M7116 Maticové populační modely

Populace strukturovaná do dvou stadií

10. 10. 2012

Fibonacciovi králíci

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t)$$

$\sigma_1 = 1, \sigma_2 = 1, \gamma = 1, \varphi = 1$

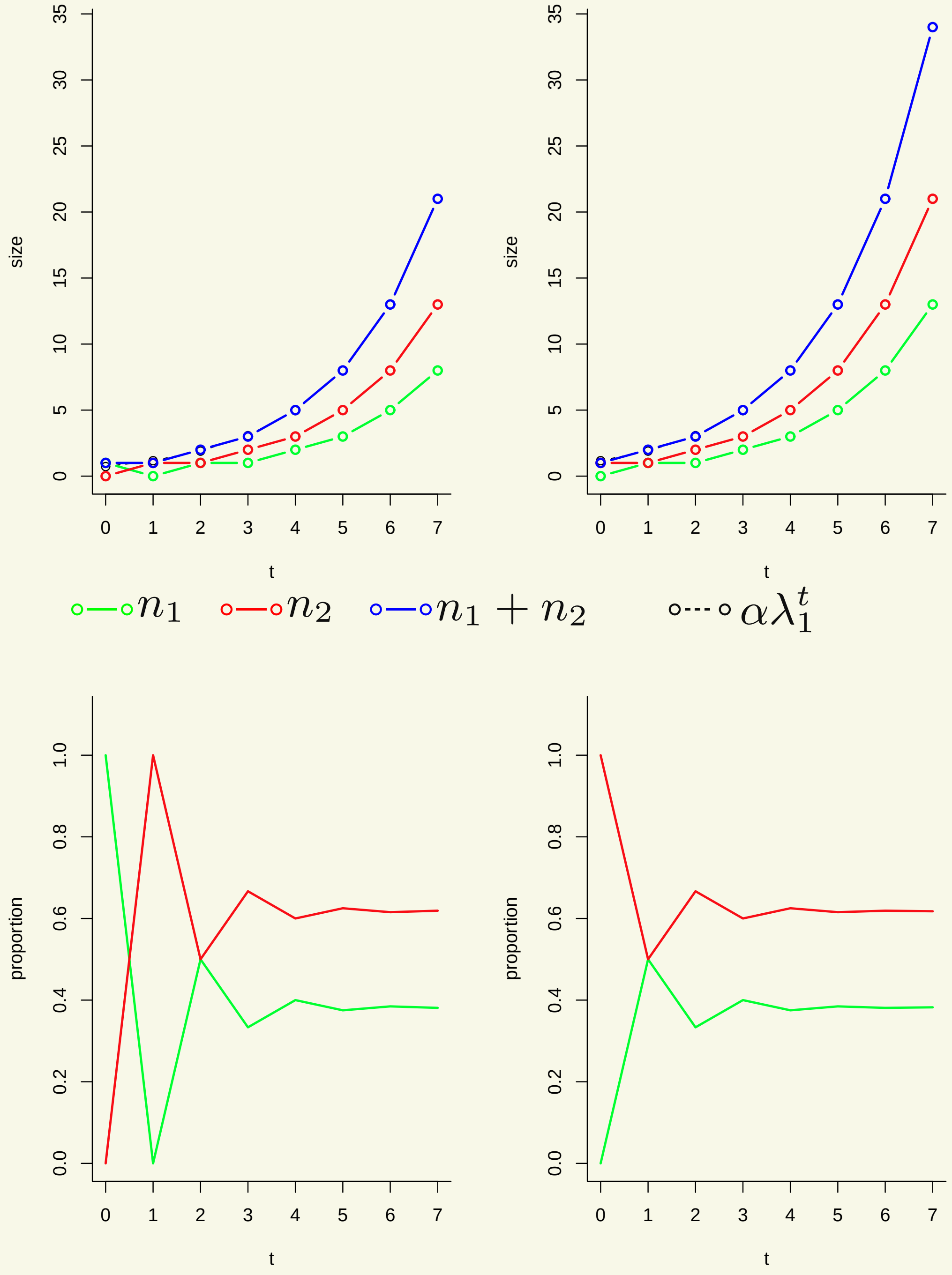
$$\mathsf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\lambda_1 = 1.61803, \qquad \boldsymbol{v}_1 = \begin{pmatrix} 0.38197 \\ 0.61803 \end{pmatrix}$$

$$\lambda_2 = -0.61803, \qquad \boldsymbol{v}_2 = \begin{pmatrix} 0.61803 \\ -0.38197 \end{pmatrix}$$

$$\mathsf{A}^2 = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\mathsf{A}^3 = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$



