

# Interspecific Interactions

*“Populační ekologie živočichů“*

Stano Pekár



# Types of interactions

Effect of species 2 on  
fitness of species 1

Effect of species 1 on fitness of species 2

	Increase	Neutral	Decrease
Increase	+ +		
Neutral	0 +	0 0	
Decrease	+ -	- 0	- -

- + + .. **mutualism** (plants and pollinators)
- 0 + .. **commensalism** (saprophytism, parasitism, phoresis)
- + .. **predation** (herbivory, parasitism), **mimicry**
- 0 .. **amensalism** (allelopathy)
- - .. **competition**

# Niche measures

## ► Niche breadth

### **Levin's index ( $D$ ):**

- $p_k$  .. proportion of individuals in class  $k$
- does not include resource availability

$$D = \frac{1}{\sum_{k=1}^n p_k^2}$$

### **Smith's index ( $FT$ ):**

- $q_k$  .. proportion of available individuals in class  $k$
- $0 < D, FT < 1$

$$FT = \sum_{k=1}^n \sqrt{p_k q_k}$$

## ► Niche overlap

### **Pianka's index ( $a$ ):**

- does not account for resource availability

$$a = \frac{\sum p_{1k} p_{2k}}{\sqrt{\sum p_{1k} \sum p_{2k}}}$$

### **Lloyd's index ( $L$ ):**

- $0 < a < 1$
- $0 < L < \infty$

$$L = \sum \frac{p_{1k} p_{2k}}{q_k}$$

# Model of competition

- ▶ based on the logistic differential model

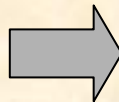
$$\frac{dN}{dt} = Nr \left( 1 - \frac{N}{K} \right)$$

- ▶ assumptions:

- all parameters are constant
- individuals of the same species are identical
- environment is homogenous, differentiation of niches is not possible
- only exact compensation is present

- ▶ model of Lotka (1925) and Volterra (1926)

**species 1:**  $N_1, K_1, r_1$



$$\frac{dN_1}{dt} = N_1 r_1 \left( 1 - \frac{N_1 + N_2}{K_1} \right)$$

**species 2:**  $N_2, K_2, r_2$

$$\frac{dN_2}{dt} = N_2 r_2 \left( 1 - \frac{N_1 + N_2}{K_2} \right)$$

- ▶ total competitive effect (intra + inter-specific)

$(N_1 + \alpha N_2)$  where  $\alpha$  .. coefficient of competition

$\alpha = 0$  .. no interspecific competition

$\alpha < 1$  .. species 2 has lower effect on species 1 than species 1 on itself

$\alpha = 0.5$  .. one individual of species 1 is equivalent to 0.5 individuals of species 2)

$\alpha = 1$  .. both species has equal effect on the other one

$\alpha > 1$  .. species 2 has greater effect on species 1 than species 1 on itself

species 1:

$$\frac{dN_1}{dt} = N_1 r_1 \left( 1 - \frac{N_1 + \alpha_{12} N_2}{K_1} \right)$$

species 2:

