T) \cdot b$$

$$\|$$

$$e_j (\tilde{A})^T \cdot b$$

$$\|$$

$$|A^b_j|$$