Jeansova nestabilita

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\, \rho \vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{1}{\rho}\nabla p - \nabla \Phi$$

$$p = c_s^2 \rho$$

$$\nabla^2 \Phi = 4\pi G \rho$$

- výchozí stav: $\rho = \rho_0,\ p = p_0,\ \vec{v} = \vec{v}_0 = 0$
  $\Phi = \Phi_0$ JEANSŮV „ŠVINDL"
- lineární porucha: $\rho = \rho_0 + \rho_1$
  $p = p_0 + p_1$
  $\Phi = \Phi_0 + \Phi_1$
  $\vec{v} = \vec{v}_1$

Linearizace rovnic s poruchou:

$$\frac{\partial \varrho_1}{\partial t} + \varrho_0 \operatorname{div} \vec{v}_1 = 0 \qquad \Big/ \frac{\partial}{\partial t}$$

$$\frac{\partial \vec{v}_1}{\partial t} = -\frac{c_s^2}{\varrho_0} \nabla \varrho_1 - \nabla \Phi_1 \qquad / \operatorname{div}$$

$$\nabla^2 \Phi_1 = 4\pi G \varrho_1$$

$$\Rightarrow \boxed{\frac{\partial^2 \varrho_1}{\partial t^2} - c_s^2 \nabla^2 \varrho_1 - \underbrace{4\pi G \varrho_0}_{\equiv \omega_J^2} \varrho_1 = 0}$$

elementární řešení

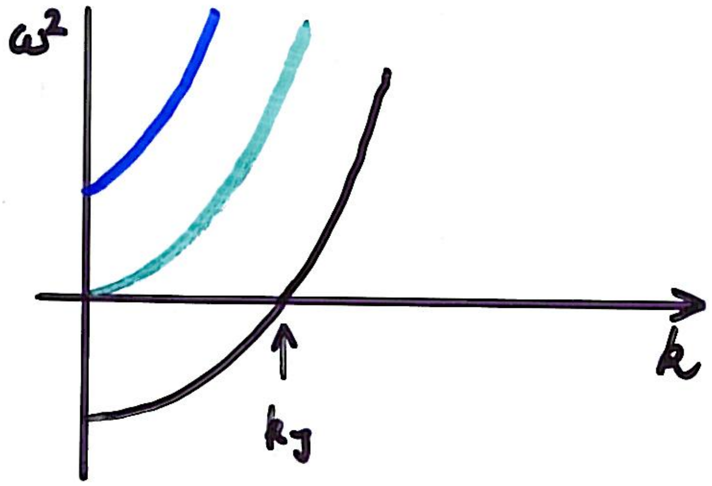
$$\varrho_1^0 = \hat{\varrho} e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

obecné řešení

$$\varrho_1 = \int_{-\infty}^{\infty} \hat{\varrho}(k) e^{i(\vec{k}\cdot\vec{r} - \omega t)} dk$$

DISPERZNÍ RELACE

$$\omega^2 = c_s^2 k^2 - \omega_J^2 = c_s^2 k^2 - 4\pi G \rho_0$$



$$k_J \equiv \frac{\omega_J}{c_s}$$

$\omega^2 = 0 \Leftrightarrow |k| = k_J$

NEUTRÁLNÍ STABILITA

$\omega^2 > 0$ STABILITA $\rho_1 \propto e^{\pm i|\omega|t}$

$\omega^2 < 0$ NESTABILITA $\rho_1 \propto e^{\pm \gamma t}$, $\gamma = i\omega$

Jeansova vlnová délka

$$\lambda_J = \frac{2\pi}{k_J} = \frac{\sqrt{\pi}\, c_s}{\sqrt{G\rho_0}}$$

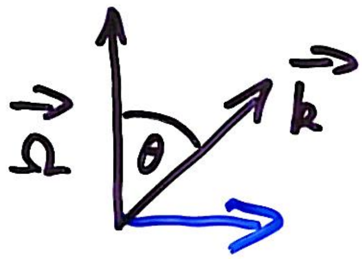
ELEKTROSTATICKÉ VLNY V PLAZMATU

$$\omega^2 = c_s^2 k^2 + \omega_p^2, \quad \omega_p^2 = \frac{4\pi n e^2}{m_e}$$

Nestabilita v rotujícím systému

$$\vec{\Omega} = \text{konst}$$

$$\varrho_0 \frac{\partial \vec{v}_1}{\partial t} = -\nabla p_1 - \varrho_0 \nabla \Phi_1 + 2\varrho_0 \vec{v}_1 \times \vec{\Omega}$$