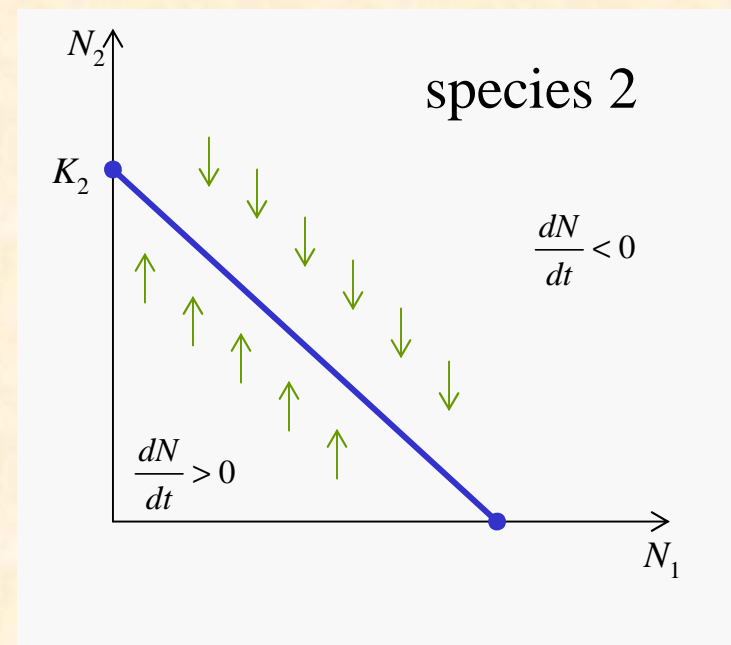
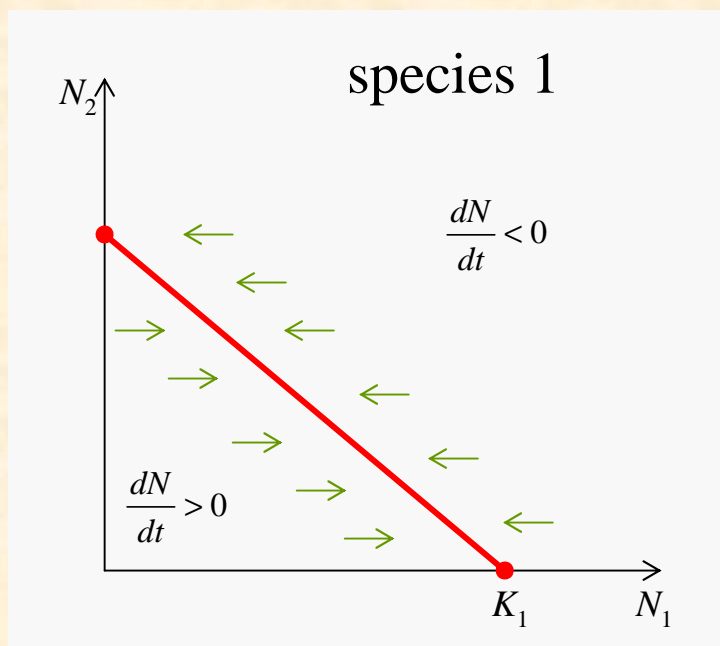
$$\frac{dN_2}{dt} = N_2 r_2 \left( 1 - \frac{\alpha_{21} N_1 + N_2}{K_2} \right)$$

- ▶ if competing species use the same resource then interspecific competition is equal to intraspecific

# Analysis of the model

- ▶ examination of the model behaviour on a phase plane
- ▶ used to describe change in any two variables in coupled differential equations by projecting orthogonal vectors
- ▶ identification of isoclines: a set of abundances for which the growth rate of at least one population is 0:

$$\frac{dN}{dt} = 0$$





# Isoclines

## ▶ species 1

$$r_1 N_1 (1 - [N_1 + \alpha_{12} N_2] / K_1) = 0$$

$$r_1 N_1 ([K_1 - N_1 - \alpha_{12} N_2] / K_1) = 0$$

$$\text{if } r_1, N_1, K_1 = 0$$

$$\text{and if } K_1 - N_1 - \alpha_{12} N_2 = 0$$

$$\text{then } N_1 = K_1 - \alpha_{12} N_2$$

$$\text{if } N_1 = 0 \text{ then } N_2 = K_1 / \alpha_{12}$$

$$\text{if } N_2 = 0 \text{ then } N_1 = K_1$$

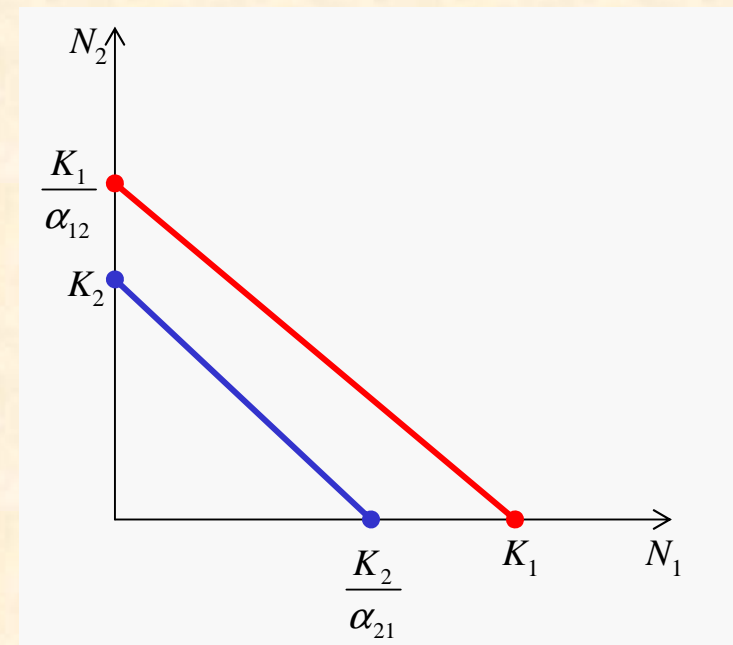
## ▶ species 2

$$r_2 N_2 (1 - [N_2 + \alpha_{21} N_1] / K_2) = 0$$

$$N_2 = K_2 - \alpha_{21} N_1$$

$$\text{if } N_2 = 0 \text{ then } N_1 = K_2 / \alpha_{21}$$

$$\text{if } N_1 = 0 \text{ then } N_2 = K_2$$



▶ above isocline  $i_1$  and below  $i_2$  competition is weak

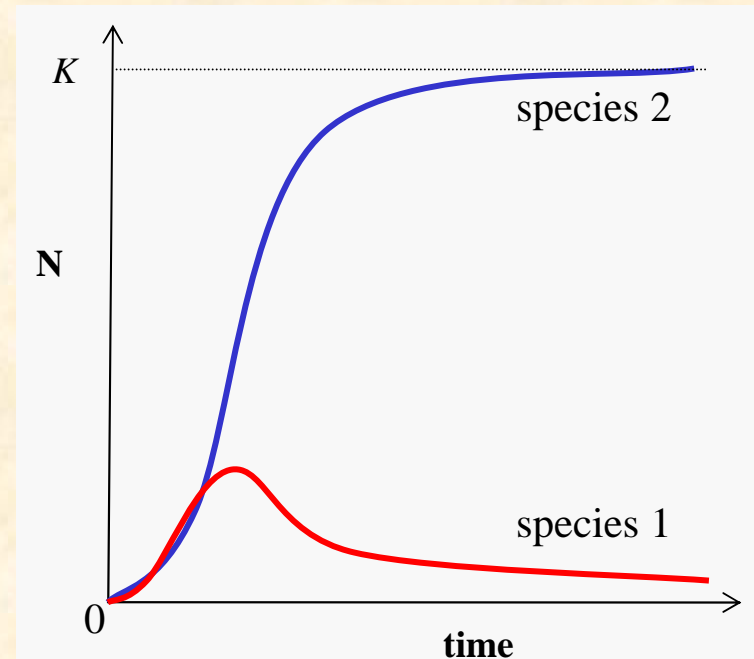
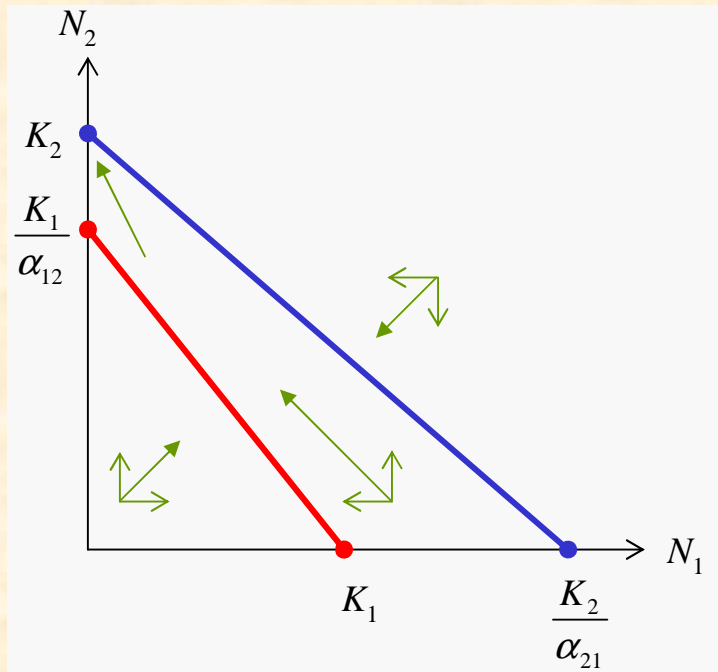
▶ in-between  $i_1$  and  $i_2$  competition is strong

# 1. Species 2 drives species 1 to extinction

- ▶  $K$  and  $\alpha$  determine the model behaviour
- ▶ disregarding initial densities species 2 (stronger competitor) will outcompete species 1 (weaker competitor)
- ▶ equilibrium  $(0, K_2)$

$$K_2 > \frac{K_1}{\alpha_{12}} \quad K_1 < \frac{K_2}{\alpha_{21}}$$

$$K_1 = K_2 \quad r_1 = r_2$$
$$\alpha_{12} > \alpha_{21} \quad N_{01} = N_{02}$$



