Matrix analysis

Net reproductive rate (R_0)

- average total number of offspring produced by a female in her lifetime

$$R_0 = \sum_{x=0}^{n} l_x m_x$$

Average generation time (T)

- average age of females when they give birth

$$T = \frac{\sum_{x=0}^{n} x l_x m_x}{R_0}$$

Expectation of life

- age specific expectation of life
- o .. oldest age

$$e_x = \frac{T_x}{l_x}$$ where $$T_x = \sum_{x}^{o} L_x$$ $$L_x = \frac{l_x + l_{x+1}}{2}$$

Growth rates

- Discrete time/generations

- estimate of λ (finite growth rate) from the life table:

$$\mathbf{A}\tilde{\mathbf{N}}_t = \lambda\tilde{\mathbf{N}}_t$$

where $\tilde{\mathbf{N}}_t$ is vector at stable age distribution

λ is dominant positive eigenvalue of **A**

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0$$

- or

$$\lambda \approx \frac{R_0}{T}$$

- Continuous time

- r can be estimated from λ

$$r = \ln(\lambda)$$

- by approximation

$$r \approx \frac{\ln(R_0)}{T}$$

or by Euler's method

$$1 = \int_0^t l(x)m(x)e^{-rx}dx$$

Stable Class distribution (SCD)

- relative abundance of different life history age/stage/size categories
- ▸ population approaches stable age distribution:

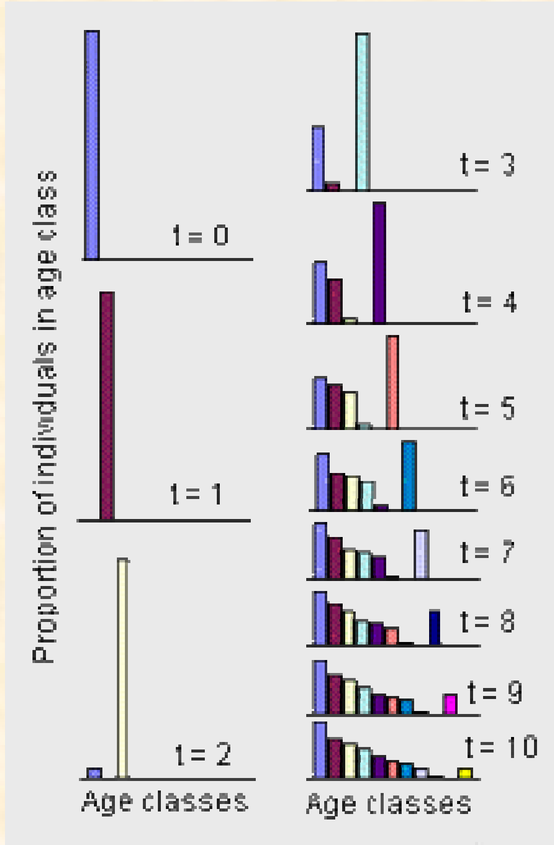
  $\boldsymbol{N_0} : \boldsymbol{N_1} : \boldsymbol{N_2} : \boldsymbol{N_3} :....: \boldsymbol{N}_s$ is stable

- once population reached SCD it grows exponentially

▸ $\mathbf{w}_1$ .. right eigenvector (vector of the dominant eigenvalue)
- provides stable age distribution
- scale $\mathbf{w}_1$ by sum of individuals

$$\mathbf{A}\mathbf{w}_1 = \lambda_1 \mathbf{w}_1$$

$$SCD = \frac{\mathbf{w}_1}{\sum_{i=1}^{S} \mathbf{w}_1}$$



Reproductive value (v_x)

- measures relative reproductive potential and identifies age class that contributes most to the population growth

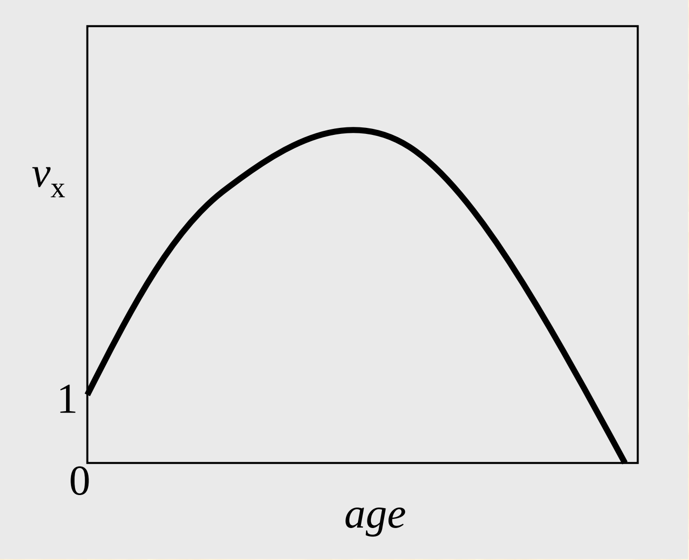
- when population increases then early offspring contribute more to v_x than older ones

$$\mathbf{v}_1\mathbf{A}' = \lambda_1\mathbf{v}_1$$

- $\mathbf{v}_1$ .. left eigenvector (vector of the dominant eigenvalue of transposed **A)**

\- $\mathbf{v}_1$ is proportional to the reproductive values and scaled to the first category

$$v_x = \frac{\tilde{u}_x}{\tilde{u}_1}$$



Sensitivity (*s*)

▸ identifies which process (*p*, *F*, *G*) has largest effect on the population increase (λ_1)
- examines change in λ_1 given small change in processes (a_{ij})
- sensitivity is larger for survival of early, and for fertility of older classes
- not used for postreproductive census with class 0

$$s_{ij} = \frac{v_{ij} w'_{ij}}{\langle \mathbf{v}, \mathbf{w} \rangle}$$

← sum of pairwise products

Elasticity (*e*)

▸ weighted measure of sensitivity
- measures relative contribution to the population increase
- impossible transitions = 0

$$e_{ij} = \frac{a_{ij}}{\lambda_1} s_{ij}$$

Conservation biology

- to adopt means for population promotion or control

Conservation/control procedure

1. Construction of a life table
2. Estimation of the intrinsic rates
3. Sensitivity analysis - helps to decide where conservation/control efforts should be focused
4. Development and application of management plan
5. Prediction of future