$$\frac{dN}{dt} = Nr$$

Interspecific Interactions

"Populační ekologie živočichů"

Stano Pekár

Types of interactions

Effect of species 1 on fitness of species 2 (columns); Effect of species 2 on fitness of species 1 (rows)

	Increase	Neutral	Decrease
Increase	+ +		
Neutral	0 +	0 0	
Decrease	+ -	- 0	- -

+ + .. **mutualism** (plants and pollinators)

0 + .. **commensalism** (saprophytism, parasitism, phoresis)

\- + .. **predation** (herbivory, parasitism)**, mimicry**

\- 0 .. **amensalism** (allelopathy)

\- - .. **competition**

Niche measures

- Niche breadth

Levin's index (D):

- p_k .. proportion of individuals in class k
- does not include resource availability

$$D = \frac{1}{\sum_{k=1}^{n} p_k^2}$$

Smith's index (FT):

- q_k .. proportion of available individuals in class k
- $0 < D, FT < 1$

$$FT = \sum_{k=1}^{n} \sqrt{p_k q_k}$$

- Niche overlap

Pianka's index (a):

- does not account for resource availability
- $0 < a < 1$

$$a = \frac{\sum p_{1k} p_{2k}}{\sqrt{\sum p_{1k} \sum p_{2k}}}$$

Lloyd's index (L):

- $0 < L < \infty$

$$L = \sum \frac{p_{1k} p_{2k}}{q_k}$$

Model of competition

- based on the logistic differential model

$$\frac{\mathrm{d}N}{\mathrm{d}t} = Nr\left(1 - \frac{N}{K}\right)$$

- assumptions:

- all parameters are constant
- individuals of the same species are identical
- environment is homogenous, differentiation of niches is not possible
- only exact compensation is present

- model of Lotka (1925) and Volterra (1926)

species 1: N_1, K_1, r_1

species 2: N_2, K_2, r_2

$$\frac{dN_1}{dt} = N_1 r_1\left(1 - \frac{N_1 + N_2}{K_1}\right)$$

$$\frac{dN_2}{dt} = N_2 r_2\left(1 - \frac{N_1 + N_2}{K_2}\right)$$

- total competitive effect (intra + inter-specific)

$(N_1 + \alpha N_2)$ where α .. coefficient of competition

$\alpha = 0$ .. no interspecific competition

$\alpha < 1$ .. species 2 has lower effect on species 1 than species 1 on itself

$\alpha = 0.5$ .. one individual of species 1 is equivalent to 0.5 individuals of species 2)

$\alpha = 1$ .. both species has equal effect on the other one

$\alpha > 1$ .. species 2 has greater effect on species 1 than species 1 on itself

species 1: $$\frac{dN_1}{dt} = N_1 r_1 \left(1 - \frac{N_1 + \alpha_{12} N_2}{K_1}\right)$$

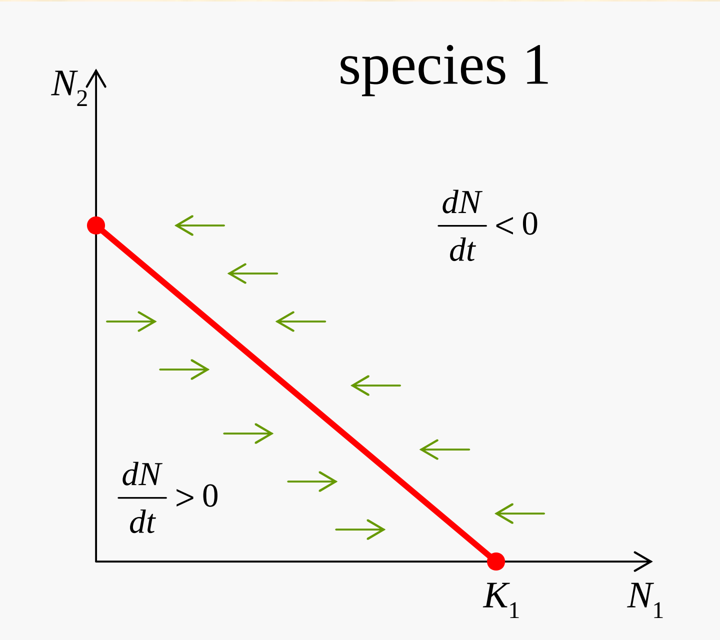
species 2: $$\frac{dN_2}{dt} = N_2 r_2 \left(1 - \frac{\alpha_{21} N_1 + N_2}{K_2}\right)$$

