$$\frac{\mathrm{d}N}{\mathrm{d}t} = Nr$$

Intraspecific Interactions

“Populační ekologie živočichů“

Stano Pekár

Density-dependent growth

- includes all mechanisms of population growth that change with density
  - population structure is ignored
  - extrinsic effects are negligible
  - response of λ and r to N is immediate

- λ and r decrease with population density either because natality decreases or mortality increases or both
  - negative feedback of the 1st order

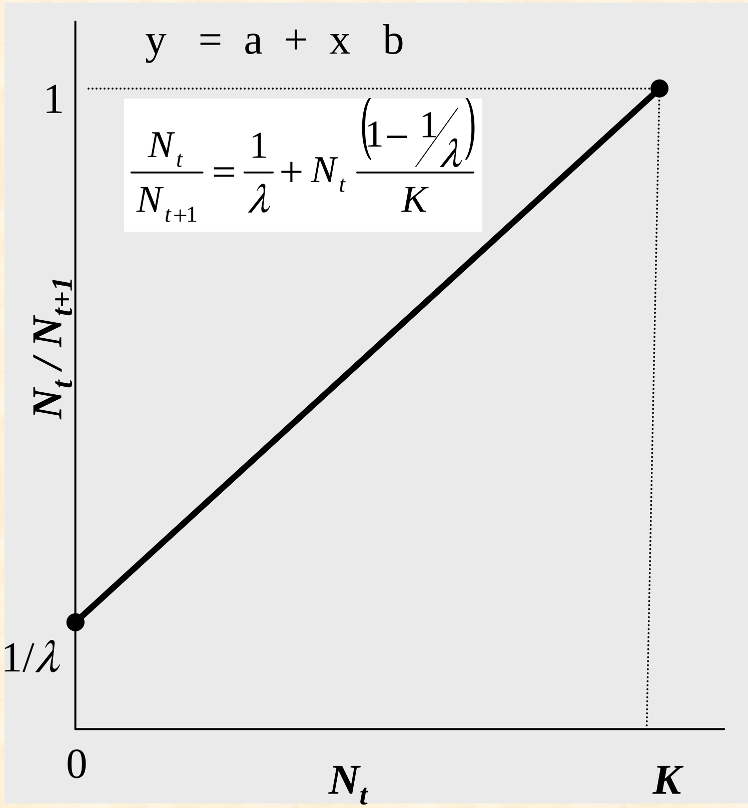
- K .. carrying capacity
  - upper limit of population growth where $\lambda = 1$ or $r = 0$
  - is constant

Discrete (difference) model

- there is linear dependence of λ on *N*

$$N_{t+1} = N_t \lambda$$

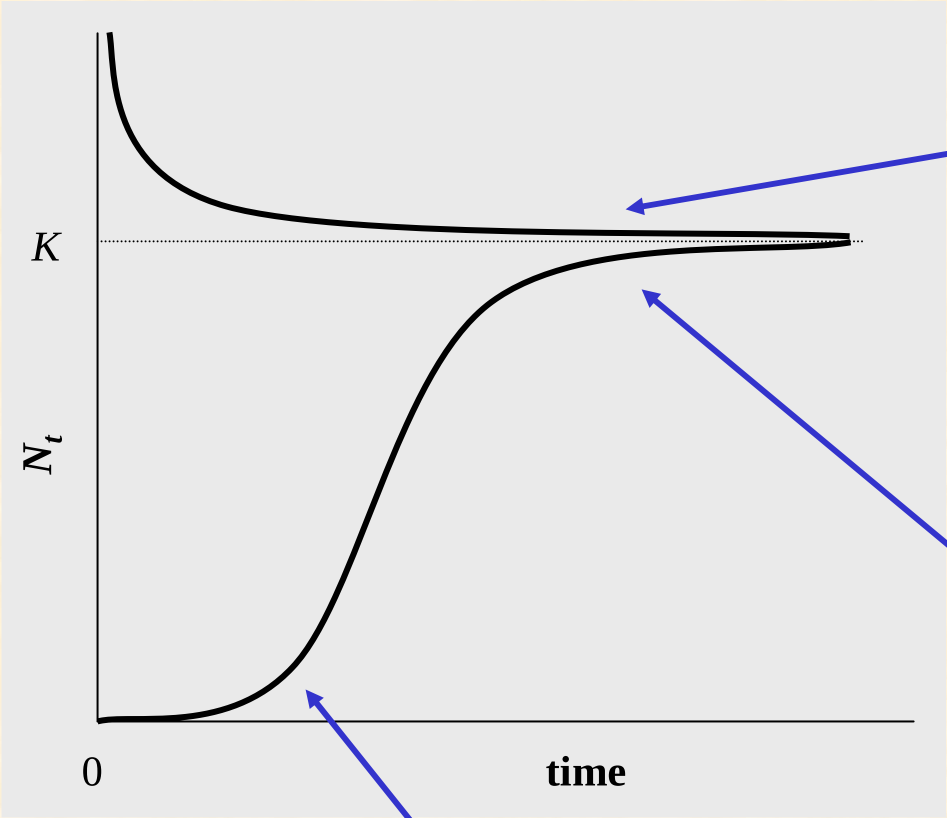
$$\frac{N_t}{N_{t+1}} = \frac{1}{\lambda}$$



