Completing the Square

http://www.brightstorm.com/math/algebra-2/quadratic-equations-and-inequalities/solving-a-quadratic-by-completing-the-square/

What is a quadratic equation?

Read the text about Solving a quadratic equation and fill in the missing verbs.

happens completed subtract take add figure include solve make

keep change foiled isolate turn

Solving a quadratic equation by completing the square. So we already know how to use these square root properties to 1)......... a quadratic. So if we're ever given with an equation with something already squared, all we want to do is isolate that term, so in this case x-3 squared is equal to 5. 2)..... the square root of both sides. So we end up with x-3 is equal to plus or minus root 5. Remember whenever you use the square root as a tool we have to 3)......... plus or minus. Then solve for x we just to add 3 to both sides leaving us with x is equal to 3 plus or minus root 5, okay.

So the square root property is a really handy property when we have something squared, okay? The problem is that we don't always have something squared, okay? So we're going to go to another example where we are going to use this completing the square in order to get it in this form. Okay.

So what we want too do is to 4)....... this problem into something squared, okay? The first step that we want to do is 5)........ all our x terms together. So what we want to do then is subtract the 10 over x squared plus 8x is equal to -10. Okay. So I now want to turn this piece into something squared, okay? The x squared and the 8x are fixed. I can't change those. Okay? I left a little space at the end because we can add something to both sides and our problem doesn't 6)..............

So what we want to do is 7)......... out what we can add there in order to make a perfect square including this 8x, okay. So I know that this has to be a x and it has to be a plus. Okay? This middle term is positive so that tells us it has to be a positive sign. But what we want to do is somehow figure out what we can put here in order to get 8 and our middle term if we 8)........ it out, okay? And the trick for that is you take this middle term and divide it by 2. Okay? So in this case 8 divided by 2 is 4. That is what is going to go right here. Okay? And what 9)......... when we foil this out what we end up getting is x squared plus 8x plus 16.

So what I've really done is by including this 4 in here I've added 16 to my initial equation. So I've added 16 to this side, I also have to add 16 to this side to 10)..... it balanced. Whatever we do to one side we also have to do to the other. Okay? So what we actually have in this case then is x+4 quantity squared is equal to 6. Okay?

So this is what's called completing the squares, okay. We took our term, figured out what we needed to 11)....... to both sides to 12)....... it a perfect square. So that middle term divided by 2, that goes in here and then that new term squared is get what gets added in both sides and then we're able to rewrite this as a perfect square.

Once we get to this point we use the same exact method we did at the very beginning in order to solve this. Take the square root of both sides x+4 is equal to plus or minus the square root of 6. And then solve for x in this case 13)......... 4 so we end up with x is equal to -4 plus or minus the square root of 6. So I 14)....... the square what we're able to do take is take a quadratic equation that couldn't be solved by using the square root property, turn it into a perfect square so that you could use the square root property in order to solve it.

Unit 8 Process 3 Cause and Effect

Section 1 Presentation

1. Read this:

Quadratic equations

A quadratic expression is an expression which contains a number raised to the power of 2 (e.g. x^2). It cannot contain numbers raised to powers greater than 2 (e.g. x^3, x^4, etc.)

Say which of the following are quadratic expressions:

a) x^2+3
b) x^2+3x+7
c) $3x^2-2$
d) x^2+x^4
e) $x+2y+z$
f) $x^3+2x-16$

2. Read this:

A quadratic expression is generally given in the form ax^2+bx+c, where x is the variable and a, b and c are constants. A quadratic equation is generally given in the form $ax^2+bx+c=0$.

Now change the following equations to the general form for quadratic equations and give the values of a, b and c. The first two are done for you:

	Given	General form	a	b	c
a)	$2x^2-3x=2$	$2x^2-3x-2=0$	2	-3	-2
b)	$3x^2=-1$	$3x^2+0x+1=0$	3	0	1
c)	$x^2=3x$				
d)	$5x-3=4x^2$				
e)	$x^2+x=1$				
f)	$5x^2+7=20$				

3. Look at this:

A quadratic equation has two solutions, called roots. If the factors of a quadratic equation can be found easily, then we can find the roots by factorising.

Example: Factorisation of $x^2+x-12=0$ gives $(x-3)(x+4)=0$. The roots of the equation are therefore 3 and -4.