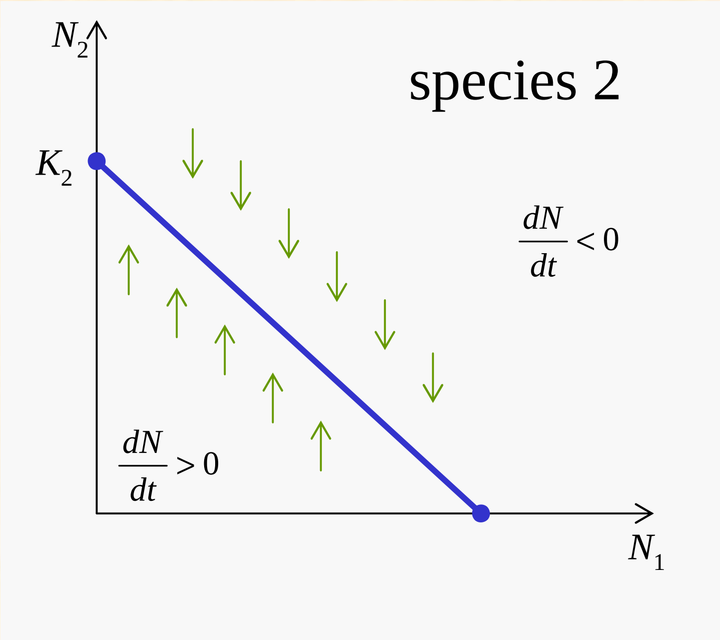
- if competing species use the same resource then interspecific competition is equal to intraspecific

Analysis of the model

- examination of the model behaviour on a phase plane
- used to describe change in any two variables in coupled differential equations by projecting orthogonal vectors
- identification of isoclines: a set of abundances for which the change in populations is 0:

$$\frac{dN}{dt} = 0$$





Isoclines

- species 1

$$r_1 N_1 (1 - [N_1 + \alpha_{12} N_2] / K_1) = 0$$
$$r_1 N_1 ([K_1 - N_1 - \alpha_{12} N_2] / K_1) = 0$$

trivial solution if $r_1, N_1, K_1 = 0$
and if $K_1 - N_1 - \alpha_{12} N_2 = 0$
then $N_1 = K_1 - \alpha_{12} N_2$

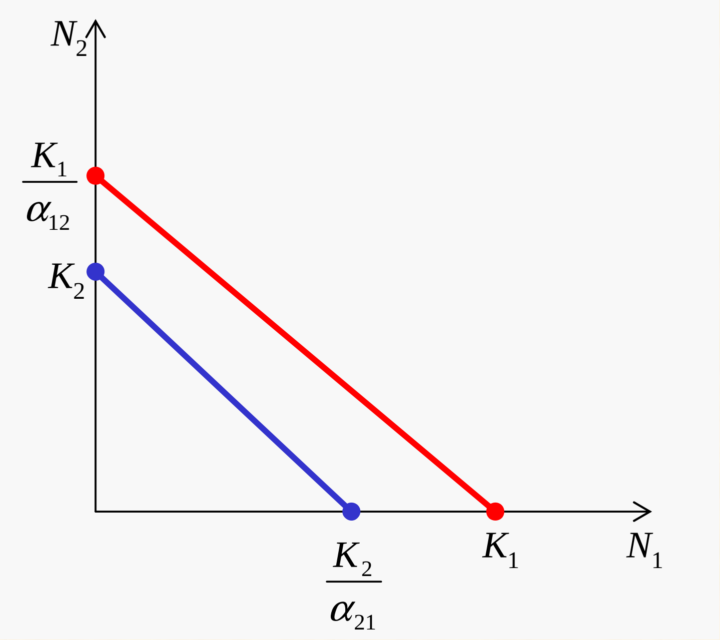
if $N_1 = 0$ then $N_2 = K_1/\alpha_{12}$
if $N_2 = 0$ then $N_1 = K_1$