$$N_{t+1} = \frac{N_t \lambda}{1 + \frac{(\lambda - 1) N_t}{K}}$$

if $a = \frac{\lambda - 1}{K}$ then

$$N_{t+1} = \frac{N_t \lambda}{1 + a N_t}$$



when $N_t > K$ then

$$\frac{\lambda}{1+aN_t} < 1$$

- population returns to K

when $N_t \rightarrow K$ then

$$\frac{\lambda}{1+aN_t} \approx 1$$

- density-dependent control
- S-shaped (sigmoid) growth

when $N_t \rightarrow 0$ then

$$\frac{\lambda}{1+aN_t} \approx \lambda$$

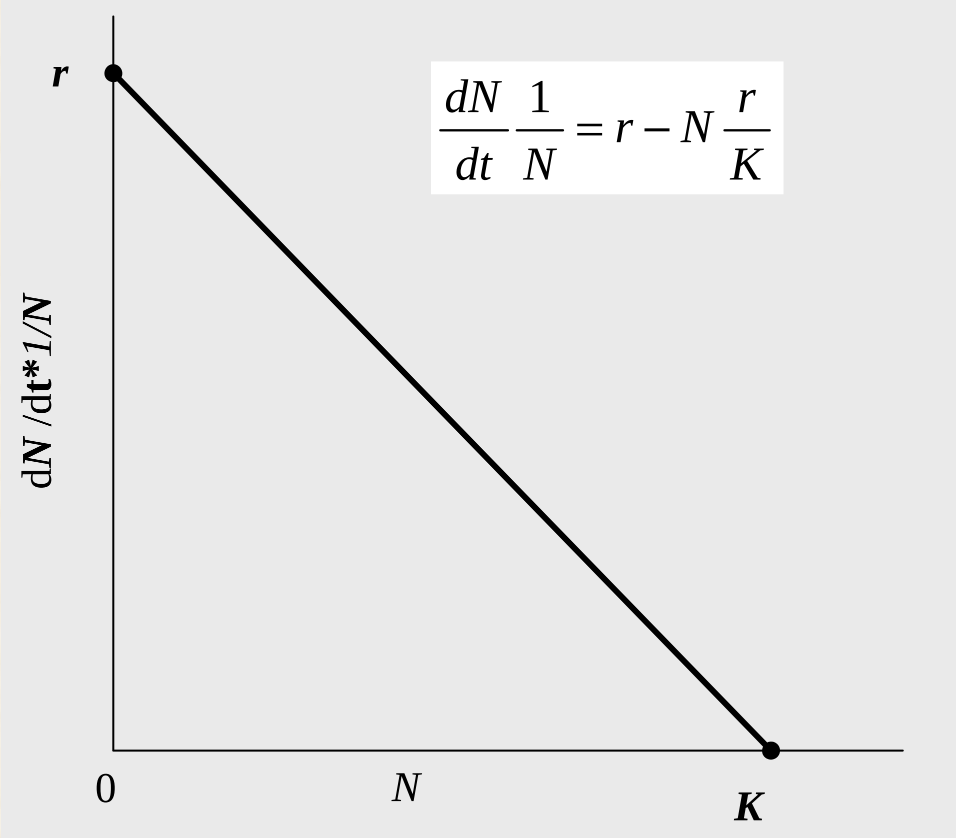
- no competition
- exponential growth

Continuous (differential) model

- logistic growth
- first used by Verhulst (1838) to describe growth of human population

