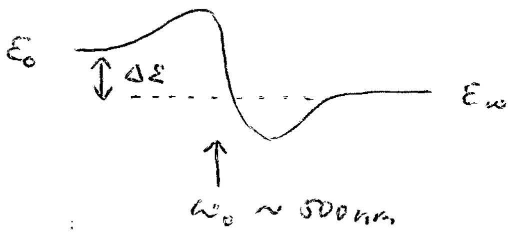
Cvičení

① - safírový krystal dopovaný Ti silně absorbuje okolo 500nm

- koncentrace absorb. atomů $n_{Ti} = 10^{25}\,m^{-3}$, nedopovaný safír má index lomu 1.77

určit rozdíl indexu lomu nad a pod absorpčním pásem

$n_\infty = 1.77$ $\qquad \varepsilon_\infty = n_\infty^2$ $\qquad \varepsilon(\omega) = \varepsilon_\infty + \frac{N}{V}\frac{e^2}{m\varepsilon_0}\frac{1}{\omega_0^2-\omega^2-i\gamma\omega}$



$$\Delta\varepsilon = \frac{N}{V}\frac{e^2}{m\varepsilon_0}\frac{1}{\omega_0^2}$$

$$\omega_0 = \frac{2\pi c}{\lambda} \doteq 3.77\cdot 10^{15}\,s^{-1}$$

$m_e = 9.11\cdot 10^{-31}\,kg$

$e = 1.602\cdot 10^{-19}\,C$

$\varepsilon_0 = 8.854\cdot 10^{-12}\,F\,m^{-1}$

odtud

$\Delta\varepsilon = 0.00224$

$n_0 = \sqrt{n_\infty^2 + \Delta\varepsilon} = 1.77063 \quad\rightarrow\quad \Delta n = 6.3\cdot 10^{-4}$

② HCl - odečteno $\quad \Delta E_{rot} = \frac{300\,cm^{-1}}{14} = 21.4\,cm^{-1} \doteq 2.66\,meV$

$\hbar\omega_0 = 2884\,cm^{-1}$

hmotnosti $\quad m_H = 1.008\,m_u$, $\quad m_{Cl} = 35.458\,m_u \qquad m_u = 1.66\cdot 10^{-27}\,kg$

redukovaná hmotnost $\quad \mu = \frac{m_H m_{Cl}}{m_H + m_{Cl}} = 0.980\,m_u$

- určit tuhost vazby a její délku

tuhost $\quad \omega_0 = 5.432\cdot 10^{14}\,s^{-1} \qquad \omega_0 = \sqrt{\frac{k}{\mu}} \rightarrow K = \mu\omega_0^2$

$K \doteq 480\,N/m$

délka vazby $\quad \Delta E_{rot} = \frac{\hbar^2}{I} = \frac{\hbar^2}{\mu r_0^2}$

$$r_0 = \frac{\hbar}{\sqrt{\mu\Delta E_{rot}}} \doteq 1.27\,\mathring{A} = 0.127\,nm$$

③

$$\mathrm{Re}\,\varepsilon(\omega) = 1 + \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\mathrm{Im}\,\varepsilon(\omega')}{\omega' - \omega}\, d\omega'$$

$$\mathrm{Im}\,\varepsilon(\omega) = \begin{cases} A & \omega_1 < \omega < \omega_2 \\ 0 & \text{jinak} \end{cases} \qquad \omega > 0$$

$$\mathrm{Im}\,\varepsilon(-\omega) = -\mathrm{Im}\,\varepsilon(\omega)$$

$$\mathrm{Re}\,\varepsilon(\omega) = 1 + \frac{1}{\pi} P \int_{0}^{\infty} \mathrm{Im}\,\varepsilon(\omega') \left(\frac{1}{\omega' - \omega} + \frac{1}{\omega' + \omega} \right) d\omega'$$

$$= 1 + \frac{1}{\pi} P \int_{\omega_1}^{\omega_2} \frac{A}{\omega' - \omega} + \frac{A}{\omega' + \omega}\, d\omega' =$$

$$= 1 + \frac{A}{\pi} \left(\ln\left| \frac{\omega_2 - \omega}{\omega_1 - \omega} \right| + \ln\left| \frac{\omega_2 + \omega}{\omega_1 + \omega} \right| \right) = 1 + \frac{A}{\pi} \ln\left| \frac{\omega_2^2 - \omega^2}{\omega_1^2 - \omega^2} \right|$$