- species 2

$$r_2 N_2 (1 - [N_2 + \alpha_{21} N_1] / K_2) = 0$$
$$N_2 = K_2 - \alpha_{21} N_1$$

trivial solution if $r_2, N_2, K_2 = 0$

if $N_2 = 0$ then $N_1 = K_2/\alpha_{21}$
if $N_1 = 0$ then $N_2 = K_2$



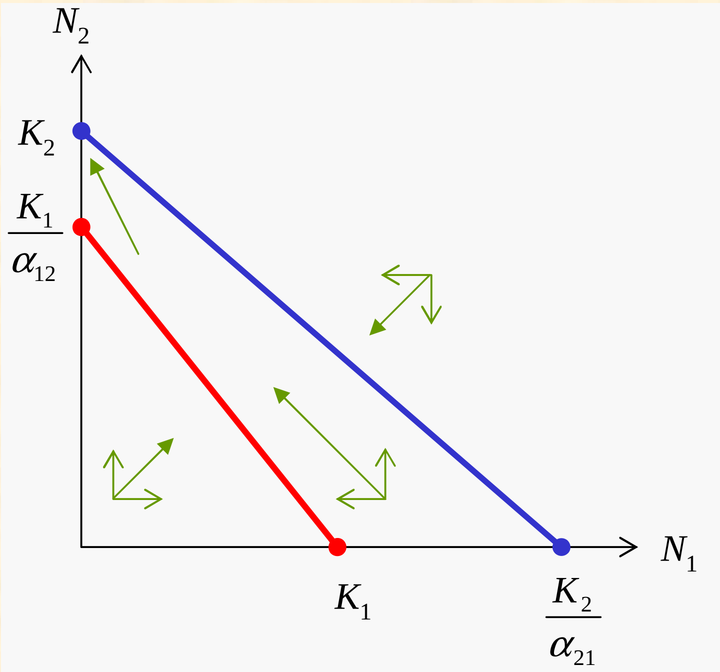
- above isocline i_1 and below i_2 competition is weak
- in-between i_1 and i_2 competition is strong

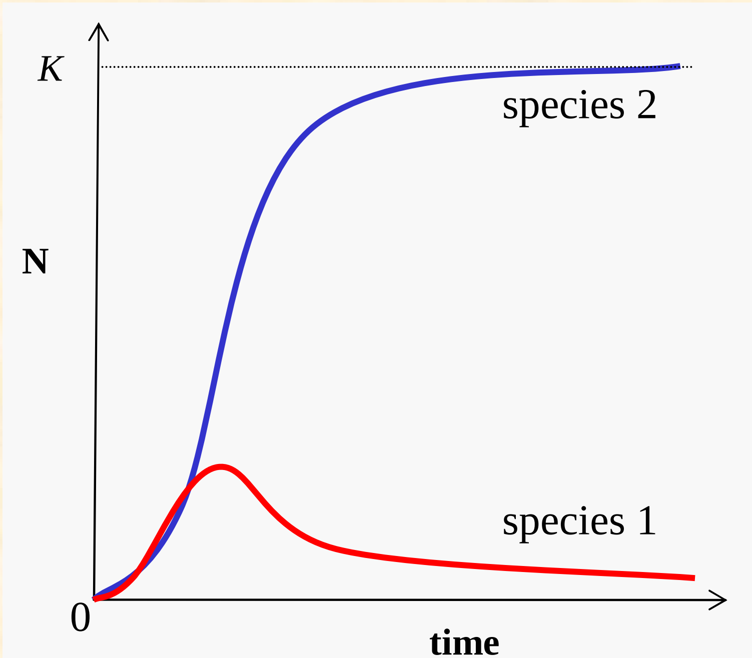
1. Species 2 drives species 1 to extinction

- K and α determine the model behaviour
- disregarding initial densities species 2 (stronger competitor) will outcompete species 1 (weaker competitor)
- equilibrium (0, K_2)

$$K_2 > \frac{K_1}{\alpha_{12}} \qquad K_1 < \frac{K_2}{\alpha_{21}}$$

$$K_1 = K_2 \qquad r_1 = r_2$$
$$\alpha_{12} > \alpha_{21} \qquad N_{01} = N_{02}$$