$$\omega^4 - (\omega_1^2 + \omega_2^2)\omega^2 + \omega_1^2\omega_2^2 = 0,$$

kde $\omega_1^2 + \omega_2^2 = 4\Omega^2 + c_s^2 k^2 - 4\pi G \varrho_0$,

$$\omega_1^2\omega_2^2 = 4\Omega^2 (c_s^2 k^2 - 4\pi G \varrho_0)\cos^2\theta$$

$\cos\theta \neq 0$

$\Rightarrow$ Jeansovo kritérium zůstává v platnosti:

nestabilita pro

$$|k| < k_J = \frac{4\pi G \varrho_0}{c_s}$$

$\cos\theta = 0$

$$\omega^2 = 4\Omega^2 + c_s^2 k^2 - 4\pi G \varrho_0$$

ROTACE ZABRAŇUJE NESTABILITĚ PRO

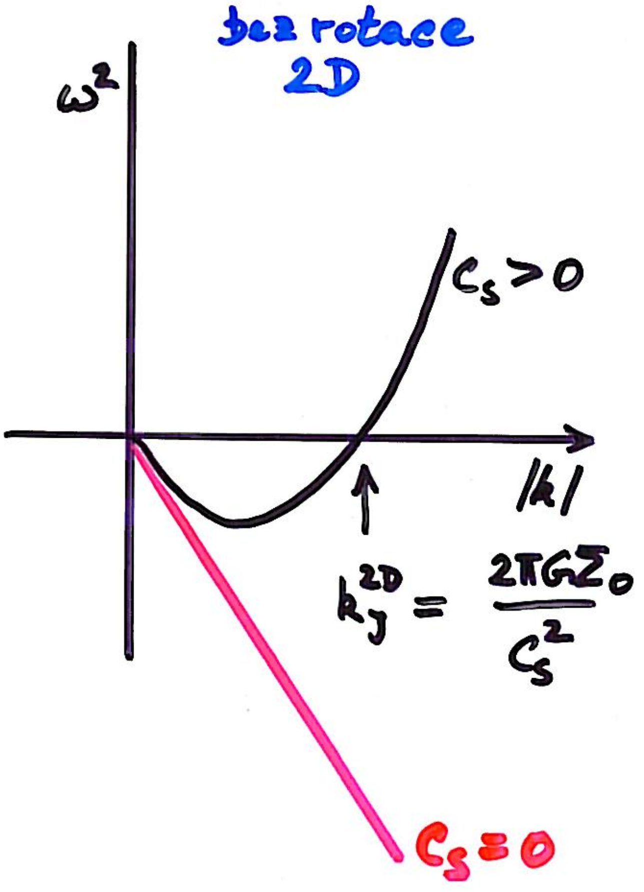
$$\Omega^2 > \pi G \varrho_0$$

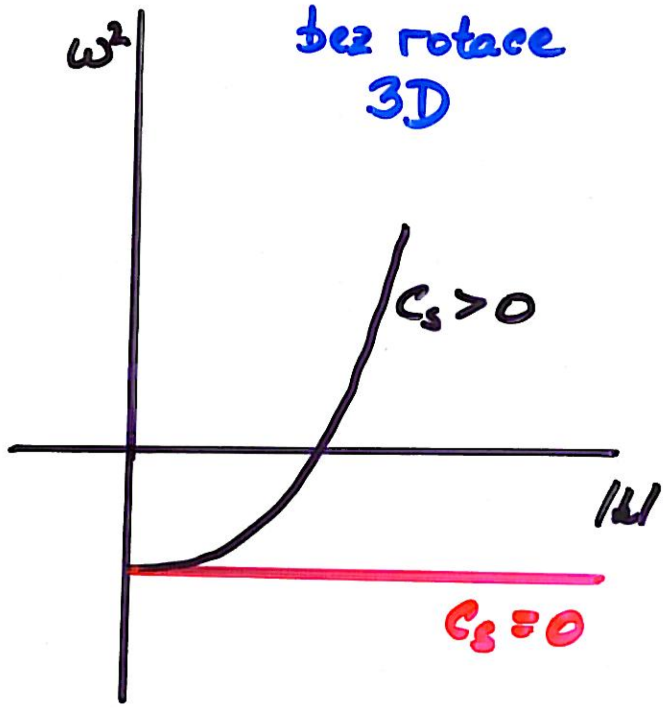
diferenciální rotace:

$$4\Omega^2 \to \kappa^2$$

Nestabilita ve 2-D systému

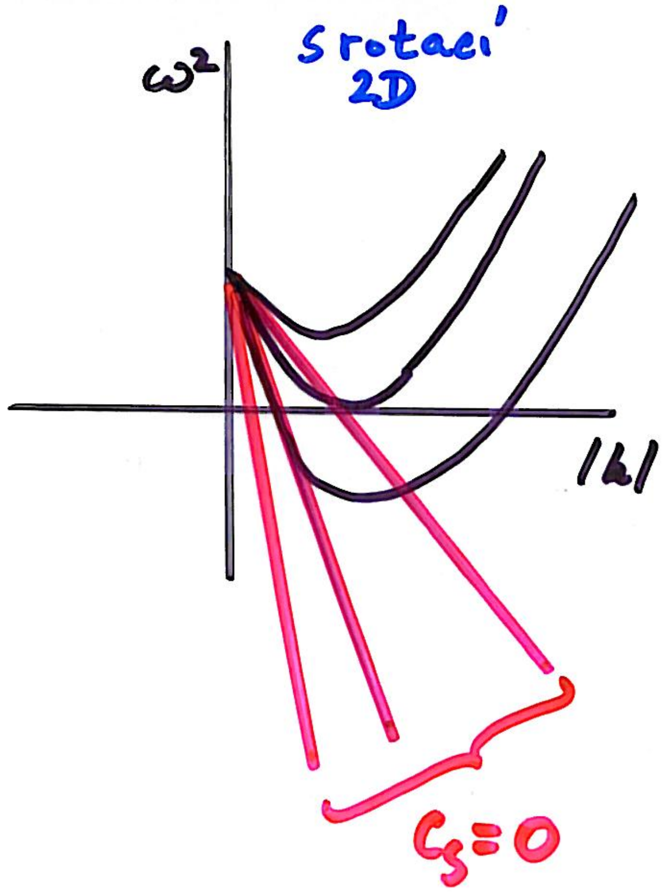
- bez rotace: $\omega^2 = k^2 c_s^2 - 2\pi G \Sigma_0 |k|$
- se stejnoměrnou rotací (Ω = konst): $\omega^2 = k^2 c_s^2 - 2\pi G \Sigma_0 |k| + 4\Omega^2$
- s diferenciální rotací: $\omega^2 = k^2 c_s^2 - 2\pi G \Sigma_0 |k| + \kappa^2$