- there is linear dependence of r on N

$$\frac{dN}{dt} = Nr \quad \rightarrow \quad \frac{dN}{dt}\frac{1}{N} = r$$



$$\frac{dN}{dt}\frac{1}{N} = r - N\frac{r}{K}$$

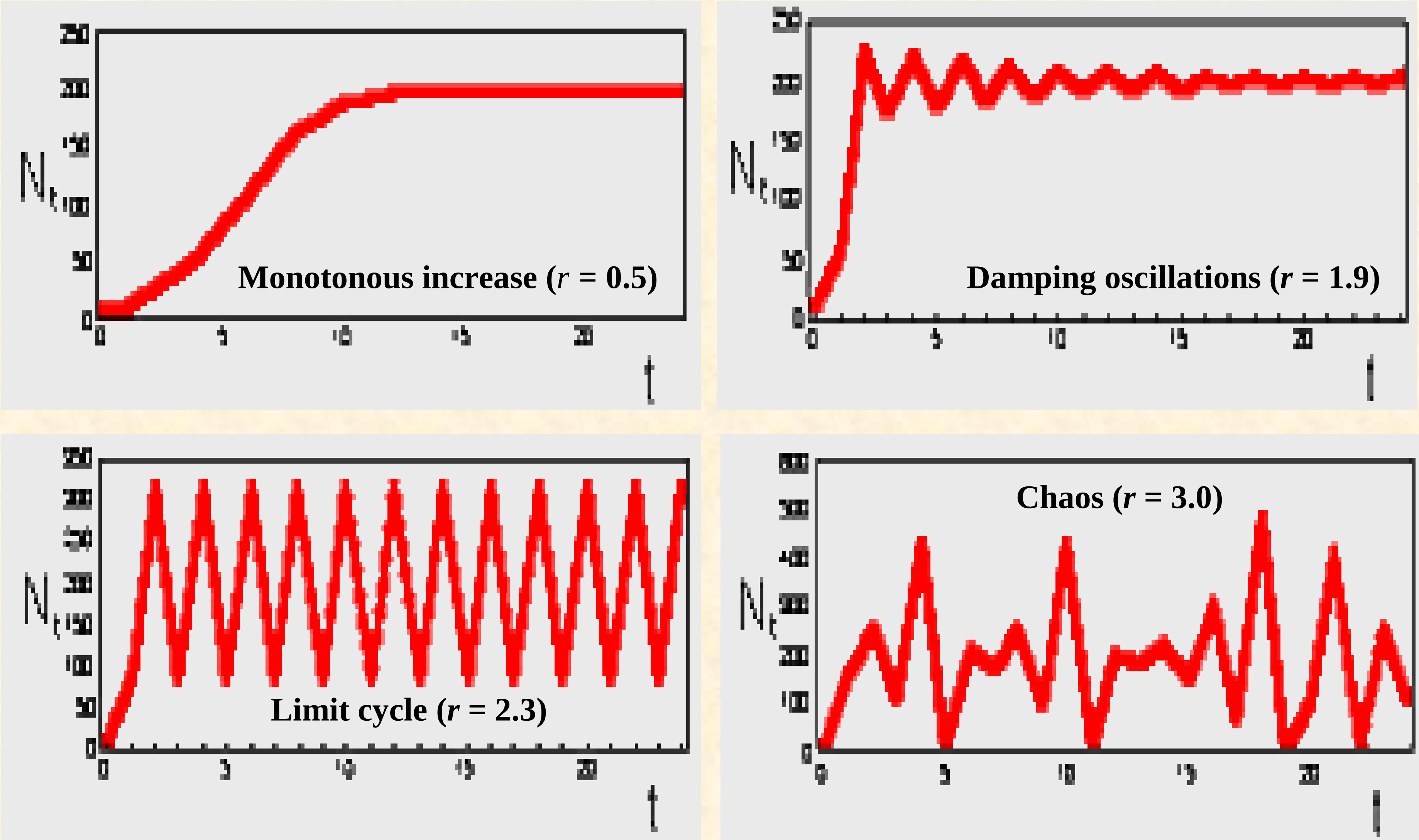
- when $N \rightarrow K$ then $r \rightarrow 0$