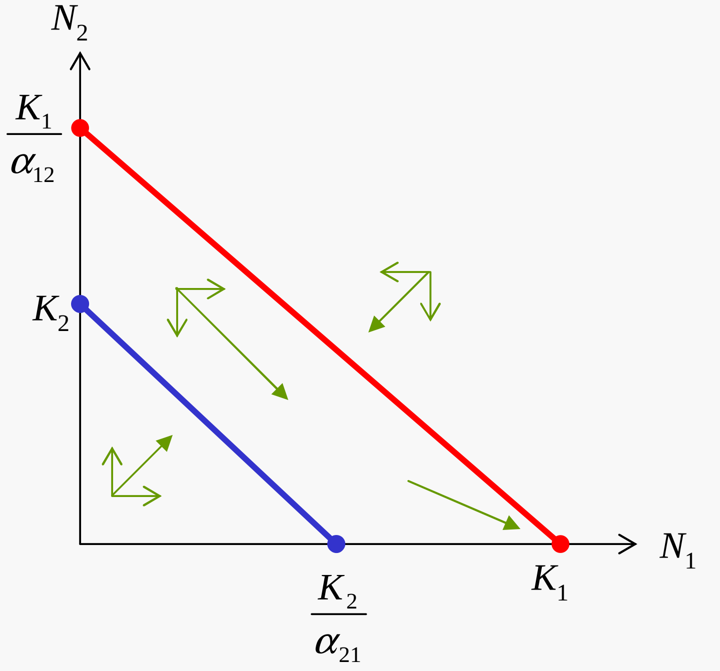


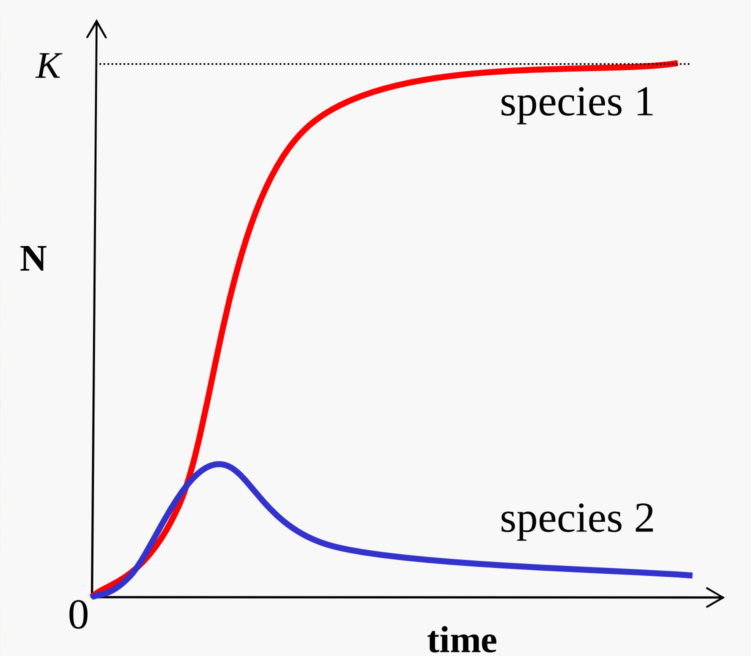
2. Species 1 drives species 2 to extinction

- species 1 (stronger competitor) will outcompete species 2 (weaker competitor)
- equilibrium (K_1, 0)

$$K_1 > \frac{K_2}{\alpha_{21}} \quad K_2 < \frac{K_1}{\alpha_{12}}$$

$$r_1 = r_2 \qquad K_1 = K_2$$
$$N_{01} = N_{02} \qquad \alpha_{12} < \alpha_{21}$$



