1. INTRODUCTION

Under normal conditions a gas is a good electrical insulator with a resistivity of some $10^{14}\Omega m$

. Development of a discharge means the transition of a gas to a conducting state with resistivity dependent on the particular conditions, for example, becoming typically 10^3 Ωm in glow discharges. Electrical discharge occurs when two conditions are satisfied. First, the applied voltage must equal or exceed a minimum value (the static discharge onset voltage V_s). Second, a free electron must be present in the gas. A free electron, accelerated by the electric field, produces more free electrons and positive ions in ionizing collisions with gas molecules. This process is described by the primary-ionization coefficient α, the average number of ionizing collisions made by one electron moving 1 cm along the direction of the electric field. An electron, departing from the cathode, creates an electron avalanche that grows exponentially to an average size $N_e = \exp(\alpha x)$ where x is the spacing of electrodes. New avalanches are begun when positive ions, photons, or excited neutrals created in previous avalanches reach the cathode and eject electrons. This process is described by the secondary-ionization coefficient γ, the average number of secondary electrons released from the cathode per positive ion produced in the gas. Although the secondary-ionization coefficient is defined relatively to the number of positive ions, its value includes the contributions of all secondary agents mentioned.

2. STATISTICS OF DISCHARGE ONSET

2.1 EFFECTIVE COEFFICIENT OF IONIZATION α AND STATISTICS OF SINGLE AVALANCHES

From a pragmatic point of view, the effective coefficient of ionization α can be defined in terms of the differential equation

$$d\,n(x) = \alpha(x).n(x).dx \qquad (1)$$

where x is a co-ordinate along a line of force in the electric field, along which electron avalanches develop. n(x) is a continuous, differentiable function which represents the actual number of electrons n participating in the collision processes. From a physical point of view it is clear than n itself can take only integer values.

As a consequence of this the definition (1) has no mathematical sense unless swarm conditions are fulfilled, i.e. n must be large enough such that the Law of Large Numbers (see for example J.S. Ventcelova:"Teoria pravdepodobnosti" STNL 1973) is operative. This ensures that, for a given value of n(x), proportionality exists between dn(x) and dx. When the Law of Large Numbers is inoperative, it is necessary to take into account the statistical nature of collision processes.

Since from a physical point of view α results from a statistical average of a number of collision processes, α cannot be related to the ionisation activity of an individual electron. In this respect, the generally accepted definition of α which is expressed in terms of ionizing collisions per primary electron per unit length in the direction of the field, is at variance with the mathematical definition (1). Thus the application of (1) to the development of an electron avalanche is restricted to the later stages of the growth. This results from the fact that initially n is so small that swarm conditions do not exits. An important consequence of this limitation is that the development of an avalanche starting at x=0 from a single electron will at first not be controlled by the equation (1), and thus the growth of the avalanche will not be exponential in character until after the avalanche has passed a finite distance.

According to the equation (1), if one electron is liberated from the cathode of a plane parallel gap to which a voltage is applied (i.e., under homogeneous field conditions) then the mean number of electrons which crosses any plane x is given by

$$N_e = \exp(\alpha .x)$$

,but because of the statistical nature of the collision processes, this mean value is subject to considerable statistical fluctuation. The average number of electrons n in an electron avalanche, initiated by a single electron at the cathode, reaching the plane x is given (see APENDIX 1):

$$\bar{n} = \frac{\sum_{n=1}^{\infty} n.p(n,x)}{\sum_{n=1}^{\infty} p(n,x)} = \exp(\alpha .x) \tag{1}$$

,where the probability distribution p for the production of an avalanche of n electrons at x is

$$p(n,x) = \frac{1}{\bar{n}}\left(1-\frac{1}{\bar{n}}\right)^{n-1} \approx \frac{1}{\bar{n}}.\exp\left(\frac{-n}{\bar{n}}\right) \qquad \text{if } n>>1 \tag{2}$$

2.2 STATISTICS OF A SEQUENCE OF AVALANCHES AND DISCHARGE ONSET

2.2.1 Townsend mechanism

Positive ions and photons, excited and metastable atoms generated during the development of avalanches can interact with the cathode and regenerate the initial electrons to set up a sequence of avalanches, but because of the statistical character of these processes there is the possibility that all these active particles may not liberate an electron. The succession of avalanches then fails to develop and the discharge is extinquished. Let Q be the probability of the interruption of a sequence of avalanches and G be the probability that the active particles, say positive ions, in the first avalanche generate m electrons at the cathode, m = 0,1,2...,etc.

If m = 0 there will be no succeeding avalanche and the probability of extinction is unity. For m = 1 the situation is the same as before when the first electron was generated , so the probability of extinction is Q again, and in general it is Q^m since the avalanches initiated by each secondary electron are mutually independent. Thus the probability of extinction of the discharge is

$$Q = G(1).Q + G(2).Q^2 + ... = \sum G(m).Q^m \qquad (3)$$

The probability G(m) may be deduced considering an avalanche containing n electrons at the anode and hence (n-1) positive ions which reach the cathode. If γ is the probability that an ion ejects an electron then the probability W(n-1,m) that an avalanche that produces n-1 positive ions results in the release of m secondary electrons at the cathode is given by the binomial distribution

$$W(n-1,m) = \binom{n-1}{m}.\gamma^m.(1-\gamma)^{n-1-m} \qquad (4)$$

Note: Binomial distribution

$$W(k,l) = \binom{k}{l}.y^l.(1-y)^{k-l}$$

express the probability W of achieving a number l of successes in k independent trials when the probability of success in a single trial is y.