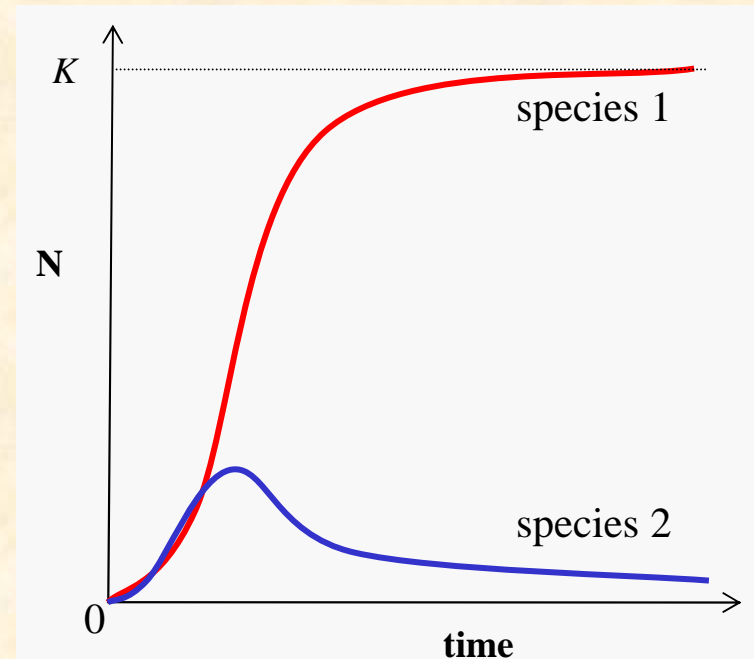
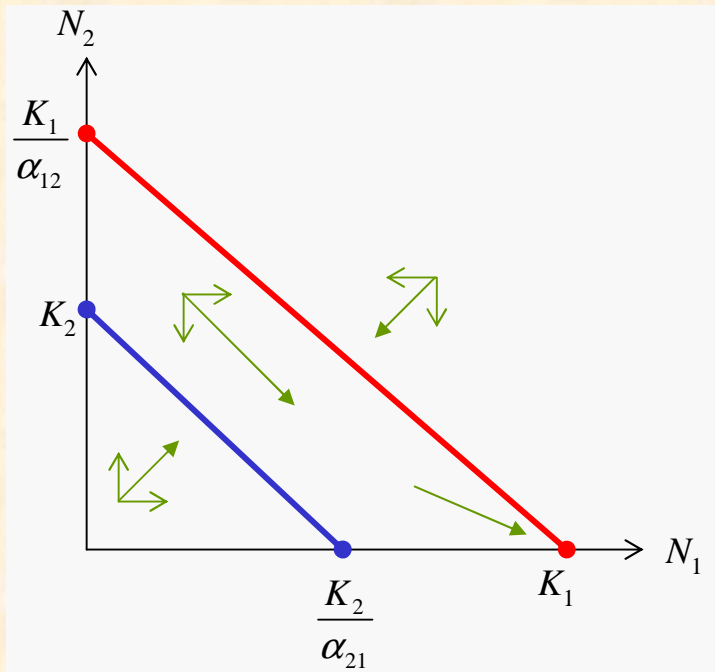


## 2. Species 1 drives species 2 to extinction

- ▶ species 1 (stronger competitor) will outcompete species 2 (weaker competitor)
- ▶ equilibrium  $(K_1, 0)$

$$K_1 > \frac{K_2}{\alpha_{21}} \quad K_2 < \frac{K_1}{\alpha_{12}}$$

$$r_1 = r_2 \quad K_1 = K_2$$
$$N_{01} = N_{02} \quad \alpha_{12} < \alpha_{21}$$



