The criterion for streamer development is determined by finding the level of ionization at which significant field distortion from space charge in the gap arises. Taking all the electrons to be in a spherical volume, with radius r, at the tip of the avalanche, the field is

$$E_r = \frac{\mathbf{e} e^{\alpha x}}{4\pi\varepsilon_0 r^2}$$

where e is the electronic charge magnitude. The radius can be approximated by the diffusion radius

$$r \approx \sqrt{3Dt}$$

where D is the diffusion coefficient. The transit time of the electrons can be found from the avalanche length x, and the drift velocity v_d or the electron mobility ke

$$t = \frac{x}{v_d} = \frac{x}{k_e E}$$

$$r = \sqrt{\frac{3Dx}{k_e E}}$$

$$E_r = \frac{\mathbf{e} e^{\alpha x}}{4\pi\varepsilon_0 \left[3Dx/\left(k_e E\right)\right]}$$

The diffusion coefficient is found from

$$\frac{D}{k_e} = \frac{\kappa T_e}{\mathbf{e}}$$

where Te is the electron temperature and κ is Boltzmann's constant. Using

$$\frac{3}{2}\kappa T_e = \mathbf{e}V$$

where V is the electron energy in volts, we have

$$D = \frac{2}{3} V k_e$$

$$r \approx \sqrt{2Vx/E}$$

$$E_r = \frac{\mathbf{e} e^{\alpha x}}{4\pi\varepsilon_0 \left(2Vx\right)} E$$

Now let x_c designate the distance at which Er becomes comparable with E. This yields Raether's avalanche-streamer criterion

$$\alpha x_c = 17.7 + \ln x_c$$

where x_c is in centimeters. In the uniform field the occurrence of a critical avalanche usually develops into a spark. Thus we can substitute $x_c = d$

$$\alpha d = \frac{\alpha}{p} p d = 17.7 + \ln\left(d/1 \text{ cm}\right)$$