Rostoucí iteroparní populace

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t)$$

$\sigma_1 = \frac{8}{9}$, $\sigma_2 = \frac{2}{3}$, $\gamma = 1$, $\varphi = 1$

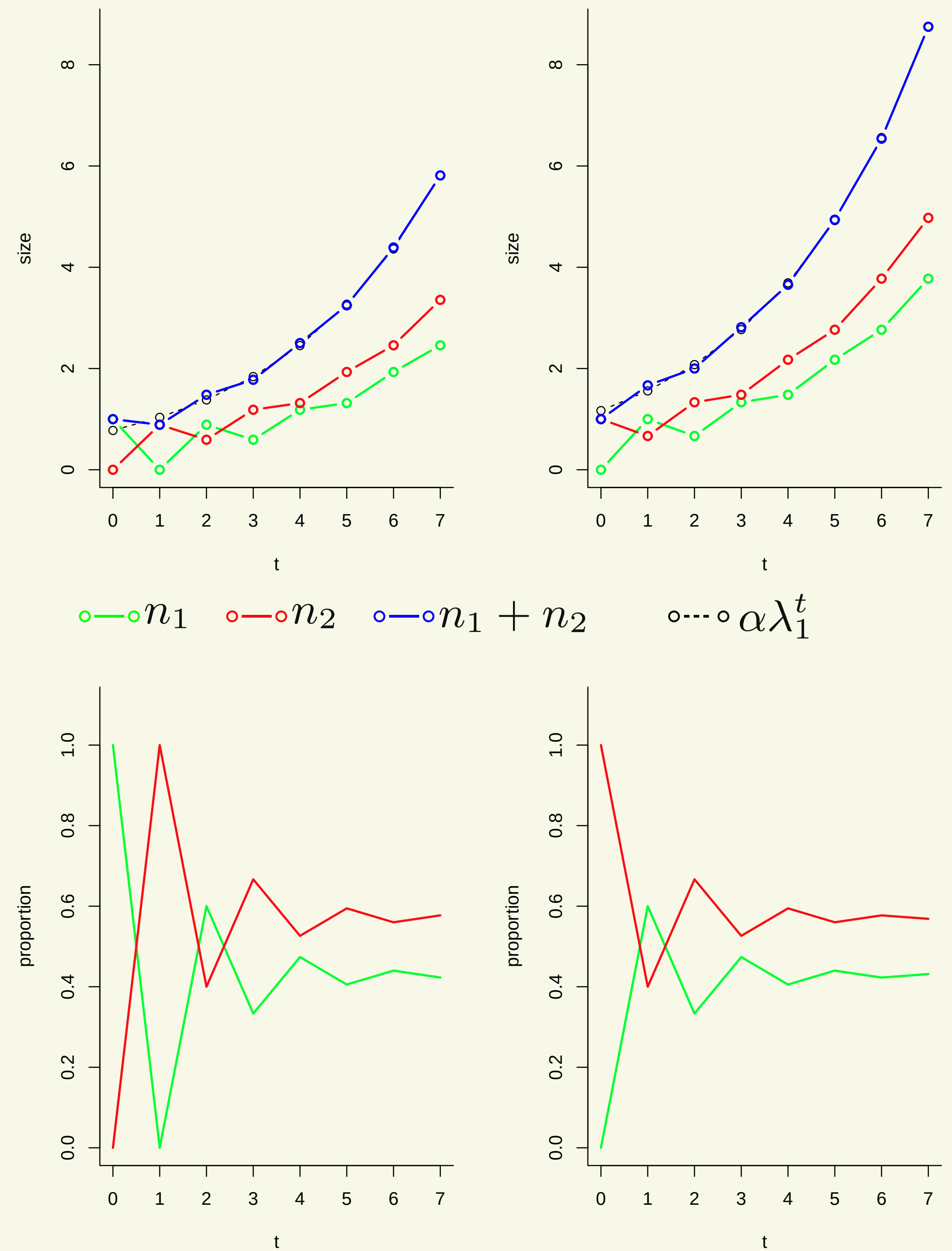
$$\mathsf{A} = \begin{pmatrix} 0 & 1 \\ \frac{8}{9} & \frac{2}{3} \end{pmatrix}$$

$$\lambda_1 = 1.33333, \qquad \boldsymbol{v}_1 = \begin{pmatrix} 0.42857 \\ 0.57143 \end{pmatrix}$$

$$\lambda_2 = -0.66667, \qquad \boldsymbol{v}_2 = \begin{pmatrix} 0.6 \\ -0.4 \end{pmatrix}$$

$$\mathsf{A}^2 = \begin{pmatrix} 0.88889 & 1.33333 \\ 0.59259 & 1.33333 \end{pmatrix}$$

$$\mathsf{A}^3 = \begin{pmatrix} 0.59259 & 1.33333 \\ 1.18519 & 1.48148 \end{pmatrix}$$



Stabilizovaná iteroparní populace (1)

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t)$$

$\sigma_1 = \frac{7}{9}, \sigma_2 = \frac{2}{3}, \gamma = \frac{1}{7}, \varphi = 1$

