EXERCISES IN CATEGORY THEORY 2

1. Products

Let $\mathcal{K}$ be a category with products.

(1) Suppose that $\mathcal{K}$ admits a terminal object 1. Show that there are isomorphisms $A \times 1 \cong A$ and $1 \times A \cong A$.
(2) Find an isomorphism $A \times B \cong B \cong A$.
(3) Define the product $A \times B \times C$ of three objects A, B and C using a universal property and show that it is unique up to isomorphism.
(4) Given objects A, B and C find an isomorphism $(A \times B) \times C \cong A \times (B \times C)$. Show that these are isomorphic (one way is to show that both have the universal product of $A \times B \times C$.)
(5) Given $f : A_1 \to A_2$ and $g : B_1 \to B_2$ find a map $f \times g : A_1 \times B_1 \to A_2 \times B_2$. *In the category of sets this is the map sending the ordered pair (a, b) to (fa, gb).*
(6) Show that in any category with products and coproducts there exists a canonical map

$$(A \times B) + (A \times C) \to A \times (B + C)$$

(To construct this, use maps of the form $f \times g$ as constructed in the previous question.)
(7) Show that, in the category of sets, the above map is an isomorphism.

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