$$\frac{dN}{dt} = Nr\left(1 - \frac{N}{K}\right)$$

<u>Solution of the differential equation</u>

$$N_t = \frac{KN_0}{(K - N_0)e^{-rt} + N_0}$$

Examination of the logistic model



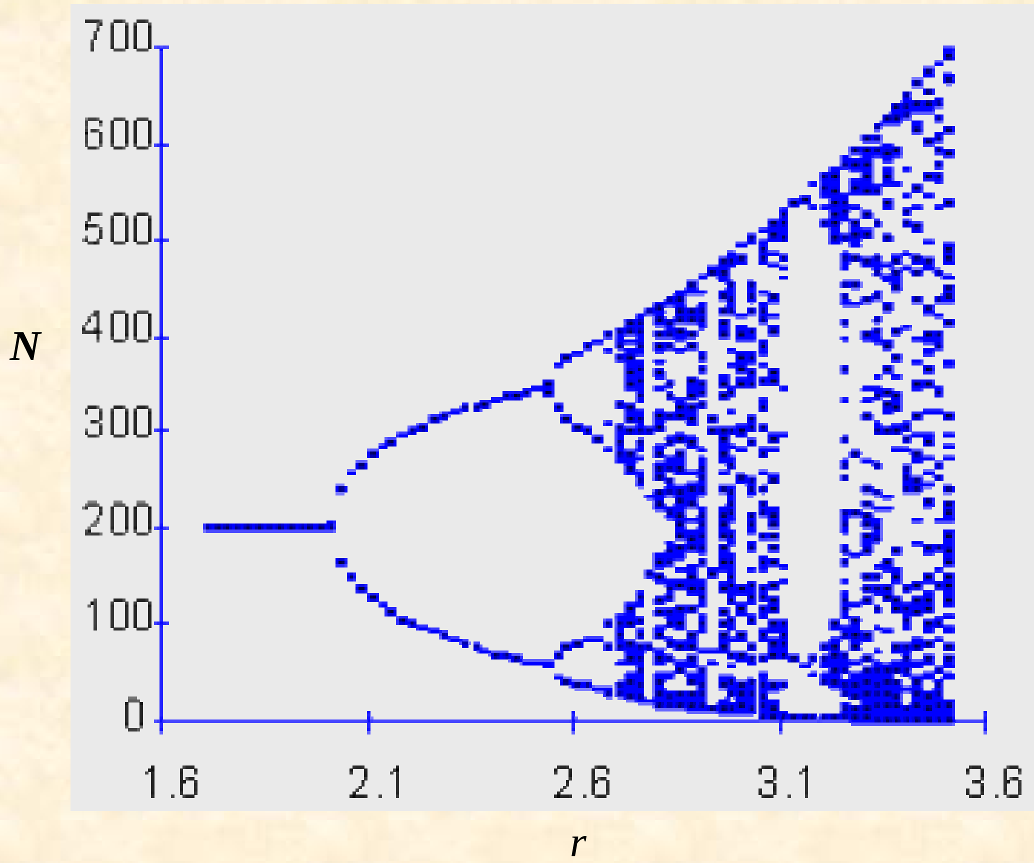
Model equilibria

1. $N = 0$ .. unstable equilibrium
2. $N = K$ .. stable equilibrium .. if $0 < r < 2$

- "Monotonous increase" and "Damping oscillations" has a stable equilibrium
- "Limit cycle" and "Chaos" has no equilibrium

$r < 2$ .. stable equilibrium
$r = 2$ .. 2-point limit cycle
$r = 2.5$ .. 4-point limit cycle
$r = 2.692$ .. chaos

- chaos can be produced by deterministic process
- density-dependence is stabilising only when r is rather low



Observed population dynamics

a) yeast (logistic curve)

