$$\lim_{x\to\frac{\pi}{4}} \operatorname{tg} 2x \ \operatorname{tg}\left(\frac{\pi}{4}-x\right) = \lim_{x\to\frac{\pi}{4}} \frac{\operatorname{tg} 2x}{\frac{1}{\operatorname{tg}\left(\frac{\pi}{4}-x\right)}} \overset{L'H}{=} \lim_{x\to\frac{\pi}{4}} \frac{\frac{1}{\cos^2 2x}\cdot 2}{-\frac{1}{\operatorname{tg}^2\left(\frac{\pi}{4}-x\right)}\cdot\frac{1}{\cos^2\left(\frac{\pi}{4}-x\right)}(-1)} = \lim_{x\to\frac{\pi}{4}} \frac{2\operatorname{tg}^2\left(\frac{\pi}{4}-x\right)\cos^2\left(\frac{\pi}{4}-x\right)}{\cos^2 2x} =$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{2\frac{\sin^2\left(\frac{\pi}{4}-x\right)}{\cos^2\left(\frac{\pi}{4}-x\right)}\cos^2\left(\frac{\pi}{4}-x\right)}{\cos^2 2x} = \lim_{x\to\frac{\pi}{4}} \frac{2\sin^2\left(\frac{\pi}{4}-x\right)}{\cos^2 2x} \overset{L'H}{=}$$

$$\overset{L'H}{=} \lim_{x\to\frac{\pi}{4}} \frac{2\cdot 2\sin\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-x\right)}{2\cdot 2\cos 2x\sin 2x} = \lim_{x\to\frac{\pi}{4}} \frac{\sin\left(\frac{\pi}{2}-2x\right)}{\sin 4x} \overset{L'H}{=} \lim_{x\to\frac{\pi}{4}} \frac{\cos\left(\frac{\pi}{2}-2x\right)(-2)}{\cos(4x)(4)} =$$

$$= \frac{-2}{-4} = \frac{1}{2}$$