GONIOMETRICKÉ ROVNICE

(Seminář z matematiky I - M1130/02 2015)

(1) Řešte v $\mathbb{R}$ rovnice:

(a) $2\sin^2 x - \sin x = 0$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{k\pi,\, \frac{\pi}{6} + 2k\pi,\, \frac{5}{6}\pi + 2k\pi\right\}\right]$

(b) $\sin^2 x + 2\sin x - 3 = 0$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{\pi}{2} + 2k\pi\right\}\right]$

(c) $\cos x\,(2\cos x + 1) = 1$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\pi + 2k\pi,\, \frac{\pi}{3} + 2k\pi,\, \frac{5}{3}\pi + 2k\pi\right\}\right]$

(d) $\sqrt{3}\,\mathrm{tg}^2 x + 2\,\mathrm{tg}\, x - \sqrt{3} = 0$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{\pi}{6} + k\pi,\, \frac{2}{3}\pi + k\pi\right\}\right]$

(e) $3\sin x = 2\cos^2 x$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{\pi}{6} + 2k\pi,\, \frac{5}{6}\pi + 2k\pi\right\}\right]$

(f) $\sin x + \cos^2 x = \frac{1}{4}$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{7}{6}\pi + 2k\pi,\, \frac{11}{6}\pi + 2k\pi\right\}\right]$

(g) $3\cos^2 x - 4\cos x - \sin^2 x - 2 = 0$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{2}{3}\pi + 2k\pi,\, \frac{4}{3}\pi + 2k\pi\right\}\right]$

(h) $\cos 2x + \sin x = 0$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{\pi}{2} + 2k\pi,\, \frac{7}{6}\pi + 2k\pi,\, \frac{11}{6}\pi + 2k\pi\right\}\right]$

(i) $\sin x = \cos x$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{\pi}{4} + k\pi\right\}\right]$

(j) $\sin x + \sqrt{3}\cos x = 0$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{2}{3}\pi + k\pi\right\}\right]$

(k) $\frac{\sin x}{\sqrt{3}} + \cos x = 1$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{2k\pi,\, \frac{\pi}{3} + 2k\pi\right\}\right]$

(l) $\sqrt{3}\sin x + \cos x = \sqrt{3}$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{\pi}{6} + 2k\pi,\, \frac{\pi}{2} + 2k\pi\right\}\right]$

(2) Řešte v $\mathbb{R}$ následující rovnice (nezapomeňte stanovit podmínky, je-li to třeba):

(a) $\sin 2x + \cos 2x = 1 + \mathrm{tg}\, x$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{k\pi,\, \frac{\pi}{8} + k\frac{\pi}{2}\right\}\right]$

(b) $\sin\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{3}\right) = 1 + \cos 2x$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{\pi}{2} + k\pi,\, \frac{\pi}{3} + 2k\pi,\, \frac{5}{3}\pi + 2k\pi\right\}\right]$

(c) $\sin^2 2x - \cos^2 2x = \cos 2x$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{\pi}{6} + k\pi,\, \frac{5}{6}\pi + k\pi,\, \frac{\pi}{2} + k\pi\right\}\right]$

(d) $\cos x + \sin x = \dfrac{\cos 2x}{1 - \sin 2x}$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{3}{4}\pi + k\pi,\ 2k\pi,\ \frac{3}{2}\pi + 2k\pi\right\}\right]$

(e) $\cos^4 x - \sin^4 x = \sin 2x$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{\pi}{8} + k\frac{\pi}{2}\right\}\right]$

(f) $\dfrac{\sin x + \sin 3x}{\cos x - \cos 3x} = \sqrt{3}$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{\pi}{6} + k\pi\right\}\right]$

(g) $\cos x \cos 2x = \cos 4x \cos 5x$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{k\frac{\pi}{6}\right\}\right]$

(h) $\sin 3x = 2\sin x$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{k\pi,\ \frac{\pi}{6} + k\pi,\ \frac{5}{6}\pi + k\pi\right\}\right]$

(i) $\mathrm{tg}\, x - \sin x + \cos x = 1$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{2k\pi,\ \frac{\pi}{4} + k\pi\right\}\right]$

(j) $2\sin^2 x + \cos x = 2\sin^2 x \cos x + 1$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{\pi}{4} + k\frac{\pi}{2},\ 2k\pi\right\}\right]$

(k) $\dfrac{\sqrt{3} + \mathrm{tg}\, x}{1 - \sqrt{3}\,\mathrm{tg}\, x} = 1$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{11}{12}\pi + k\pi\right\}\right]$

(l) $\sin^2 2x + \sin^2 4x = \dfrac{3}{2}$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{\pi}{6} + k\frac{\pi}{2},\ \frac{\pi}{3} + k\frac{\pi}{2},\ \frac{\pi}{8} + k\frac{\pi}{4}\right\}\right]$

(m) $|\sin x| = \sin x + 2$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{3}{2}\pi + 2k\pi\right\}\right]$

(n) $|\mathrm{tg}\, x + \mathrm{cotg}\, x| = \dfrac{4}{\sqrt{3}}$ $\left[K = \bigcup_{k\in\mathbb{Z}} \left\{\frac{\pi}{6} + k\pi,\ \frac{\pi}{3} + k\pi,\ \frac{2}{3}\pi + k\pi,\ \frac{5}{6}\pi + k\pi\right\}\right]$