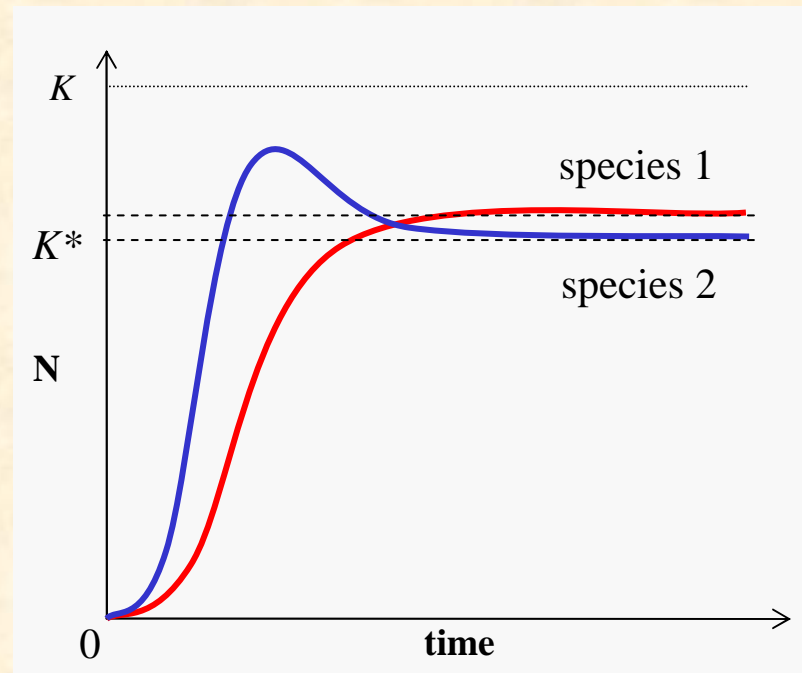
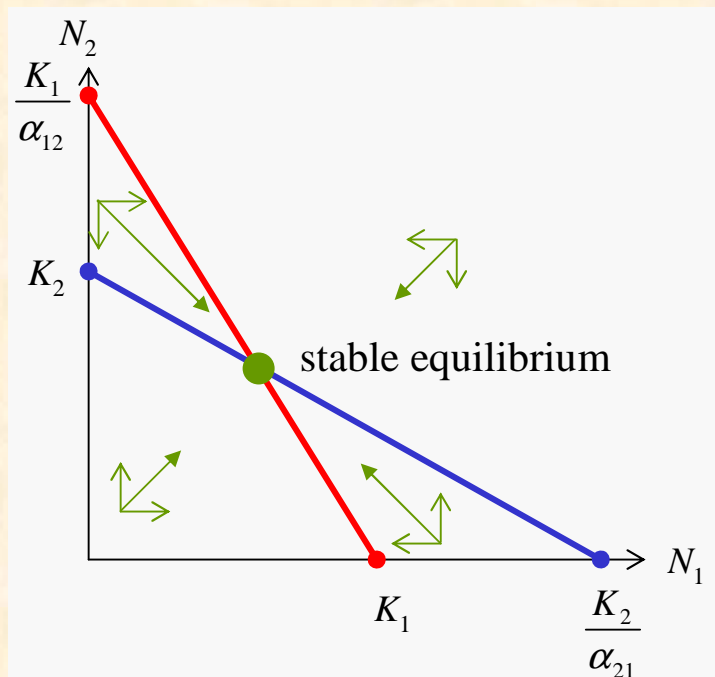
### 3. Stable coexistence of species

- ▶ disregarding initial densities both species will coexist at stable equilibrium (where isoclines cross)
- ▶ at equilibrium population density of both species is reduced
- ▶ both species are weak competitors
- ▶ equilibrium  $(K_1^*, K_2^*)$