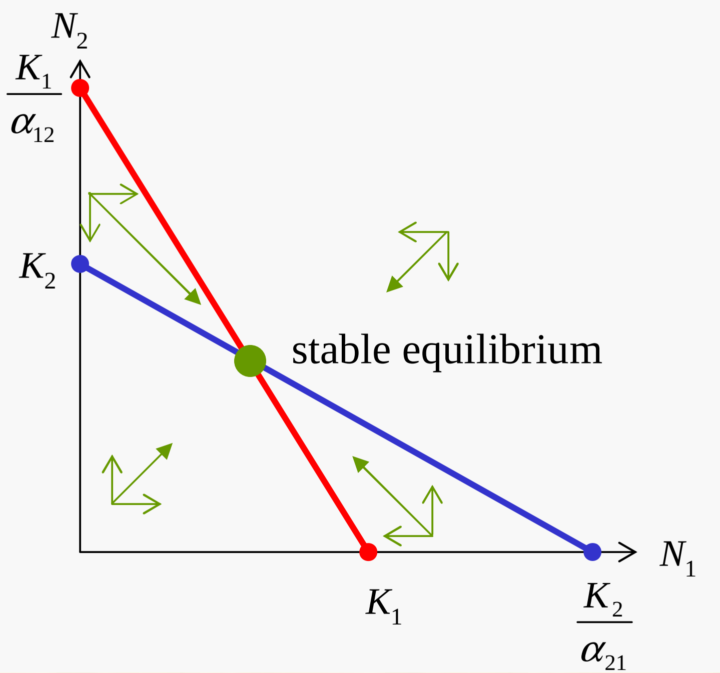


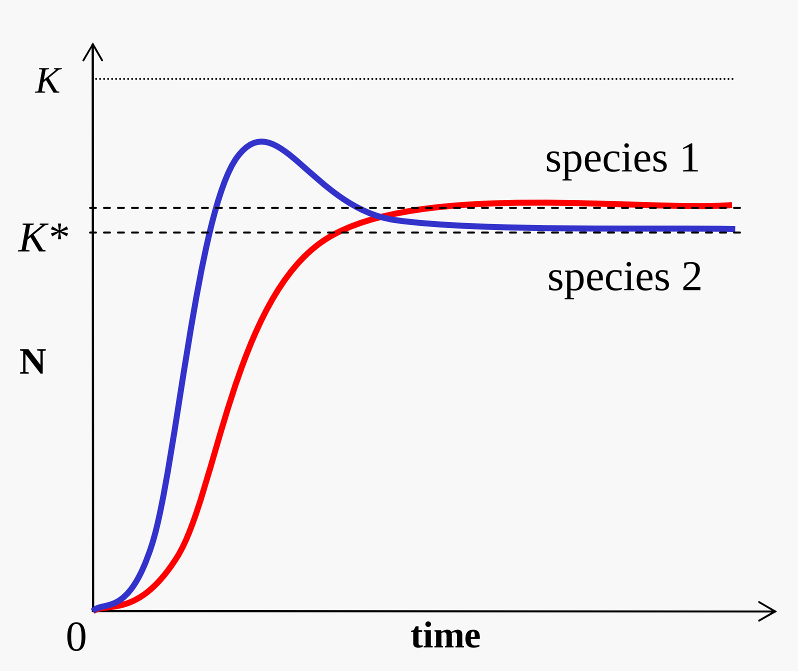
3. Stable coexistence of species

- disregarding initial densities both species will coexist at stable equilibrium (where isoclines cross)
- at at equilibrium population density of both species is reduced
- both species are weak competitors
- equilibrium (K_1*, K_2*)

$$K_1 < \frac{K_2}{\alpha_{21}} \qquad K_2 < \frac{K_1}{\alpha_{12}}$$

$$r_1 < r_2 \qquad K_1 = K_2$$
$$N_{01} = N_{02} \qquad \alpha_{12}, \alpha_{21} < 1$$

