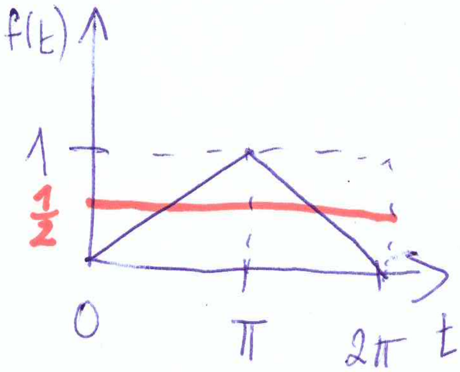


$[0,0]$ $[\pi,1]$ $[2\pi,0]$

1

$T = 2\pi \Rightarrow \omega = \frac{2\pi}{T} = 1$

$$f(t) = \begin{cases} \frac{1}{\pi}t & t \in \langle 0 + k2\pi, \pi + k2\pi) \quad k \in \mathbb{Z} \\ 2 - \frac{1}{\pi}t & t \in \langle \pi + 2k\pi, 2\pi + 2k\pi) \end{cases}$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

$f(t) = At + B$

$$\left.\begin{aligned} 0 &= A \cdot 0 + B \\ 1 &= A\pi + B \end{aligned}\right\} \begin{aligned} B &= 0 \\ A &= \frac{1}{\pi} \end{aligned}$$

$f(t) = \frac{1}{\pi}t$

$f(t) = At + B$

$$\left.\begin{aligned} 1 &= A \cdot \pi + B \quad /(-2) \\ 0 &= A2\pi + B \end{aligned}\right\} f(t) = 2 - \frac{1}{\pi}t$$

$1 = -A\pi$

$-2 = -B$

Předpoklady: • funkce je sudá, nenulové budou koeficienty u cos
• a_0 bude nenulové

výpočet:

$$a_0 = \frac{2}{T}\int_0^T f(t)\,dt = \frac{2}{2\pi}\left[\int_0^{\pi} \frac{1}{\pi}t\,dt + \int_{\pi}^{2\pi}\left(2 - \frac{1}{\pi}t\right)dt\right] =$$

$$= \frac{1}{\pi}\left[\frac{1}{\pi}\left[\frac{t^2}{2}\right]_0^{\pi} + 2\left[t\right]_{\pi}^{2\pi} - \frac{1}{\pi}\left[\frac{t^2}{2}\right]_{\pi}^{2\pi}\right] =$$

$$= \frac{1}{\pi}\left[\underbrace{\frac{1}{\pi} \cdot \frac{\pi^2}{2}}_{\frac{\pi}{2}} - \underbrace{\frac{1}{\pi} \cdot 0}_{0} + 2 \cdot \underbrace{(2\pi - \pi)}_{\pi} - \frac{1}{\pi}\underbrace{\left[\frac{4\pi^2}{2} - \frac{\pi^2}{2}\right]}_{\frac{3\pi^2}{2}}\right]$$

$$= \frac{1}{\pi}\left[-\pi + 2\pi\right] = 1 \Rightarrow \underline{\frac{a_0}{2} = \frac{1}{2}}$$

• b_n mají být nulové, zkontrolujeme:

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)\,dt = \frac{1}{\pi}\left[\int_0^{\pi}\frac{1}{\pi}t\sin(nt)\,dt + \int_{\pi}^{2\pi}\left(2-\frac{1}{\pi}t\right)\sin(nt)\,dt\right] = \ldots$$

$$\int t\sin(nt)\,dt = \frac{-t\cos(nt)}{n} + \int\frac{\cos(nt)}{n}\,dt = -\frac{t}{n}\cos(nt) + \frac{1}{n}\cdot\frac{\sin(nt)}{n}$$

$u = t \quad u' = 1$

$v' = \sin(nt) \quad v = \frac{-\cos(nt)}{n}$

$$\int t\sin(nt)\,dt = \frac{1}{n}\left[-t\cos(nt) + \frac{1}{n}\sin(nt)\right]$$

$$b_n = \left(\frac{1}{\pi}\right)^2\left[\frac{1}{n}\left[-t\cos(nt) + \frac{1}{n}\sin(nt)\right]\right]_0^{\pi} + \int_{\pi}^{2\pi}\frac{2}{\pi}\sin(nt)\,dt - \left(\frac{1}{\pi}\right)^2\frac{1}{n}\left[-t\cos(nt) + \frac{1}{n}\sin(nt)\right]_{\pi}^{2\pi} =$$

$$= \left(\frac{1}{\pi}\right)^2\left[\frac{1}{n}\left[-\pi\cos(n\pi) + \frac{1}{n}\underbrace{[\sin(n\pi) - 0]}_{0}\right]\right] + \frac{2}{\pi}\,\frac{-\cos n2\pi + \cos n\pi}{n} -$$

$$-\left(\frac{1}{\pi}\right)^2\left[-\frac{2\pi}{n}\cos(2\pi n) + \frac{\pi}{n}\cos n\pi + \frac{1}{n}\left[\underbrace{\sin 2n\pi - \sin n\pi}_{0}\right]\right] =$$

$$= \underline{-\frac{1}{n\pi}\cos(n\pi)} - \frac{2}{n\pi}\cos(2n\pi) + \underline{\frac{2}{n\pi}\cos(n\pi)} + \frac{2}{n\pi}\cos(2n\pi) - \underline{\frac{1}{n\pi}\cos n\pi} = 0$$