$$K_1 < \frac{K_2}{\alpha_{21}} \quad K_2 < \frac{K_1}{\alpha_{12}}$$

$$r_1 < r_2 \quad K_1 = K_2$$

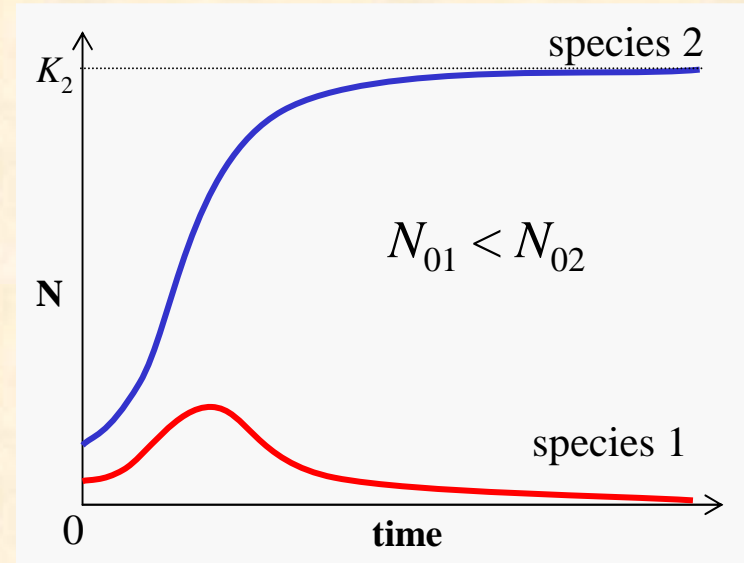
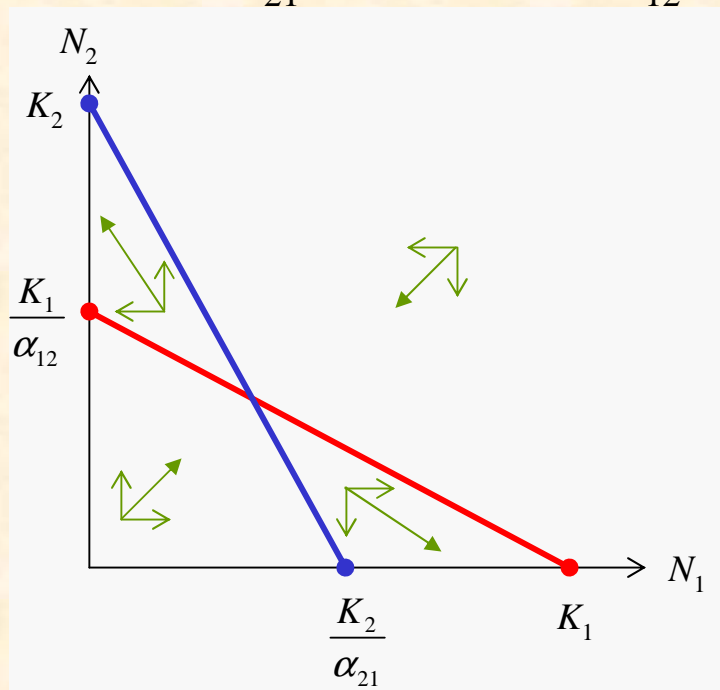
$$N_{01} = N_{02} \quad \alpha_{12}, \alpha_{21} < 1$$



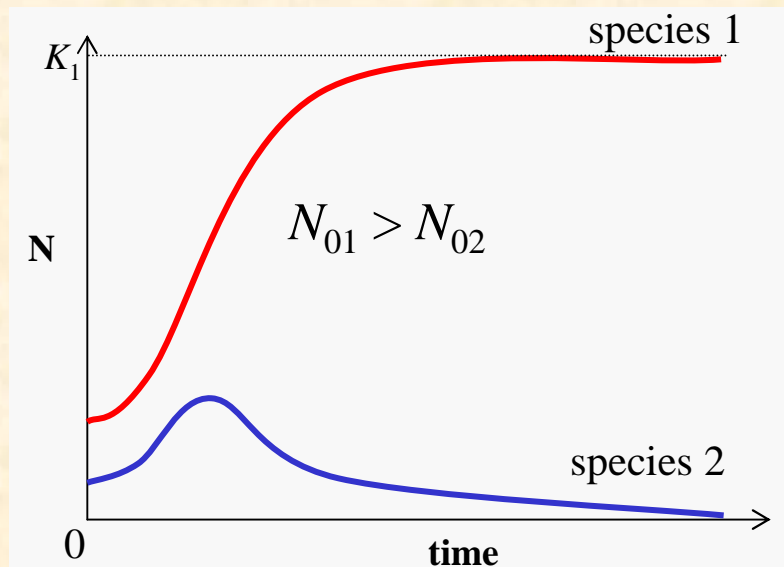
# 4. Competitive exclusion

- ▶ one species will drive other to extinction depending on the initial conditions
- ▶ coexistence only for a short time
- ▶ both species are strong competitors
- ▶ equilibrium  $(K_1, 0)$  or  $(0, K_2)$