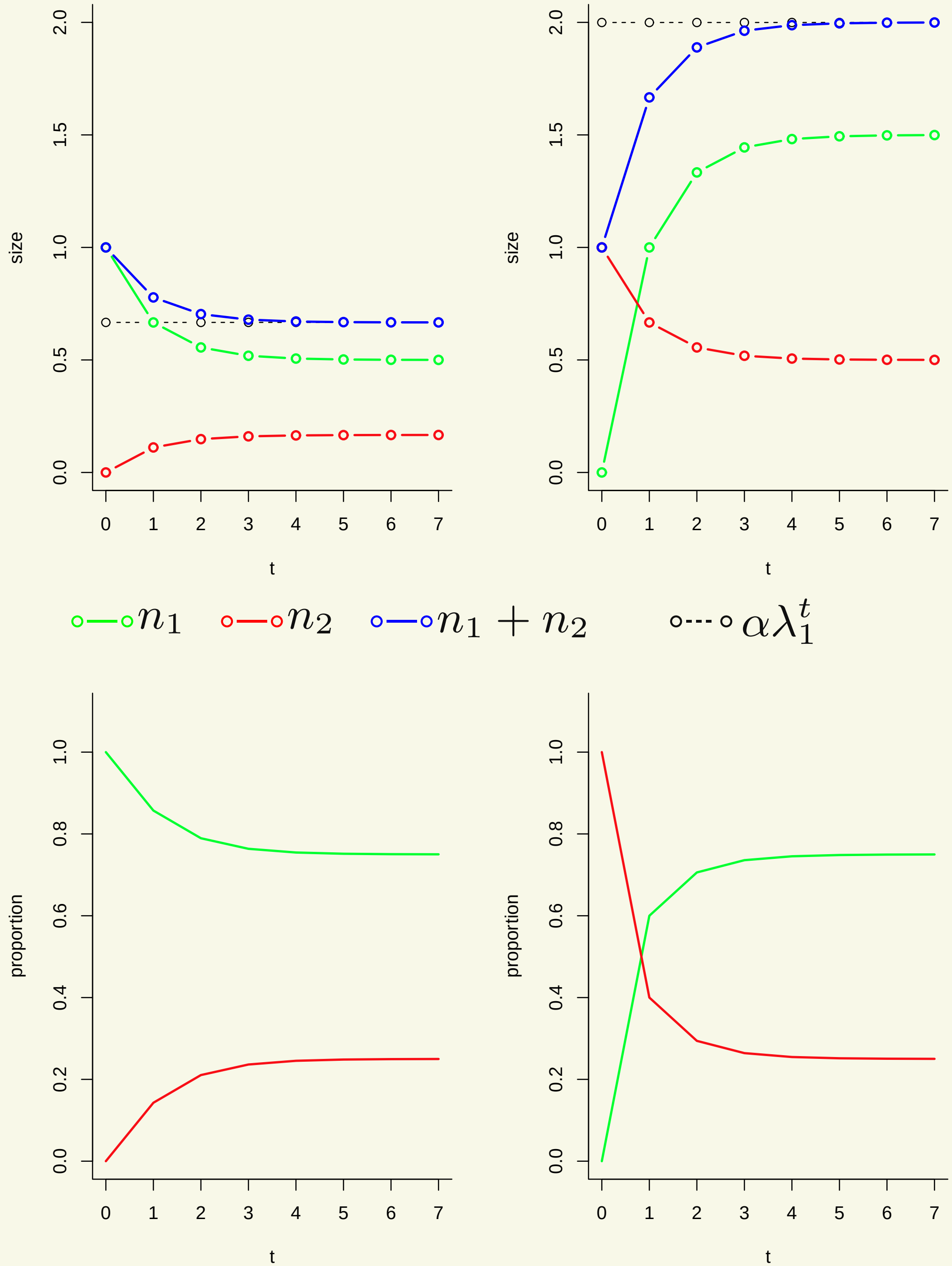
$$\mathsf{A} = \begin{pmatrix} \frac{2}{3} & 1 \\ \frac{1}{9} & \frac{2}{3} \end{pmatrix}$$

$$\lambda_1 = 1, \qquad \boldsymbol{v}_1 = \begin{pmatrix} 0.75 \\ 0.25 \end{pmatrix}$$

$$\lambda_2 = 0.33333, \quad \boldsymbol{v}_2 = \begin{pmatrix} 0.75 \\ -0.25 \end{pmatrix}$$

$$\mathsf{A}^2 = \begin{pmatrix} 0.555556 & 1.33333 \\ 0.14815 & 0.55556 \end{pmatrix}$$

$$\mathsf{A}^3 = \begin{pmatrix} 0.51852 & 1.44444 \\ 0.16049 & 0.51852 \end{pmatrix}$$



Stabilizovaná iteroparní populace (2)

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t)$$

$$\sigma_1 = \tfrac{3}{4},\ \sigma_2 = \tfrac{1}{2},\ \gamma = \tfrac{1}{3},\ \varphi = 1$$

