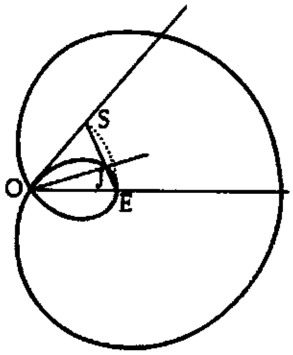
WildTrig29: Trisecting angles and Hadley's theorem

http://www.youtube.com/watch?v=eAmPFKU1rvk&feature=related



Listen to the video and answer Qs.

1) Which equipment does the speaker use to trisect a segment?............................
2) How can you divide a segment into three pieces? Describe the process.

   ....

3) Using the Euclidean construct, how many parts can you divide an angle into?...
4) How many trisectors are there between line 1 and line 2?..........................
5) What is a spread?..
6) What does Q stand for? ....
7) What is the proof based on?...
8) Who is Frank Hadley? ....

Read these mathematical expressions:

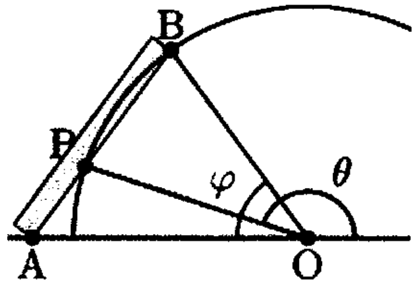
$$\sin^2(n\theta) = S_n(\sin^2\theta).$$

$$S_n(s) = s\sum_{k=0}^{n-1}\frac{n}{n-k}\binom{2n-1-k}{k}(-4s)^{n-1-k}.$$

$$\int_0^1\left(S_n(s)-\frac{1}{2}\right)\left(S_m(s)-\frac{1}{2}\right)\frac{ds}{\sqrt{s(1-s)}} = 0.$$

$$\sum_{n=1}^{\infty}\frac{S_n(s)}{n!}x^n = \frac{1}{2}e^x\left[1-e^{-2sx}\cos\left(2x\sqrt{s(1-s)}\right)\right].$$

Trisecting an angle Article by: *J J O'Connor* and *E F Robertson*



Pre-reading 1) Apart from angle trisection, there were two more famous problems of the Greek geometry – squaring the circle and doubling the cube. What do you know about these problems?

2) Do you know how to bisect angles? Describe it.

3) Try to explain the following terms:

trisector amateur mathematician radius equilateral triangle

compass regular polygon arbitrary angle

Reading. 1) Read the text and correct these wrong statements.

a) Angle trisection was the most important problem in ancient Greek times.

b) During his career, the author created many false proofs concerning the problem of trisecting an arbitrary angle.

c) When we know that a proof is wrong, we can easily find out why.

d) Trisecting an angle is special because it has never been studied in history.

There are three classical problems in Greek mathematics which were extremely influential in the development of geometry. These problems were those of squaring the circle, doubling the cube and trisecting an angle. The present article studies the problem of trisecting an arbitrary angle. In some sense this is the least famous of the three problems. Certainly in ancient Greek times doubling of the cube was the most famous, then in more modern times the problem of squaring the circle became the more famous, especially among amateur mathematicians.

The problem of trisecting an arbitrary angle, which we examine here, is the one for which I have been sent the largest number of false proofs during my career. It is an easy task to tell that a 'proof' one has been sent 'showing' that the trisector of an arbitrary angle can be constructed using ruler and compasses must be incorrect since no such construction is possible. Of course knowing that a proof is incorrect and finding the error in it are two different matters and often the error is subtle and hard to find.

There are a number of ways in which the problem of trisecting an angle differs from the other two classical Greek problems. Firstly it has no real history relating to the way that the problem first came to be studied. Secondly it is a problem of a rather different type. One cannot square any circle, nor can one double any cube. However, it is possible to trisect certain angles.

2) Read the second part and try to draw two constructions following the instructions.

a) For example there is a fairly straightforward method to trisect a right angle. For given the right angle *CAB* draw a circle to cut *AB* at *E*. Draw a second circle (with the same radius) with centre *E* and let it intersect the first circle at *D*. Then *DAE* is an equilateral triangle and so the angle *DAE* is 60° and *DAC* is 30°. So the angle *CAB* is trisected.

b) Although it is difficult to give an accurate date as to when the problem of trisecting an angle first appeared, we do know that Hippocrates, who made the first major contribution to the problems of squaring a circle and doubling a cube, also studied the problem of trisecting an angle. There is a fairly straightforward way to trisect any angle which was known to Hippocrates. It works as follows. Given an angle *CAB* then draw *CD* perpendicular to *AB* to cut it at *D*. Complete the rectangle *CDAF*. Extend *FC* to *E* and let *AE* be drawn to cut *CD* at *H*. Have the point *E* chosen so that $HE = 2AC$. Now angle *EAB* is $^1/_3$ of angle *CAB*. To see this let *G* be the midpoint of *HE* so that $HG = GE = AC$. Since *ECH* is a right angle, $CG = HG = GE$. Now angle EAB = angle CEA = angle ECG. Also since $AC = CG$ we have angle CAG = angle CGA. But angle CGA = angle GEC + angle $ECG = 2 \times CEG = 2 \times EAB$ as required.

The proof of the impossibility had to await the mathematics of the 19th century. The final pieces of the argument were put together by Pierre Wantzel. In 1837 Wantzel published proofs in *Liouville's Journal.* Gauss had stated that the problems of doubling a cube and trisecting an angle could not be solved with ruler and compasses but he gave no proofs. In this 1837 paper Wantzel was the first to prove these results.

3) Do you think there are other means to trisect angles by going outside the Greek framework?