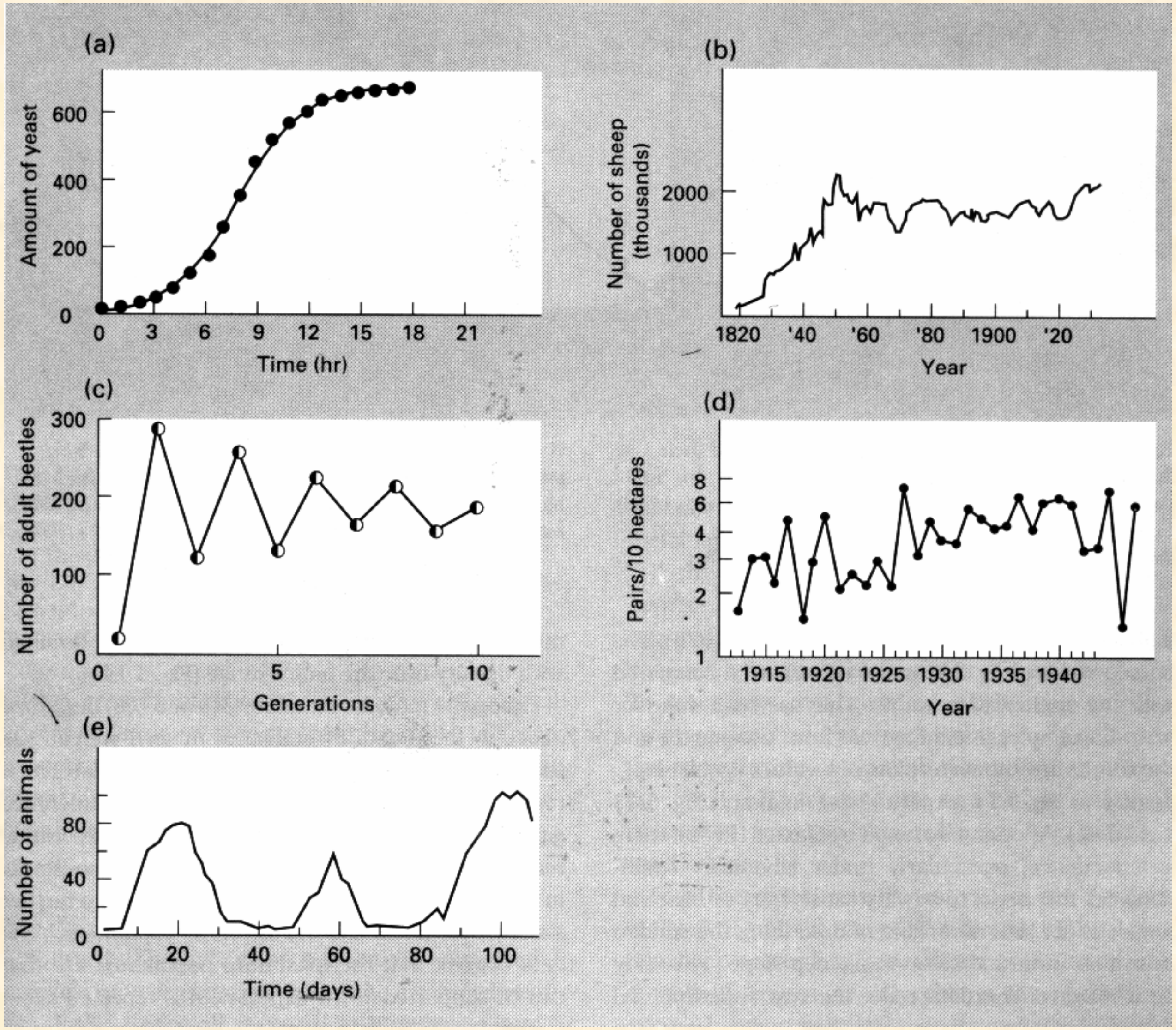
b) sheep (logistic curve with oscillations)

c) *Callosobruchus* (damping oscillations)

d) *Parus* (chaos)

e) *Daphnia*

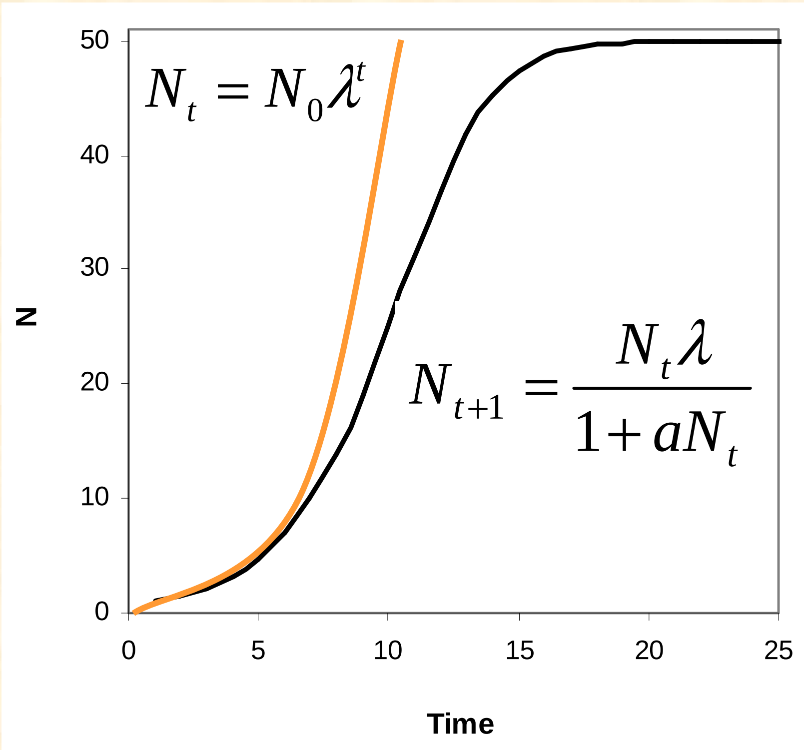
- of 28 insect species in one species chaos was identified, one other showed limit cycles, all other were in stable equilibrium