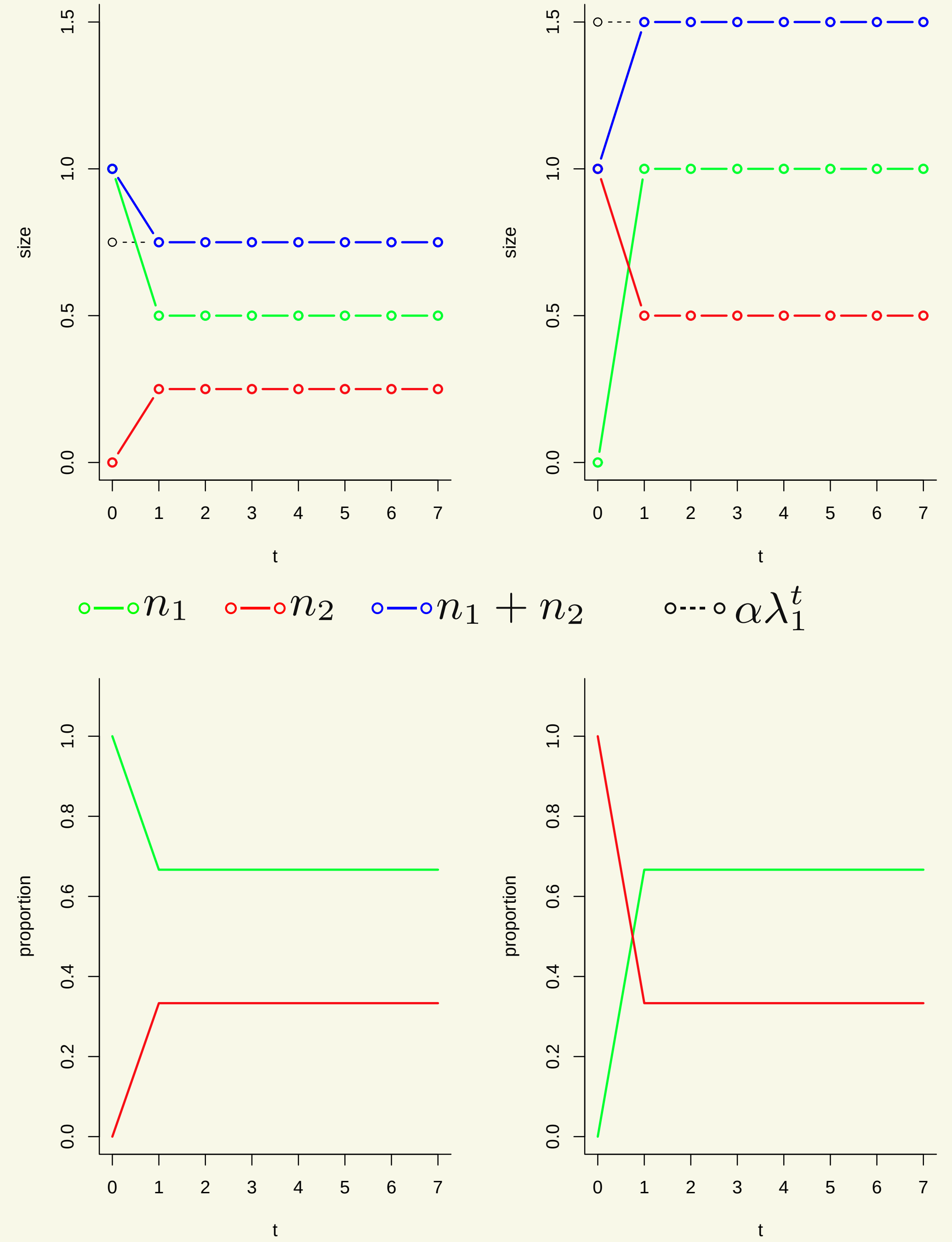
$$\mathsf{A} = \begin{pmatrix} \frac{1}{2} & 1 \\ \frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

$$\lambda_1 = 1, \quad \boldsymbol{v}_1 = \begin{pmatrix} 0.66667 \\ 0.33333 \end{pmatrix}$$

$$\lambda_2 = 0, \quad \boldsymbol{v}_2 = \begin{pmatrix} 0.66667 \\ -0.33333 \end{pmatrix}$$

$$\mathsf{A}^2 = \begin{pmatrix} 0.5 & 1 \\ 0.25 & 0.5 \end{pmatrix}$$

$$\mathsf{A}^3 = \begin{pmatrix} 0.5 & 1 \\ 0.25 & 0.5 \end{pmatrix}$$



Stabilizovaná iteroparní populace (3)

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t)$$

$\sigma_1 = \frac{8}{9}$, $\sigma_2 = \frac{1}{9}$, $\gamma = 1$, $\varphi = 1$

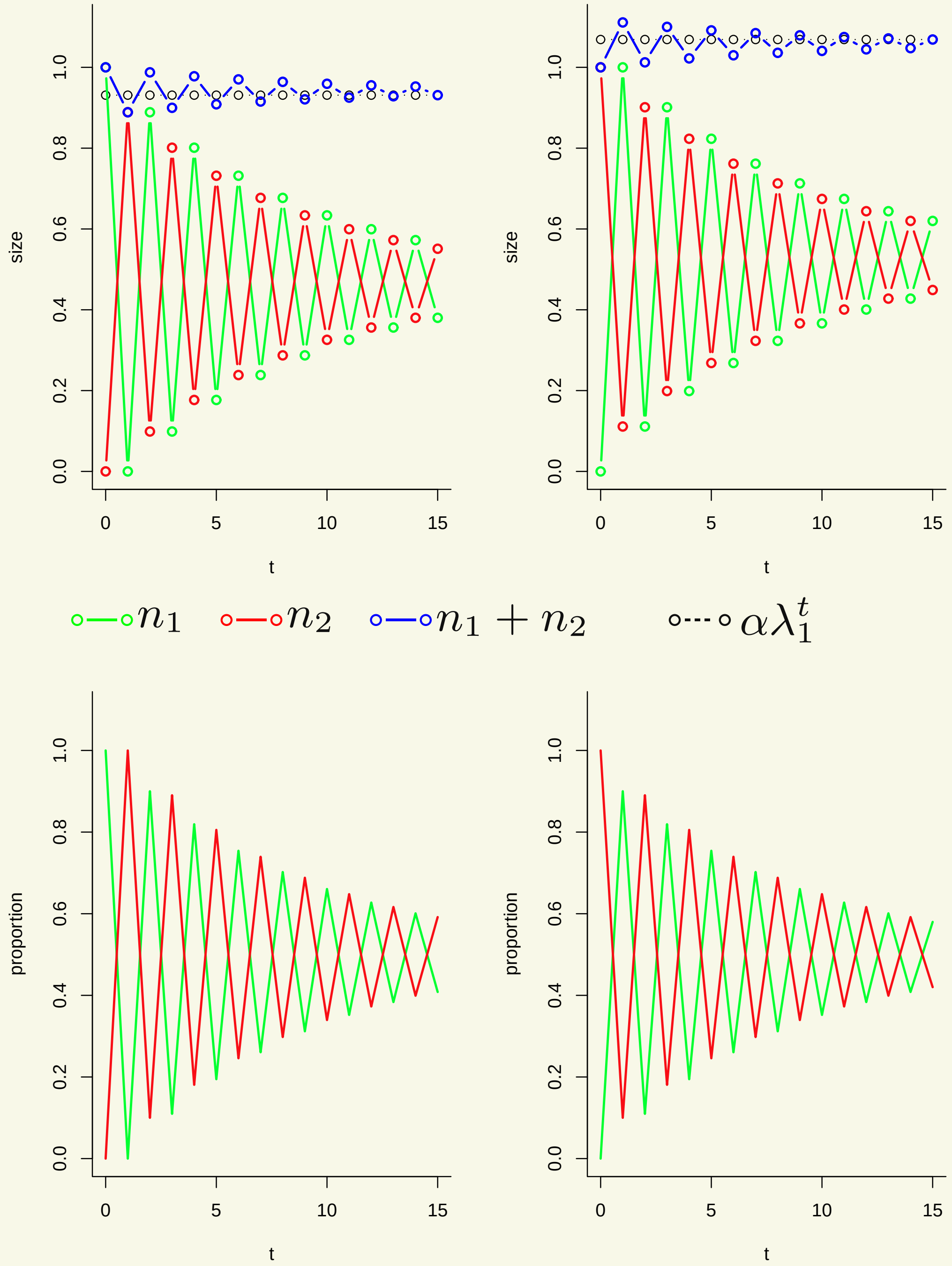
$$\mathsf{A} = \begin{pmatrix} 0 & 1 \\ \frac{8}{9} & \frac{1}{9} \end{pmatrix}$$

