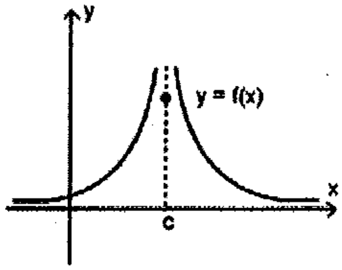
Continuous functions

http://www.brightstorm.com/math/calculus/limits-and-continuity/continuous-functions



1) **What kind of a function is a continuous function?**
2) **Can you give an example of a continuous function?**
3) **What is a domain of a function?**

Listening. Listen to the video and fill in the missing information.

1) A function is continuous at x=a if ....
2) If a function is continuous at .... in its ...., it is a ....function.
3) f(x)=x3 is a ...., therefore it is ....
4) g(x)=x-2 is continuous for ....
5) The third example of an exponential function the presenter gives is ....
6) Logarithmic functions are defined only for ....
7) Natural log is ....

Reading Continuous functions Qs.

1) What does the writer say about the behaviour of a reasonable function?
2) What do the graphs of continuous function look like?
3) What does the third definition say?
4) Why is 1/x a continuous function?
5) How are the functions whose domains are closed intervals defined?
6) Can you name some continuous function in trigonometry?
7) Why is the function x3/sin x continuous?
8) What is the property of the composition of continuous functions?

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UNIT 3

FOCUS A

CONTINUOUS FUNCTIONS

Imagine that you had the information shown in the table about some function *f*. What would you expect the output *f (1)* to be?

x	0.9	0.99	0.999
$f(x)$	2.93	2.9954	2.9999997

It would be quite a shock to be told that $f(1)$ is , say, 625. A reasonable function should present no such surprise. The expectation is that $f(1)=3$. More generally, we expect the output of a function at the input a to be closely connected with the outputs of the function at inputs that are near a. The functions of interest in calculus usually behave in the expected way; they offer no spectacular gaps or jumps. The graphs of these functions consist of curves or lines, not wildly scattered points. The technical term for these functions is "continuous", which will be defined in this section.

The following three definitions express in terms of limits our expectation that $f(a)$ is determined by values of $f(x)$ for x near a.

Definition *Continuity from the right at a number a.* Assume that $f(x)$ is defined at a and in some open interval (a, b). Then the function f is <u>continuous at a from the right</u> if $\lim_{x\to a+} f(x)=f(a)$. This means that

1 $\lim\limits_{x\to a+} f(x)$ exists and

2 that limit is $f(a)$.

Definition *Continuity from the left at a number a.* Assume that $f(x)$ is defined at a and in some open interval (c, a). Then the function f is <u>continuous at a from the left</u> if $\lim_{x\to a-} f(x)=f(a)$. This means that

1 $\lim\limits_{x\to a-} f(x)$ exists and

2 that limit is $f(a)$.

The next definition applies if the function is defined in some open interval that includes the number a. It essentially combines the first two definitions.

Definition *Continuity at a number a.* Assume that $f(x)$ is defined in some open interval (b,c) that contains the number a. Then the function f is <u>continuous at a</u> if $\lim_{x\to a} f(x)=f(a)$. This means that

1 $\lim\limits_{x\to a} f(x)$ exists and

2 that limit is $f(a)$.

This third definition amounts to asking that the function be continuous both from the right and from the left at a.

The following definitions define the notion of "continuous function"; they depend on the type of domain of the function.

Definition *Continuous function.* Let f be a function whose domain is the x axis or is made up of open intervals. Then f is a <u>continuous function</u> if it is continuous at each number a in its domain.

Thus x^2 is a continuous function. So is $1/x$, whose domain consists of the intervals $(-\infty, 0)$ and $(0, \infty)$. Although this function explodes at 0, this does not prevent it from being a continuous function. *The key to being continuous is that the function is continuous at each number in its domain.* The number 0 is not in the domain of $1/x$.

Only a slight modification of the definition is necessary to cover functions whose domains involve closed intervals. We will say that a function whose domain is the closed interval $[a, b]$ is *continuous* if it is continuous at each point in the open interval (a, b), continuous from the right at a, and continuous from the left at b. Thus $\sqrt{1-x^2}$ *is* continuous on the interval $[-1, 1]$.

In a similar spirit, we say that a function with domain $[a,\infty)$ is continuous if it is continuous at each point in (a, ∞) and continuous from the right at a. Thus x is a continuous function. A similar definition covers functions whose domains are of the form $(-\infty, b]$.

Many of the functions met in algebra and trigonometry are continuous. For instance, 2^x, x, $\sin x$, $\tan x$ and any polynomial are continuous. So is any rational function (the quotient of two polynomials). Moreover, algebraic combinations of continuous functions are continuous. For example, since x^3 and $\sin x$ are continuous, so are $x^3+\sin x$, $x^3-\sin x$, and $x^3 \sin x$. The function $x^3/\sin x$, which is not defined when $\sin x=0$, is continuous on its domain. The following definitions are needed to make these statements general.

Definition *Sum, difference, product, and quotient of functions.* Let f and g be two functions. The functions, $f+g$, $f-g$, fg, and f/g are defined as follows.

$(f+g)(x)=f(x)+g(x)$ for x in the domains of both f and g.
$(f-g)(x)=f(x)-g(x)$ for x in the domains of both f and g.
$(fg)(x)=f(x)\,g(x)$ for x in the domains of both f and g.
$(f/g)(x)=f(x)/g(x)$ for x in the domains of both f and g, $g(x)\neq 0$.