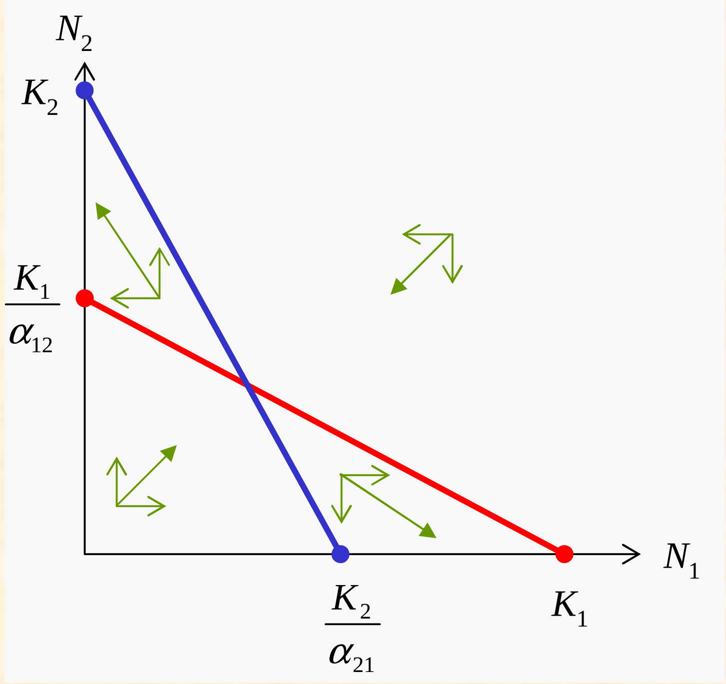


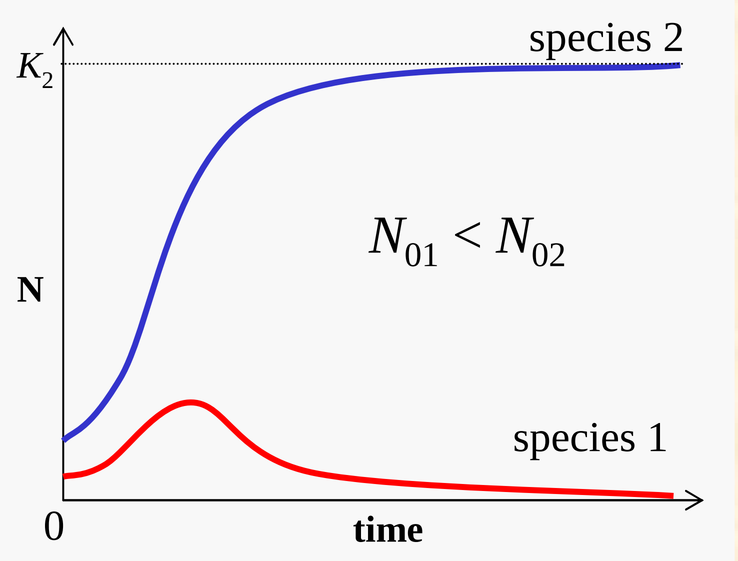


4. Competitive exclusion

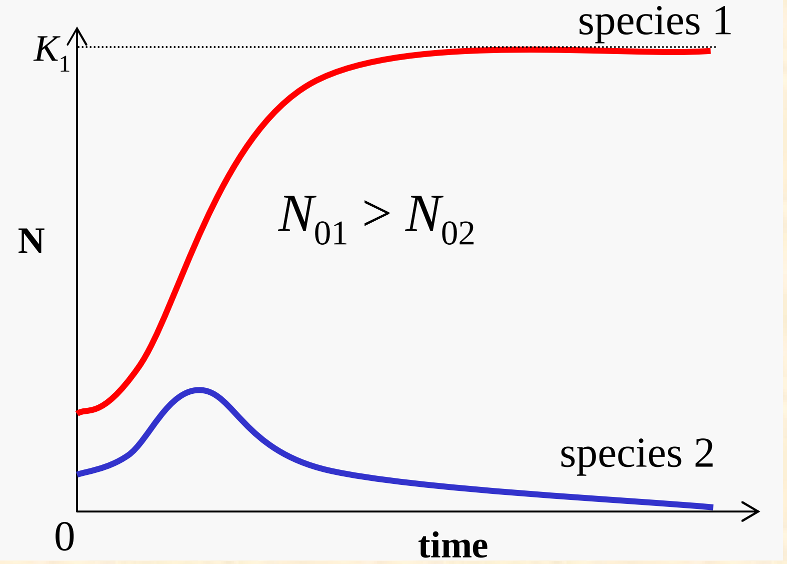
- one species will drive other to extinction depending on the initial conditions
- coexistence only for a short time
- both species are strong competitors
- equilibrium (K_1, 0) or (0, K_2)

$$K_1 > \frac{K_2}{\alpha_{21}} \qquad K_2 > \frac{K_1}{\alpha_{12}}$$





$r_1 = r_2$
$K_1 = K_2$ $\qquad \alpha_{12}, \alpha_{21} > 1$



Stability analysis

- Jacobian matrix of partial derivations

$$\mathbf{J} = \begin{pmatrix} \dfrac{\partial\, \mathrm{d}N_1/\mathrm{d}t}{\partial N_1} & \dfrac{\partial\, \mathrm{d}N_1/\mathrm{d}t}{\partial N_2} \\ \dfrac{\partial\, \mathrm{d}N_2/\mathrm{d}t}{\partial N_1} & \dfrac{\partial\, \mathrm{d}N_2/\mathrm{d}t}{\partial N_2} \end{pmatrix}$$