$$\lambda_1 = 1, \qquad \boldsymbol{v}_1 = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

$$\lambda_2 = -0.88889, \qquad \boldsymbol{v}_2 = \begin{pmatrix} 0.52941 \\ -0.47059 \end{pmatrix}$$

$$\mathsf{A}^2 = \begin{pmatrix} 0.88889 & 0.11111 \\ 0.09877 & 0.90123 \end{pmatrix}$$

$$\mathsf{A}^3 = \begin{pmatrix} 0.09877 & 0.90123 \\ 0.80110 & 0.19890 \end{pmatrix}$$



Vymírající iteroparní populace

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t)$$

$$\sigma_1 = \tfrac{7}{9},\ \sigma_2 = \tfrac{1}{9},\ \gamma = 1,\ \varphi = 1$$

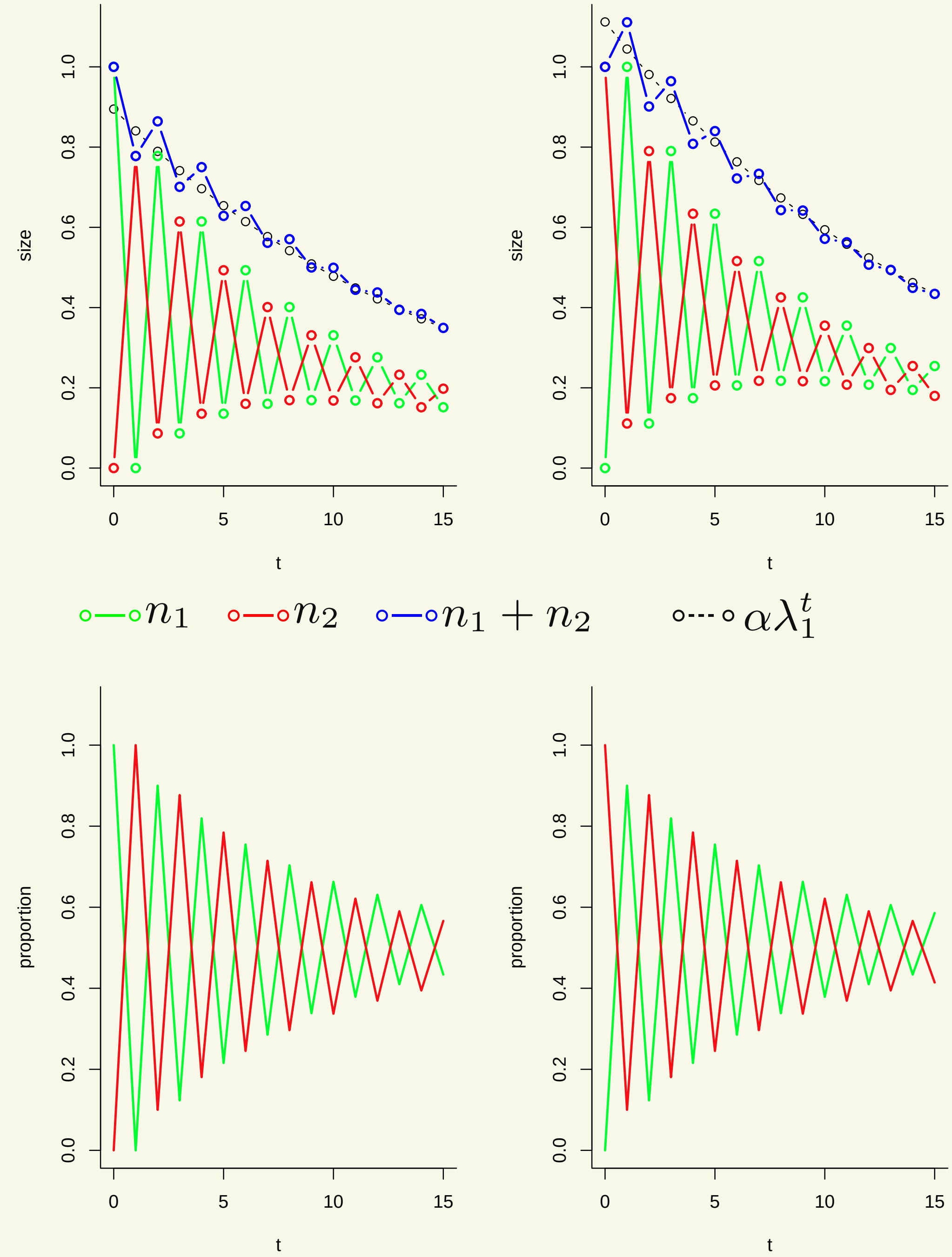
$$\mathsf{A} = \begin{pmatrix} 0 & 1 \\ \frac{7}{9} & \frac{1}{9} \end{pmatrix}$$