$\underline{\underline{b_n = 0}}$, to jsme čekali

• Počítáme a_n:

$$a_n = \frac{2}{T}\int f(t)\cos(n\omega t)\,dt = \frac{1}{\pi}\left[\int_0^{\pi}\frac{1}{\pi}t\cos(nt) + \int_{\pi}^{2\pi}\left(2-\frac{t}{\pi}\right)\cos(nt)\,dt\right] = \ldots$$

$$\int t\cos(nt) = \frac{t\sin(nt)}{n} - \int\frac{\sin(nt)}{n}\,dt = \frac{t}{n}\sin(nt) + \frac{1}{n^2}\cos(nt)$$

$u = t \quad u' = 1$

$v' = \cos nt \quad v = \frac{\sin(nt)}{n}$

$$\int t\cos(nt) = \frac{1}{n}\left[t\sin(nt) + \frac{1}{n}\cos(nt)\right]$$

$$a_n = \frac{1}{\pi^2}\left[\frac{1}{n}\left[t\sin(nt) + \frac{1}{n}\cos(nt)\right]\right]_0^{\pi} + \int_{\pi}^{2\pi}\frac{2}{\pi}\cos(nt)\,dt - \frac{1}{\pi^2}\frac{1}{n}\left[t\sin(nt) + \frac{1}{n}\cos(nt)\right]_{\pi}^{2\pi} =$$

$$= \frac{1}{\pi^2}\left[\frac{1}{n}\left[\pi\underbrace{\sin(n\pi)}_{0} + \frac{1}{n}\cos n\pi - \frac{1}{n}\right]\right] + \frac{2}{n\pi}\left[\underbrace{\sin(2\pi n)}_{0} - \underbrace{\sin(\pi n)}_{0}\right]$$

$$-\frac{1}{\pi^2}\cdot\left[\frac{1}{n}\left[2\pi\underbrace{\sin(2\pi n)}_{0} - \pi\underbrace{\sin\pi n}_{0} + \frac{1}{n}\underbrace{\cos 2\pi n}_{1} - \frac{1}{n}\cos n\pi\right]\right] =$$

$$= \frac{1}{\pi^2}\left[\frac{1}{n^2}\left[\cos(n\pi) + \cos(n\pi)\right] - \frac{1}{n^2} - \frac{1}{n^2}\right] + \cancel{\frac{2}{\pi}\left[1 - \cos(n\pi)\right]} =$$

$$= \frac{2}{n^2\pi^2}\left(\cos(n\pi) - 1\right)$$

rozdělme na sudé a liché:

$n = 2l$ sudé $\quad a_n = a_{2l} = \frac{2}{\pi^2 (2l)^2} \left(\underbrace{\cos(2l\pi)}_{1} - 1 \right) = 0$

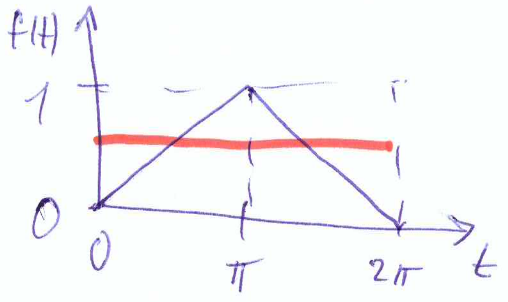
$n = 2l-1$ liché $\quad a_n = a_{2l-1} = \frac{2}{\pi^2 (2l-1)^2} \left(\underbrace{\cos (2l-1)\pi}_{-1} - 1 \right) = \frac{-4}{\pi^2 (2l-1)^2}$

Celkový výsledek:

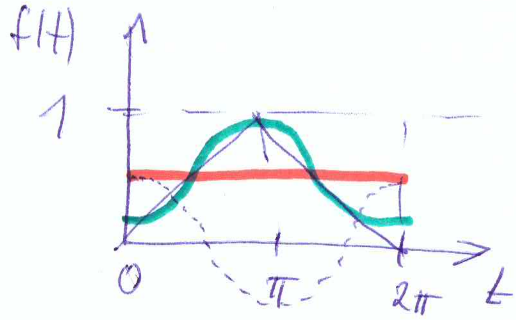
$$f(t) = \frac{1}{2} - \sum_{l=1}^{\infty} \frac{4}{\pi^2 (2l-1)^2} \cos[(2l-1)t]$$

$$f(t) = \frac{1}{2} - \frac{4}{\pi^2} \sum_{l=1}^{\infty} \frac{1}{(2l-1)^2} \cos[(2l-1)t]$$

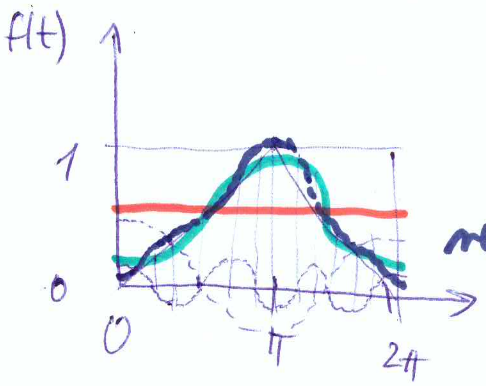
Zakreslíme:



$f_1(t) = \frac{1}{2}$ 1. člen



$f_2(t) = \frac{1}{2} - \underbrace{\frac{4}{\pi^2} \cdot \frac{1}{1}}_{\frac{4}{9} < \frac{5}{10} = \frac{1}{2}} \cos t$ 2. člen



velmi přibližně

řada bude určena správně

$f_3(t) = \frac{1}{2} - \frac{4}{\pi^2} \cos t - \frac{4}{\pi^2} \cdot \frac{1}{9} \cos 3t$