TRANSFORMACE LAPLACEHO OPERÁTORU Z KARTÉZSKÝCH DO SFÉRICKÝCH SOUŘADNIC

$$(x,y,z)\rightarrow(r,\theta,\varphi)$$

$$\Delta=\nabla^2=\vec{\nabla}.\vec{\nabla}=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

$$\Delta=\nabla^2=\vec{\nabla}.\vec{\nabla}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

$$x=r\sin\theta\cos\varphi;\;\; y=r\sin\theta\sin\varphi;\;\; z=r\cos\theta;$$

$$r=\sqrt{x^2+y^2+z^2}\;\;\text{nebo}\; r^2=x^2+y^2+z^2;$$

$$\theta=\begin{cases}\arctan(y/x) & \text{pro} \quad x>0;\\ \pi/2 & \text{pro} \quad x=0;\, y\geq 0;\\ \pi-\arctan(y/-x) & \text{pro} \quad x<0;\\ 3\pi/2 & \text{pro} \quad x=0;\, y\leq 0;\end{cases}\quad \text{nebo}\;\sin\theta=\frac{\sqrt{x^2+y^2}}{r};$$

$$\varphi=\arccos(z/r)\;\;\text{nebo}\;\tan\varphi=y/x;$$

$$\left(\frac{\partial}{\partial x}\right)=\left(\frac{\partial r}{\partial x}\right)\left(\frac{\partial}{\partial r}\right)+\left(\frac{\partial\theta}{\partial x}\right)\left(\frac{\partial}{\partial\theta}\right)+\left(\frac{\partial\varphi}{\partial x}\right)\left(\frac{\partial}{\partial\varphi}\right);$$

$$\left(\frac{\partial}{\partial y}\right)=\left(\frac{\partial r}{\partial y}\right)\left(\frac{\partial}{\partial r}\right)+\left(\frac{\partial\theta}{\partial y}\right)\left(\frac{\partial}{\partial\theta}\right)+\left(\frac{\partial\varphi}{\partial y}\right)\left(\frac{\partial}{\partial\varphi}\right);$$

$$\left(\frac{\partial}{\partial z}\right)=\left(\frac{\partial r}{\partial z}\right)\left(\frac{\partial}{\partial r}\right)+\left(\frac{\partial\theta}{\partial z}\right)\left(\frac{\partial}{\partial\theta}\right)+\left(\frac{\partial\varphi}{\partial z}\right)\left(\frac{\partial}{\partial\varphi}\right);$$

$$\frac{\partial^2}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\right);\;\;\frac{\partial^2}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\right);\;\;\frac{\partial^2}{\partial z^2}=\frac{\partial}{\partial z}\left(\frac{\partial}{\partial z}\right);$$

http://skisickness.com/2009/11/20/