G(m) depends jointly on W and the probability p of the development of an avalanche containing (n-1) ions , summed over all possible values of n, i.e.

$$G(m) = \sum_{n=m+1}^{\infty} p(n,x).W(n-1,m) = \frac{[\gamma(n-1)]^m}{[\gamma(n-1)+1\}]^{m+1}} =$$

$$= q^m / (q+1)^{m+1} \qquad (5)$$

where $q = \gamma(e^{\alpha.x} - 1)$

Thus

$$\mathbf{Q = \sum Q^m.q^m / (q+1)^{m+1} = \frac{1}{q + 1 - qQ}}$$

This equation gives

$$Q = 1 \text{ or } Q = 1/q \qquad (6)$$

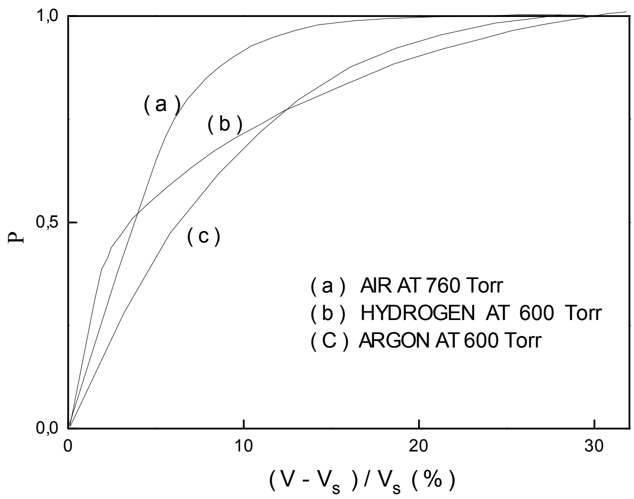
Thus the probability P = 1 - Q that the sequence is uninterrupted is

$$P = 0 \text{ for } q<1, \quad \text{and} \quad P = 1 - 1/q \text{ for } q>1 \qquad (7)$$

Now q=1 represents the condition for the onset of breakdown which is satisfied when V=Vs . If V<Vs , then q<1 and the sequence of avalanches is certainly interrupted. Even when V>Vs , so that q>1, P will be such that the discharge current will not flow indefinitely. P is appreciably less than unity for voltages which are considerably greater than Vs . Thus to ensure the establishment of an uninterrupted flow of discharge current initiated by a single electron a large overvoltage must be applied. This is shown in Figure 1 for the case of discharge between parallel aluminium electrodes set 0.3 mm apart in air, hydrogen , and argon at pressures of 100 kPa and 78 kPa. It can be seen that at least 25% overvoltage in needed before P approaches close to unity.

The probability of interruption decreases if more than one electron is used to initiate avalanches. Thus if N electrons are used simultaneously to initiate N avalanches the probability of an uninterrupted sequence is 1-(1-P) = 1-Q . This shows very clearly the need for a large number of initiatory electrons and a large overvoltage to ensure reliable and rapid discharge initiation.

FIG.1. Experimentally measured probability of an uninterrupted sequence of electron



avalanches initiated by a single electron as a function of overvoltage. (Taken from Grey Morgan,C. and Harcombe,D. (1953). Proc. Phys. Soc.,B66, 673.)

2.2.2. Statistics of the discharge onset according to streamer mechanism.

The Townsend mechanism successfully accounted for the dependence of the breakdown voltage on gas density and electrode spacing. Other experimental evidence, however, was obtained which appeared to be inconsistent with the Townsend mechanism. The time between application of voltage and electrical breakdown was measured in conditions where secondary action by photons could be neglected. In spark gaps at atmospheric pressure with electrode spacing `1 cm, very short delay times (< 1 μs) were obtained. Since positive ion transit times across the gap are 10^{-5} s, this time was too short to have involved a series of successive avalanches produced by ions arriving on the cathode. The streamer theory was developed to account for these observations.

The discharge onset in streamer theory can be defined as the occurrence of one avalanche that achieves the critical size $n_c = 10^8$. The probability can be calculated from the integral of the avalanche-size distribution

$$P = \int_{n_c}^{\infty} p(n).dn = \exp(-n_c / \bar{n}) \qquad (8)$$

The streamer mechanism neglects secondary electrons produced at the cathode. The Townsend mechanism neglects field distortion by space charge. The Townsend mechanism is valid for $n \ll n_c$ and the streamer mechanism for $n \geq n_c$. In the transition region between these two mechanisms the breakdown is due to a buil-dup of space charge in a sequence of avalanches linked by the cathode secondary emission processes in which the total number of positive ions produced achieves a critical value.