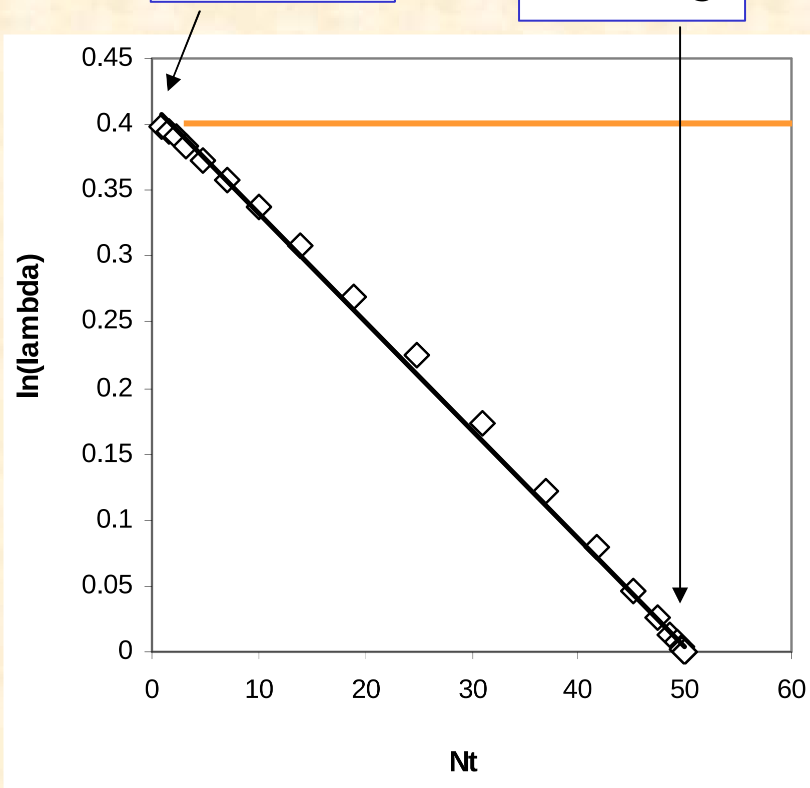


Evidence of DD

- in case of density-independence λ is constant – independent of N
- in case of DD λ is changing with N: $\ln(\lambda) = a - bN_t$
- plot ln(λ) against N_t
- estimate λ and K

$$\lambda_{\max} = e^a \qquad K = -\frac{a}{b}$$





General logistic model

- Hassell (1975) proposed general model for DD
- r is not linearly dependent on N

$$N_{t+1} = \frac{N_t \lambda}{(1 + aN_t)^{\theta}}$$

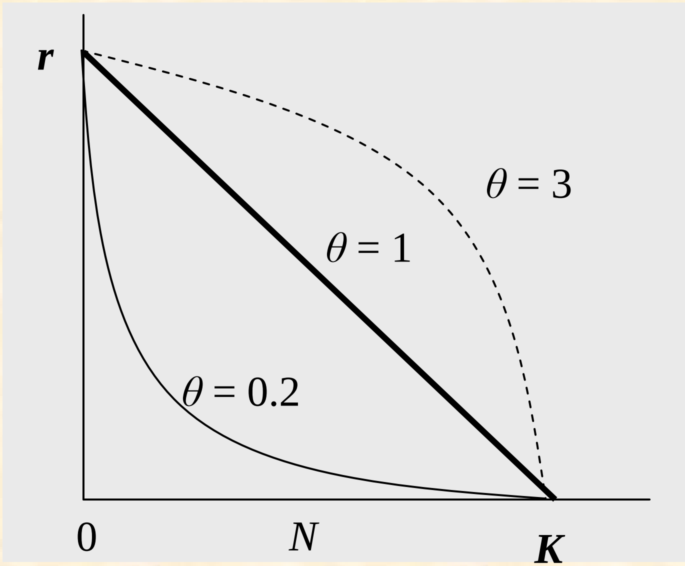
$$\frac{dN}{dt} = rN\left(1 - \left(\frac{N}{K}\right)^{\theta}\right)$$

where θ.. the strength of competition

- $\theta >> 1$ .. scramble competition (over-compensation), strong DD, leads to fluctuations around K