Now make similar sentences about the following:

a) $x^2+7x+10=0$
b) $x^2-9x+18=0$
c) $x^2-100=0$
d) $x^2+5x-6=0$

4. Read this:

Factorisation of $x^2+12x+36$ gives $(x+6)^2$. Therefore the expression is known as a perfect square.

x^2+ax can be made into a perfect square by adding $\left(\frac{a}{2}\right)^2$.

For example, x^2+20x can be made into a perfect square by adding 100.
$x^2+20x+100$ factorises into $(x+10)^2$.

Write similar sentences about the following expressions:

a) x^2-12x
b) x^2+3x
c) x^2+7x

(Note: This operation is known as *completing the square*).

Section 2 Development

5. Look and read:

If the factors of a quadratic equation cannot be found easily, then we can find the roots by using the formula

$$\frac{-b\pm\sqrt{(b^2-4ac)}}{2a}$$

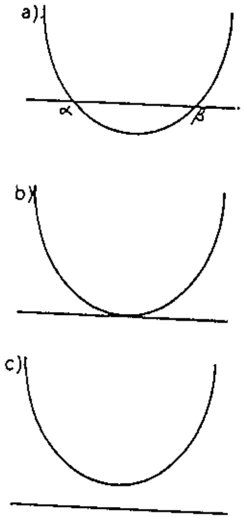
The two roots are at $\frac{-b+\sqrt{(b^2-4ac)}}{2a}$ and $\frac{-b-\sqrt{(b^2-4ac)}}{2a}$

If (b^2-4ac) is negative, then $\sqrt{(b^2-4ac)}$ is imaginary and no real roots satisfy the equation.

Complete these two sentences:
a) If (b^2-4ac) is positive, ...
b) If (b^2-4ac) is zero, ...

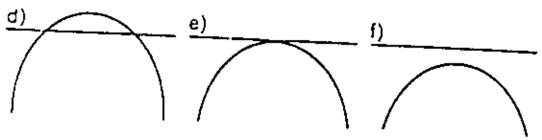
6. Look and read:

The two roots of a quadratic equation are denoted by α and β. We can show the six possible cases by drawing graphs.



- If (b^2-4ac) is positive and a is positive, then there are two real roots at α and β and the quadratic expression is only negative for values of x between α and β.
- If (b^2-4ac) is zero and a is positive, then the two real roots α and β *coincide*, and the quadratic expression is positive for all other values of x.
- If (b^2-4ac) is negative and a is positive, then there are no real roots, and the quadratic expression is always positive.

Now describe the other three cases:



7. **Look at these examples:**

$x^2-2x-3=0$ i) Factorisation of the left-hand side gives $(x-3)(x+1)=0$.
ii) Factorising the left-hand side gives $(x-3)(x+1)=0$.

Now change the following examples to form (ii):

a) $x^2-2x-3=0$ Addition of 4 to both sides gives a perfect square.
b) $\frac{25}{10}$ Reduction of the fraction gives $\frac{5}{2}$.
c) $9x=18y$ Division of both sides by 9 gives $x=2y$.
d) $x^2+10x+32$ Subtraction of 7 from this expression gives a perfect square.
e) $x^2-5x+6=0$ Solution of this equation gives roots at 2 and 3.
f) $a-\frac{a}{x^2}=0$ Multiplication of both sides by x^2 gives $ax^2-a=0$.

8. **Look at this example:**

$x^2-10x-200=0$.
Factorising, we obtain $(x+10)(x-20)=0$

Use expressions from this list to complete the calculation below.

completing the square,
dividing ......,
factorising,
subtracting ......,
subtracting ......,
taking the square root ......,

Given $ax^2+bx+c=0$

...... we obtain $x^2+\frac{b}{a}x+\frac{c}{a}=0$.

...... we obtain $x^2+\frac{b}{a}x=-\frac{c}{a}$

...... we obtain $x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}$

...... we obtain $\left(x+\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}$

...... we obtain $x+\frac{b}{2a}=\frac{\pm\sqrt{(b^2-4ac)}}{2a}$

...... we obtain $x=\frac{-b\pm\sqrt{(b^2-4ac)}}{2a}$

This gives the formula for finding the roots of a quadratic equation.

Section 3 Reading

9. **Look and read:**