$$\lambda_1 = 0.93922, \qquad \boldsymbol{v}_1 = \begin{pmatrix} 0.51567 \\ 0.48433 \end{pmatrix}$$

$$\lambda_2 = -0.82811, \qquad \boldsymbol{v}_2 = \begin{pmatrix} 0.54701 \\ -0.45299 \end{pmatrix}$$

$$\mathsf{A}^2 = \begin{pmatrix} 0.77778 & 0.11111 \\ 0.08642 & 0.79012 \end{pmatrix}$$

$$\mathsf{A}^3 = \begin{pmatrix} 0.08642 & 0.79012 \\ 0.61454 & 0.17421 \end{pmatrix}$$



Stabilizovaná semelparní populace

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t)$$

$\sigma_1 = 1$, $\sigma_2 = 0$, $\gamma = 1$, $\varphi = 1$

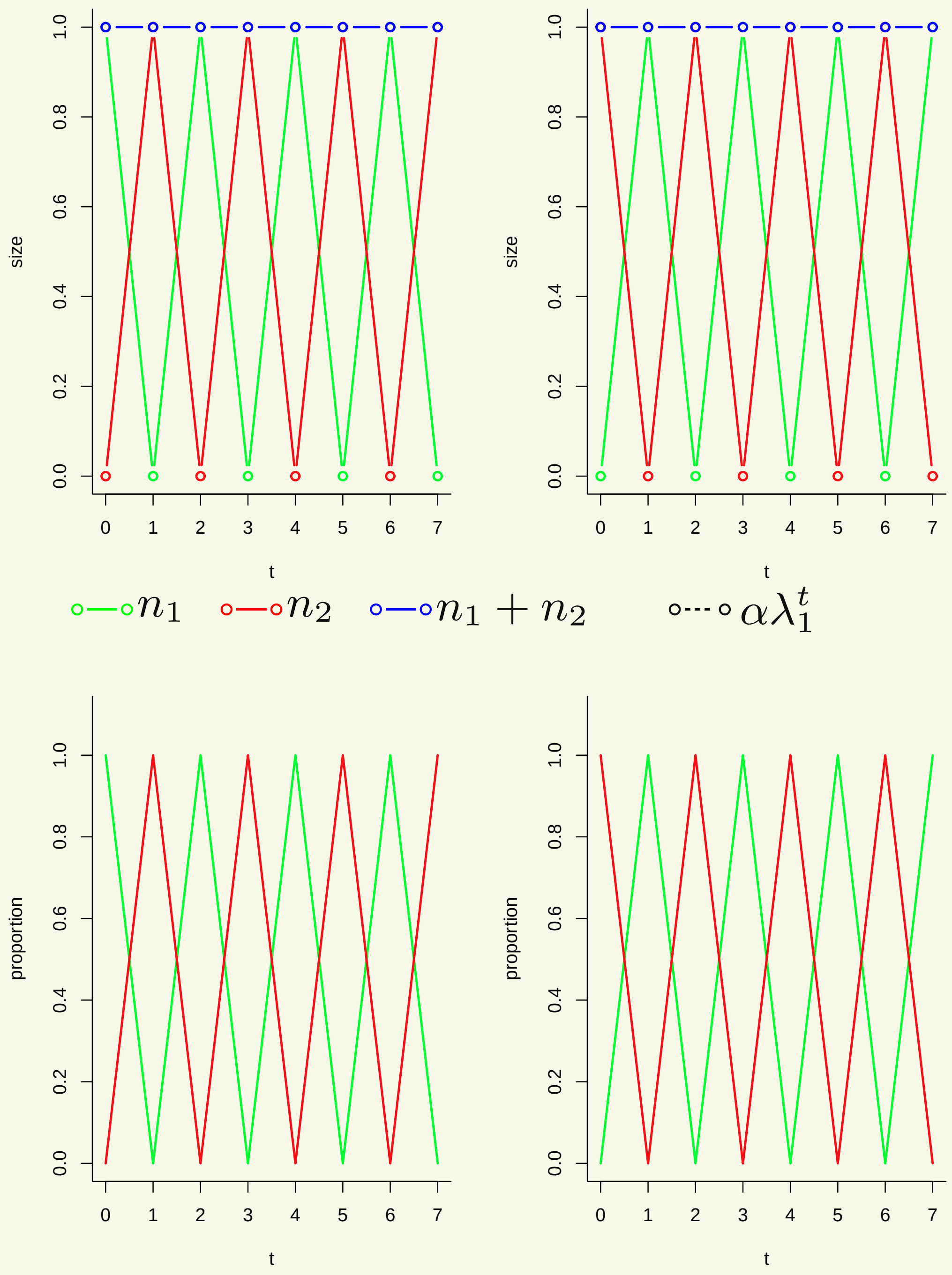
$$\mathsf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\lambda_1 = 1, \qquad \boldsymbol{v}_1 = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

$$\lambda_2 = -1, \qquad \boldsymbol{v}_2 = \begin{pmatrix} 0.5 \\ -0.5 \end{pmatrix}$$

$$\mathsf{A}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathsf{A}^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