- $\theta = 1$ .. contest competition (exact compensation), stable density

- $\theta < 1$ .. under-compensation - weak DD, population will return to K



Models with time-lags

- species response to resource change is not immediate but delayed due to maternal effect, seasonal effect, predator pressure
- appropriate for species with long generation time where reproductive rate is dependent on density of a previous generation
- time lag (*d* or τ) .. negative feedback of the 2nd order

discrete model

$$N_{t+1} = \frac{N_t \lambda}{1 + aN_{t-d}}$$

continuous model

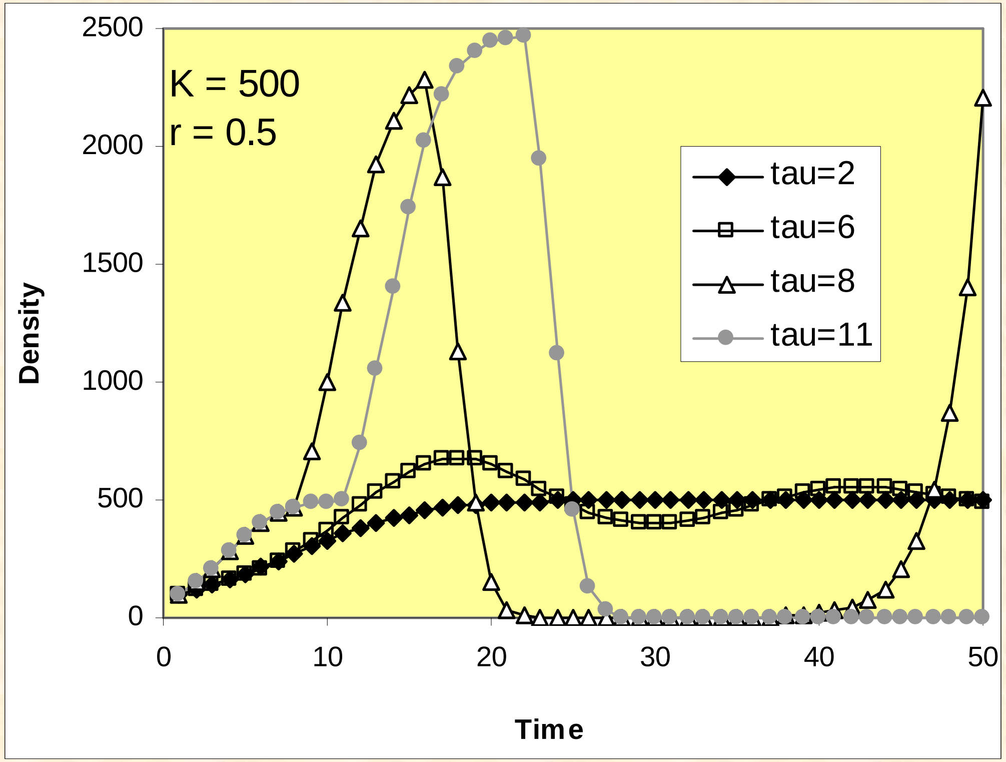
$$\frac{dN}{dt} = N_t r \left(1 - \frac{N_{t-\tau}}{K}\right)$$

- many populations of mammals cycle with 3-4 year periods
- time-lag provokes fluctuations of certain amplitude at certain periods
- period of the cycle in continuous model is always 4τ

Solution of the continuous model:

$$N_{t+1} = N_t e^{r\left(1-\frac{N_{t-\tau}}{K}\right)}$$

$r\,\tau < 1 \rightarrow$ monotonous increase
$r\,\tau < 3 \rightarrow$ damping fluctuations
$r\,\tau < 4 \rightarrow$ limit cycle fluctuations
$r\,\tau > 5 \rightarrow$ extinction