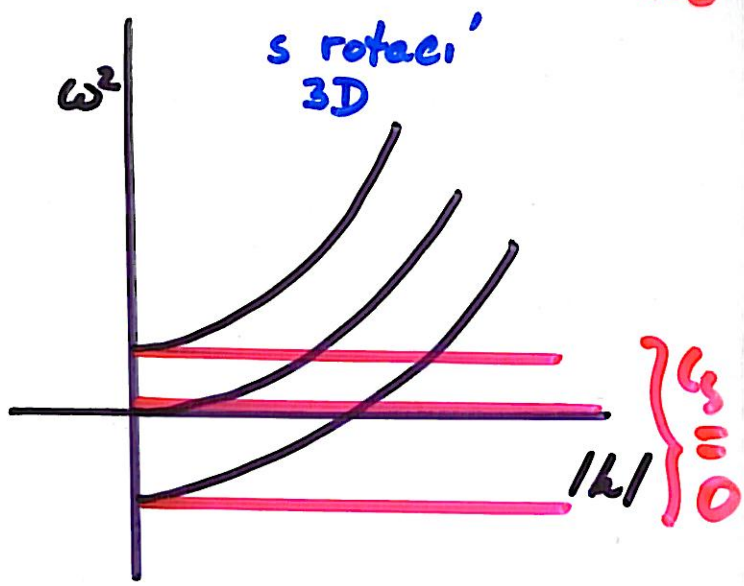




$c_s = 0$: prostředí bez tlaku $\Rightarrow \rho_1 \propto e^{\pm \gamma t}$,

kde $\gamma^2 = \begin{cases} 4\pi G \rho_0 & (3D) \\ 2\pi G \Sigma_0 |k| & (2D) \end{cases}$





⇒ ve 2D nelze self-gravitující systém stabilizovat pouhou rotací

- 2D systém bez tlaku, ale s rotací (Ω = konst) je nestabilní pro

$$|k| > \frac{2\Omega^2}{\pi G \Sigma_0}, \text{ tj. pro } \lambda < \frac{\pi^2 G \Sigma_0}{\Omega^2}$$

- TLAK + ROTACE mohou 2D systém stabilizovat na všech vlnových délkách

$$\omega^2 = 0 \quad \wedge \quad \frac{d\omega^2}{dk} = 0$$

$$\Rightarrow \boxed{\frac{2\Omega c_s}{\pi G \Sigma_0} = 1 \quad \text{Toomre (1964)}}$$

- s diferenciální rotací:

$$4\Omega^2 \Rightarrow \varkappa^2$$

$\Rightarrow$ • nestabilita v disku bez tlaku pro $\lambda < \lambda_{crit} = \frac{4\pi^2 G \Sigma_0}{\varkappa^2}$

- úplná stabilizace TLAKEM + ROTACÍ:

$$\boxed{Q \equiv \frac{\varkappa c_s}{\pi G \Sigma_0} = 1 \quad \text{Toomre (1964)}}$$

1-8

(1998)
from Binney & Tremaine

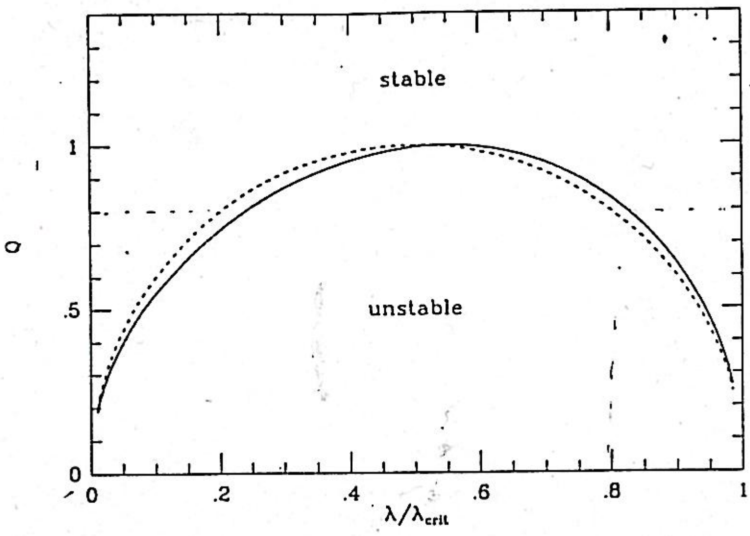


Figure 6-13. Neutral stability curves for short-wavelength axisymmetric perturbations in a gaseous disk [dashed line, from eq. (6-48)] and a stellar disk [solid line, from eq. (6-52)].