If f and g are defined at least in an open interval that includes the number a and if f and g are continuous at a, then so are $f+g$, $f-g$, and fg. Moreover, if $g(a)\neq 0$, f/g is also continuous at a.

A function obtained by the composition of continuous functions is also continuous. That is, if the function g is continuous at a and the function f is continuous at $g(a)$, then the composition, $f\circ g$, is continuous at a. For instance, the function $\sqrt[3]{1+x^2}$ is continuous since both the polynomial $1+x^2$ and the cube root function are continuous.

Notice:
- *subjunctive* (čes. konjunktiv) in a sentence: "*This third definition amounts to asking that the function **be** continuous both from the right and from the left at a.*" (having the meaning: "... the function ***should be*** continuous ...").

Subjunctive is used in formal style; usually expresses wish, request, necessity, possibility, likelihood, condition, will etc.

- The present subjunctive has the same form as the infinitive (the present subjunctive of ***to be*** is ***be*** for all persons, and the present subjunctive for all other verbs is the same as their present tense – "*s*" is not added for the third person singular). *The king lives here.* (simple present tense)
  Long live the king! (subjunctive)
- The past subjunctive has the same form as the simple past tense in all verbs except ***to be***, whose past subjunctive is ***were*** for all persons. The past subjunctive is used in conditional sentences and after certain structures (if/ if only, as if/ though, wish, it's time, etc.).
  I wish I were at home.

- *inversion* of the subject and the verb : "... x^2 *is a continuous function.* ***So is*** $1/x$ ..." (see the explanation of inversion in Unit 6)
- *since* – typical of formal style; meaning *because, as* (čes. protože)
- *also* – used in a formal style; its position in a sentences is the same as that of frequency adverbs (after the simple tenses of *to be* but before the simple tenses of all other verbs; with tenses consisting of more than one verb, they are placed after the first auxiliary). *Too,* and *as well* have the same meaning, but they are placed at the end of a sentence.

Exercises

1. What kinds of infinitives can you find in the text above?

2. Replace the group of words in italics with an infinitive or an infinitive construction:

a) He got to the top *and was very disappointed when he found* that someone else had reached it first.
b) There are a lot of sheets *that need mending.*
c) I was surprised *when I heard* that he had left the faculty.
d) It is necessary *that everyone should know* the truth.
e) There was no place *where we could sit.*
f) *It is expected that he will broadcast* a statement tonight. (He is expected ...)
g) *It* is likely *that he will arrive* before six. (He is ...)
h) *It is said that* he was sitting there all day. (He is said ...)
i) *They believe that he is* one of the best Polish mathematicians. (He ...)
j) He was the only one *who understood* the question.

3. Insert "to" where necessary before infinitives in brackets:

a) He made me (do) it all over again.
b) I used (live) in London.
c) You needn't (say) anything. Just nod your head and they will (understand).
d) I want (see) the college where you live.
e) May I (use) your phone?
f) He is expected (arrive) in the afternoon.
g) Please let me (know) your decision soon.
h) Need I (come)? I' d much rather (stay) at home.
i) He was made (sign) a paper admitting his guilt.
j) I heard the door (open) and saw a shadow (move) across the floor.

4. Use the perfect infinitive of the verb in italics with the appropriate auxiliary verb:

a) I realized that our department was on fire. – That (*be*) a terrible moment.
b) I saw Einstein last night. – You (*not see*) Einstein; he died many years ago. You (*dream*) it.
c) I've had a headache for two days. – You (*go*) to the doctor when it started.
d) As I was standing in the hall I saw Prof. Brown. – It (*not be*) Prof. Brown; he had left for Spain. It (*be*) his brother Joseph.
e) I've brought my own sandwiches. – You (*not bring*) them. I have enough for two.
f) The president (*unveil*) the statue, but he is ill so his wife is doing it instead.
g) He (*not catch*) the 7.45 bus because he didn't leave home till 7.35.
h) He said that censorship of news was absurd and it (*abolish*) years ago. (passive voice)
i) People used to work much harder. – They (*have*) a lot of energy in those days.
j) We (*set*) out today, but the weather is so bad that we decided to postpone our start till tomorrow.

5. Read the following passage and then write the sentences using an appropriate infinitive form** (to do, to have done, to be doing, to have been doing)**:

> *SUNKEN TREASURE*
> *Experts from the British Museum have announced the discovery of a ship which sank in a storm off the Scottish coast over 400 years ago. Divers have found gold bars on the sea bed near the wreck, which the experts believe are only a small part of the ship's precious cargo. According to the British Museum, the ship is in good condition and the cargo is worth millions of pounds.*

a) Divers are reported (find) a Spanish ship.
b) Experts are reported (study) objects from it.
c) The ship is thought (sink) 400 years ago.
d) The ship is thought (return) to Spain.
e) The ship is believed (carry) gold bars.
f) The ship is believed (lie) on the sea bed.
g) The ship is said (be) in good condition.
h) The gold is said (be) worth millions of pounds.

6. Put in the right kind of infinitive:

a) I ought (work) right now.
b) Your watch will (repair) by Tuesday.
c) I'd like (go) home early today.
d) I'd like (see) her face when she opened the letter.
e) She must (have) a shower – I can hear the water running.
f) It's important (listen) to people.
g) She hopes (choose) for the national team.
h) Try (not be) back late.
i) You should (tell) me you were ill.
j) He doesn't like (interrupt) while he's working.

7. Read out the following:

a) $a - c = m - y$
b) $4 a^2b + 8 ab^2 = 3c$
c) $1 + 2x = y^3 + q^3 = 1$
d) $(a + c) d = ca$
e) $mh^2 = 3$