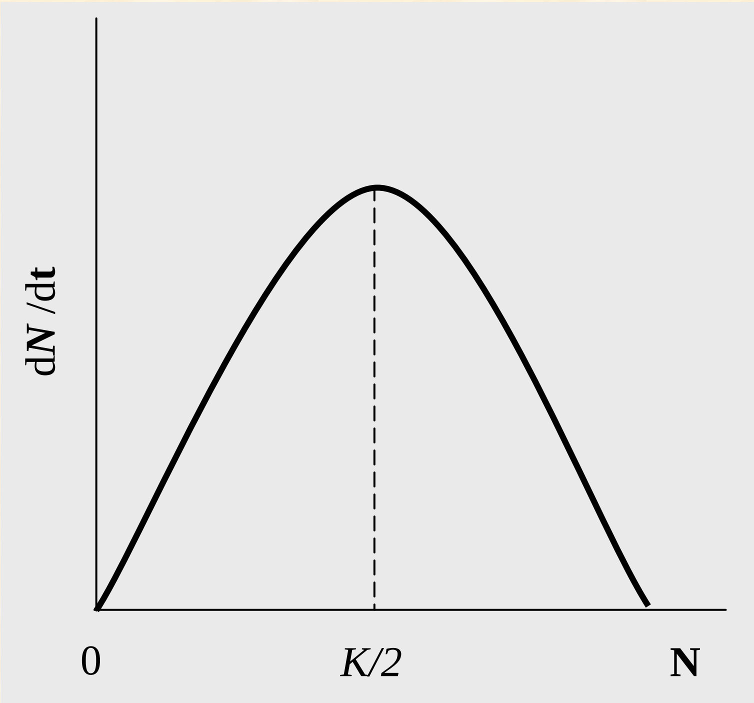


Harvesting

- Maximum Sustainable Harvest (MSH)

- to harvest as much as possible with the least negative effect on *N*
- ignore population structure
- ignore stochasticity

$$\frac{dN}{dt} = Nr\left(1 - \frac{N}{K}\right) = 0$$



local maximum: $N^* = \frac{K}{2}$

Amount of MSH (V_{max}):
- replace N^* by $K/2$

$$\text{MSH} = \frac{rK}{4}$$

- Robinson & Redford (1991)
- Maximum Sustainable Yield (MSY)

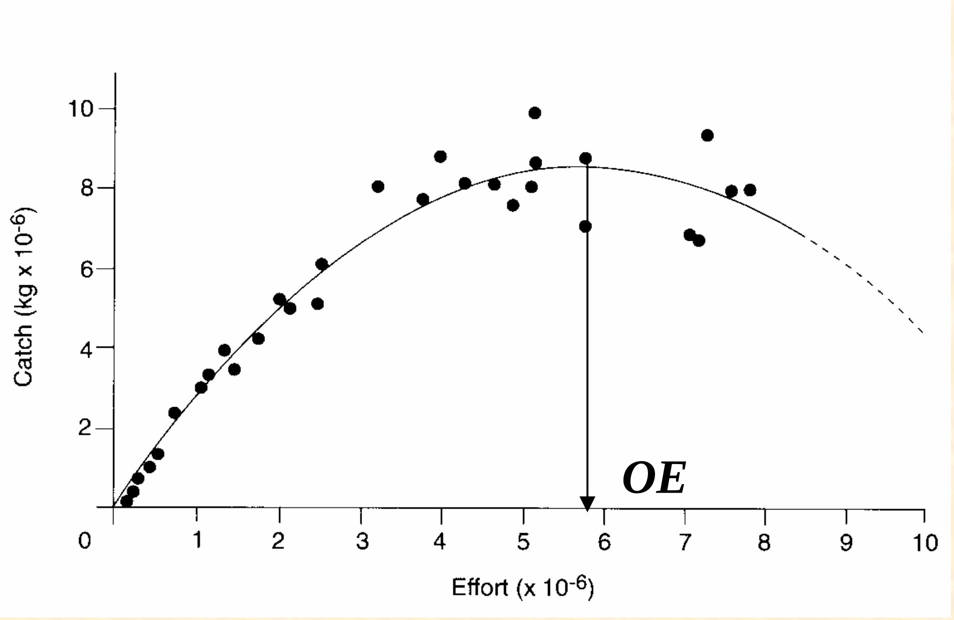
$$\text{MSY} = a\left(\frac{\lambda K - K}{2}\right)$$

where $a = 0.6$ for longevity < 5
$a = 0.4$ for longevity $= (5,10)$
$a = 0.2$ for longevity > 10

- Surplus production (catch-effort) models
- when r, λ and K are not known
- effort and catch over several years is known
- Schaefer quadratic model

$$catch = \alpha + \beta E + \gamma E^2$$

- local maximum of the function identifies optimal effort (*OE*)



Alee effect

- individuals in a population may cooperate in hunting, breeding – positive effect on population increase
- Allee (1931) – discovered inverse DD

- genetic inbreeding – decrease in fertility
- demographic stochasticity – biased sex ratio
- small groups – cooperation in foraging, defence, mating, thermoregulation

- K_2 .. extinction threshold,

- unstable equilibrium

- population increase is slow at low density but fast at higher density

$$\frac{dN}{dt} = Nr\left(1 - \frac{N}{K_1}\right)\left(\frac{N}{K_2} - 1\right)$$