Vymírající semelparní populace

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t)$$

$\sigma_1 = \frac{8}{9}$, $\sigma_2 = 0$, $\gamma = 1$, $\varphi = 1$

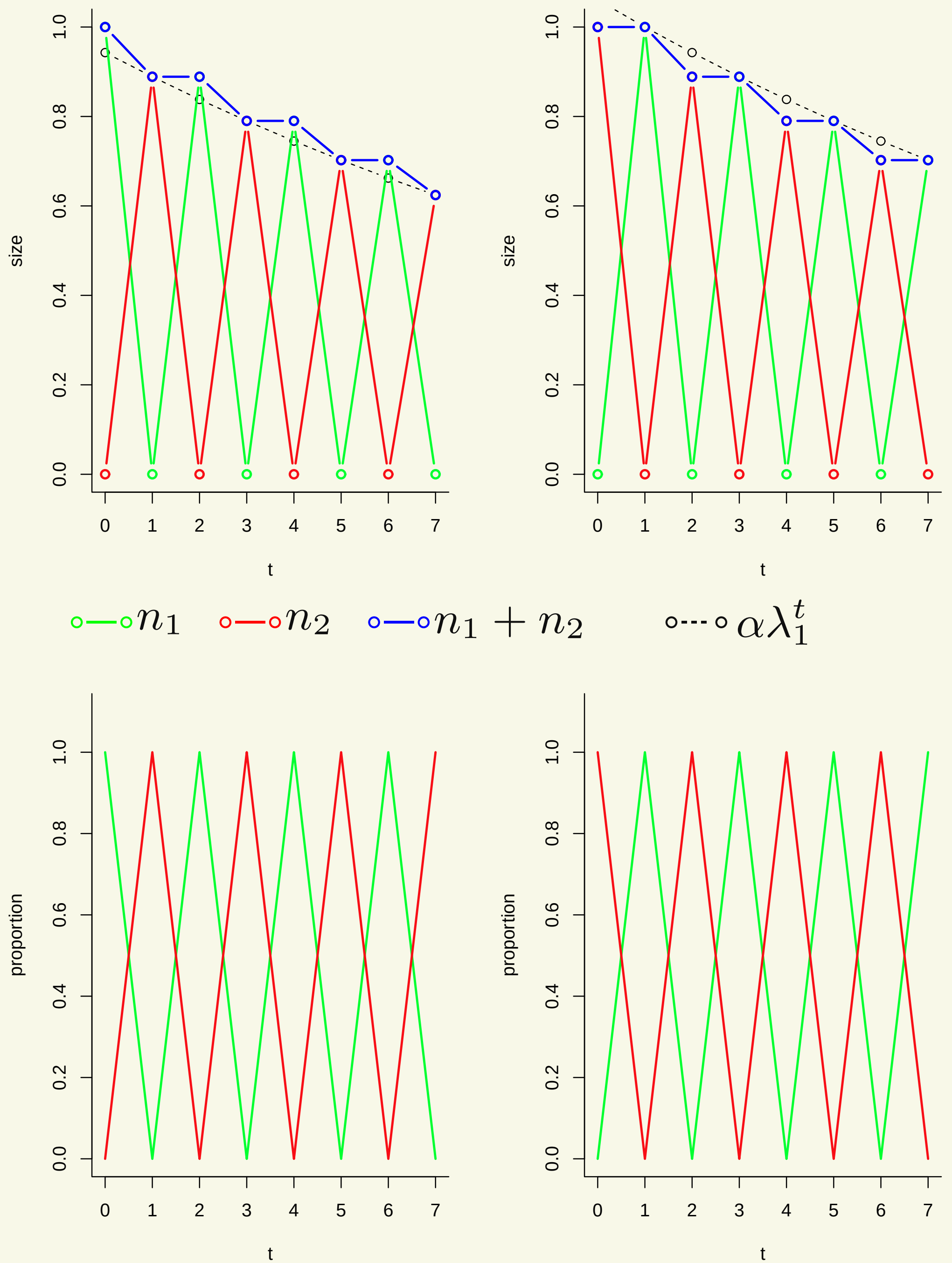
$$\mathsf{A} = \begin{pmatrix} 0 & 1 \\ \frac{8}{9} & 0 \end{pmatrix}$$

$$\lambda_1 = 0.94281, \qquad \boldsymbol{v}_1 = \begin{pmatrix} 0.51472 \\ 0.48528 \end{pmatrix}$$

$$\lambda_2 = -0.94281, \qquad \boldsymbol{v}_2 = \begin{pmatrix} 0.51472 \\ -0.48528 \end{pmatrix}$$

$$\mathsf{A}^2 = \begin{pmatrix} 0.88889 & 0 \\ 0 & 0.88889 \end{pmatrix}$$

$$\mathsf{A}^3 = \begin{pmatrix} 0 & 0.88889 \\ 0.79012 & 0 \end{pmatrix}$$



Neplodná populace (1)