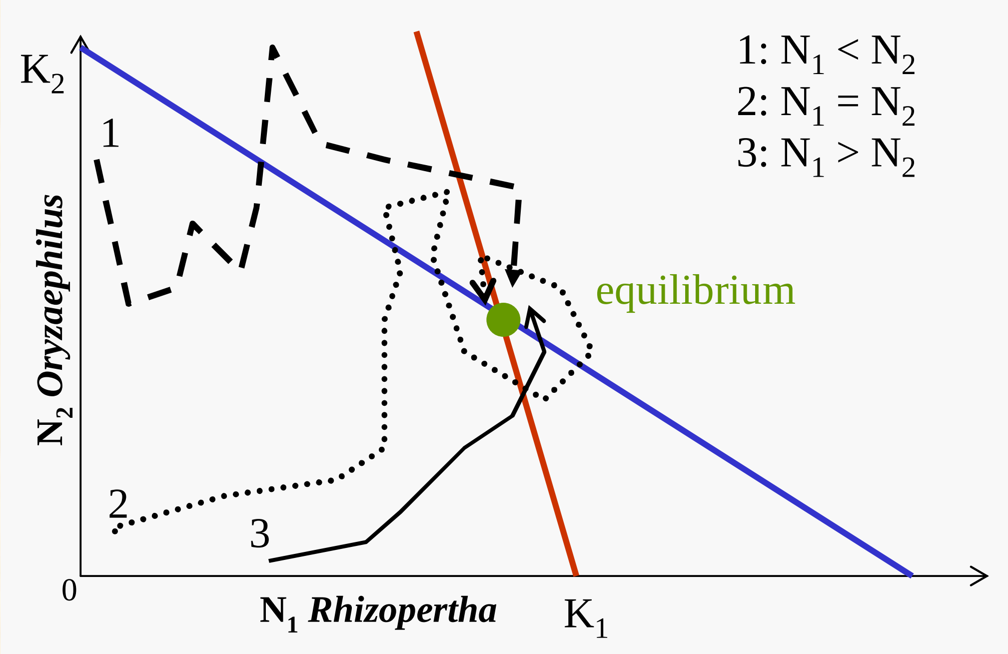
- evaluation of the derivations for densities close to equilibrium
- estimate eigenvalues of the matrix

- if all eigenvalues < 0 .. locally **stable**

- Lotka-Volterra system is stable for $\alpha_{12}\alpha_{21} < 1$

Test of the model

- when *Rhizopertha* and *Oryzaephilus* were reared separately both species increased to 420-450 individuals (= *K*)
- when reared together *Rhizopertha* reached K_1 = 360, while *Oryzaephilus* K_2 = 150 individuals
- combination resulted in more efficient conversion of grain (K_{12} = 510 individuals)
- three combinations of densities converged to the same stable equilibrium
- prediction of Lotka-Volterra model is correct



Crombie (1947)

System for discrete generations

- solution of the differential model – Ricker's model:

$$N_{1,t+1} = N_{1,t} e^{r_1 \left(\frac{K_1 - N_{1,t} - \alpha_{12} N_{2,t}}{K_1} \right)} \qquad N_{2,t+1} = N_{2,t} e^{r_2 \left(\frac{K_2 - N_{2,t} - \alpha_{21} N_{1,t}}{K_2} \right)}$$

- dynamic (multiple) regression is used to estimate parameters from a series of abundances

$$\ln\left(\frac{N_{1,t+1}}{N_{1,t}}\right) = r_1 - N_{1,t}\frac{r_1}{K_1} - N_{2,t}\frac{r_1 \alpha_{12}}{K_1}$$

$$\ln\left(\frac{N_{2,t+1}}{N_{2,t}}\right) = r_2 - N_{1,t}\frac{r_2}{K_2} - N_{1,t}\frac{r_2 \alpha_{21}}{K_2}$$

$$r = a \qquad \alpha = \frac{Kc}{r} \qquad K = \frac{r}{b}$$