$$K_1 > \frac{K_2}{\alpha_{21}} \quad K_2 > \frac{K_1}{\alpha_{12}}$$



$$r_1 = r_2 \quad K_1 = K_2 \quad \alpha_{12}, \alpha_{21} > 1$$



## Stability analysis of a system of differential equations

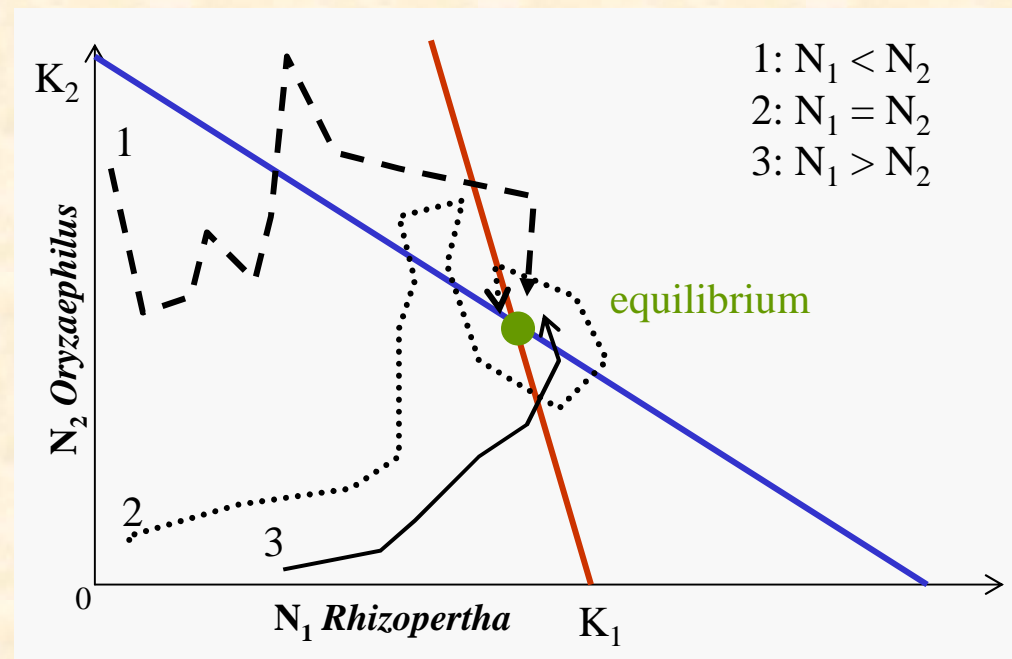
- ▶ Jacobi matrix of partial derivations

$$\mathbf{J} = \begin{pmatrix} \frac{\partial dN_1/dt}{\partial N_1} & \frac{\partial dN_1/dt}{\partial N_2} \\ \frac{\partial dN_2/dt}{\partial N_1} & \frac{\partial dN_2/dt}{\partial N_2} \end{pmatrix}$$

- ▶ evaluation of the derivations for densities close to equilibrium
- ▶ eigenvalue of the matrix
  - if all real parts of eigenvalues  $< 0$  .. locally **stable**
  - if at least one real part of an eigenvalue  $> 0$  .. **unstable**
- ▶ Lotka-Volterra system is stable for  $\alpha_{12}\alpha_{21} < 1$

# Test of the model

- ▶ when *Rhizopertha* and *Oryzaephilus* were reared separately both species increased to 420-450 individuals (=  $K$ )
- ▶ when reared together *Rhizopertha* reached  $K_1 = 360$ , while *Oryzaephilus*  $K_2 = 150$  individuals
- ▶ combination resulted in more efficient conversion of grain ( $K_{12} = 510$  individuals)
- ▶ three combinations of densities converged to the same stable equilibrium
- ▶ prediction of Lotka-Volterra model is correct



# Model for discrete generations

- ▶ solution of the differential model – Ricker's model:

$$N_{1,t+1} = N_{1,t} e^{r_1 \left( \frac{K_1 - N_{1,t} - \alpha_{12} N_{2,t}}{K_1} \right)} \quad N_{2,t+1} = N_{2,t} e^{r_2 \left( \frac{K_2 - N_{2,t} - \alpha_{21} N_{1,t}}{K_2} \right)}$$

- ▶ dynamic (multiple) regression is used to estimate parameters from series of abundances

$$\ln \left( \frac{N_{1,t+1}}{N_{1,t}} \right) = r_1 - N_{1,t} \frac{r_1}{K_1} - N_{2,t} \frac{r_1 \alpha_{12}}{K_1}$$

$$\ln \left( \frac{N_{2,t+1}}{N_{2,t}} \right) = r_2 - N_{1,t} \frac{r_2}{K_2} - N_{1,t} \frac{r_2 \alpha_{21}}{K_2}$$

$$r = a \quad \alpha = \frac{Kc}{r} \quad K = \frac{r}{b}$$