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t)$$

$\sigma_1 = \frac{1}{2}$, $\sigma_2 = \frac{3}{8}$, $\gamma = \frac{3}{4}$, $\varphi = 0$

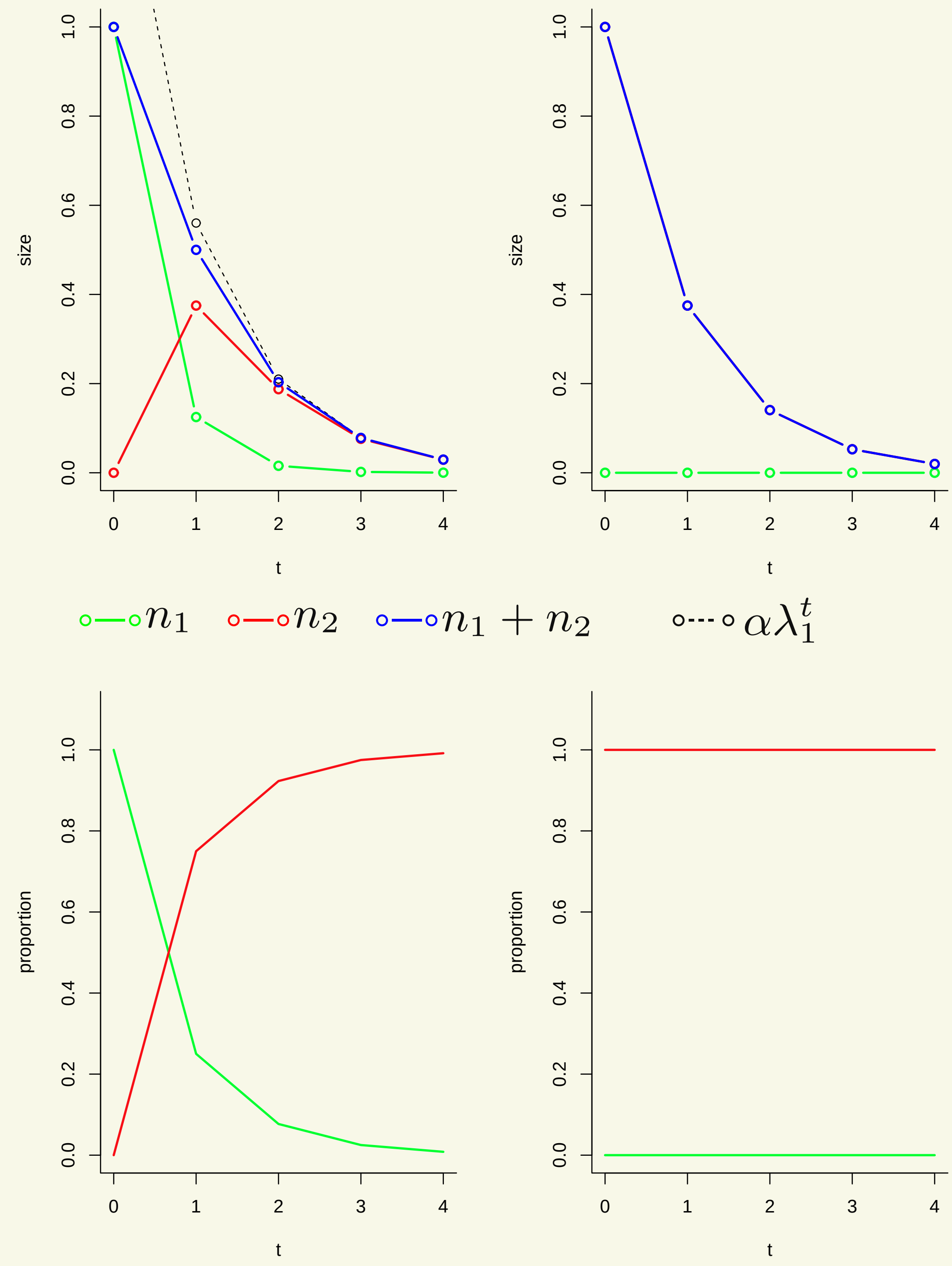
$$\mathsf{A} = \begin{pmatrix} \frac{1}{8} & 0 \\ \frac{3}{8} & \frac{3}{8} \end{pmatrix}$$

$$\lambda_1 = 0.375, \quad \boldsymbol{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\lambda_2 = 0.125, \quad \boldsymbol{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathsf{A}^2 = \begin{pmatrix} 0.01563 & 0 \\ 0.18750 & 0.14063 \end{pmatrix}$$

$$\mathsf{A}^3 = \begin{pmatrix} 0.00195 & 0 \\ 0.07617 & 0.05273 \end{pmatrix}$$



Neplodná populace (2)

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t)$$

$\sigma_1 = \frac{1}{2}$, $\sigma_2 = \frac{1}{8}$, $\gamma = \frac{3}{4}$, $\varphi = 0$

$$\mathsf{A} = \begin{pmatrix} \frac{1}{8} & 0 \\ \frac{3}{8} & \frac{1}{8} \end{pmatrix}$$

$$\lambda_{1,2} = 0.125, \quad \boldsymbol{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\boldsymbol{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathsf{A}^2 = \begin{pmatrix} 0.01563 & 0 \\ 0.09375 & 0.01563 \end{pmatrix}$$

$$\mathsf{A}^3 = \begin{pmatrix} 0.00195 & 0 \\ 0.01758 & 0.00195 \end{pmatrix}$$