Ze str. 8, příklad 4: Zjednodušte výraz

$$\left(\log_a b + \log_b a + 2\right)\left(\log_a b - \log_{ab} b\right)\log_b a - 1.$$

Řešení:

$$\begin{aligned}
\left(\log_a b + \log_b a + 2\right)\left(\log_a b - \log_{ab} b\right)\log_b a - 1 &= \left(\frac{\log_b b}{\log_b a} + \log_b a + 2\right)\left(\frac{\log_b b}{\log_b a} - \frac{\log_b b}{\log_b ab}\right)\log_b a - 1 = \\
&= \left(\frac{1}{\log_b a} + \log_b a + 2\right)\left(\frac{1}{\log_b a} - \frac{1}{\log_b a + \log_b b}\right)\log_b a - 1 = \\
&= \left(\frac{\log_b a}{\log_b a} + (\log_b a)^2 + 2\log_b a\right)\left(\frac{1}{\log_b a} - \frac{1}{\log_b a + 1}\right) - 1 = \\
&= \left(1 + (\log_b a)^2 + 2\log_b a\right)\left(\frac{\log_b a + 1 - \log_b a}{\log_b a(\log_b a + 1)}\right) - 1 = \\
&= \left(1 + \log_b a\right)^2\left(\frac{1}{\log_b a(\log_b a + 1)}\right) - 1 = \\
&= \frac{\left(1 + \log_b a\right)^2}{\log_b a(\log_b a + 1)} - 1 = \frac{1 + \log_b a}{\log_b a} - 1 = \frac{1}{\log_b a} + 1 - 1 = \\
&= \underline{\underline{\log_a b}}
\end{aligned}$$