Exponenciální a logaritmické rovnice
(Seminář z matematiky I - M1130/02 2016)

(1) Řešte v $\mathbb{R}$ exponenciální rovnice:

(a) $2^{3x-1}\cdot 4 = 8^{x+1}\cdot\left(\frac{1}{2}\right)^{x}$ [2]

(b) $\frac{1}{3^x} = \frac{1}{\sqrt{3}}\cdot\sqrt[6]{27^{3-3x}}\cdot\left(\frac{1}{9}\right)^{x+3}$ [−2]

(c) $2^{2x}\cdot 5^x - 2^{2x-1}\cdot 5^{x+1} = -600$ [2]

(d) $2\cdot 0{,}5^{x^2+\frac{8}{3}x} = \frac{8}{\sqrt[3]{4}}$ $\left[-2;-\frac{2}{3}\right]$

(e) $7\cdot 4^{-x+2} = 3\cdot 4^{-x+3} - 5$ [2]

(f) $9^{x-0{,}5} + 9^{0{,}5-x} = \frac{10}{3}$ [0; 1]

(g) $2^{x-1} - 2^{x-2} = 5^{x-3} + 2^{x-3}$ [3]

(h) $4^x + 6^x = 2\cdot 9^x$ [0]

(i) $2\cdot 4^x + 5^{x-\frac{1}{2}} = 5^{x+\frac{1}{2}} - 2^{2x-1}$ $\left[\frac{3}{2}\right]$

(j) $\left(\sqrt{4+\sqrt{15}}\right)^x + \left(\sqrt{4-\sqrt{15}}\right)^x = 8$ [−2; 2]

(k) $2^{3x} + 2^{3x-1} + 2^{3x-2} + \ldots = \sqrt{12\cdot 2^{3x} - 8}$ $\left[0;\frac{1}{3}\right]$

(l) $2^x + 2^{x-1} + 2^{x-2} + \ldots = 2\sqrt{3\cdot 2^x + 4}$ [2]

(2) Řešte v $\mathbb{R}$ logaritmické rovnice:

(a) $\log(x-9) + 2\cdot\log\sqrt{2x-1} = 2$ [13]

(b) $\log\sqrt{3x-5} + \log\sqrt{7x-3} = 1 + \log\sqrt{0{,}11}$ [2]

(c) $\frac{\log 7x}{\log(2x-7)} = 2$ [7]

(d) $\log\left(x^2\right)\log\sqrt{x} - \log\frac{1}{x} = 2$ [10; 0,01]

(e) $\log\log\log x = 0$ $\left[10^{10}\right]$

(f) $\log_5(2x+9) + \log_5(4-3x) = 2 + \log_5(4+x)$ $[-2]$

(g) $(\log_4 x - 2) \cdot \log_4 x = \frac{3}{2} \cdot (\log_4 x - 1)$ $[2; 64]$

(h) $\log 2 + \log\left(4^{x-2} + 9\right) = 1 + \log\left(2^{x-2} + 1\right)$ $[2; 4]$

(i) $\log_3\left(3^{x^2-13x+28} + \frac{2}{9}\right) = \log_5 0{,}2$ $[3; 10]$

(j) $\sqrt{x^{\log\sqrt{x}}} = 10$ $\left[\frac{1}{100}; 100\right]$

(k) $x^{\frac{3}{8}\log^3 x - \frac{3}{4}\log x} = 1000$ $[0{,}01; 100]$

(l) $\log_3 x - 2\log_{\frac{1}{3}} x = 6$ $[9]$

(m) $\log_{x-1} 3 = 2$ $\left[1 + \sqrt{3}\right]$

(n) $\log_x\left(9x^2\right) \cdot \log_3^2 x = 4$ $\left[\frac{1}{9}; 3\right]$

(o) $\log_8 x + \log_8^2 x + \log_8^3 x + \ldots = \frac{1}{2}$ $[2]$

(p) $x^{\frac{\log x + 5}{3}} = 10^{5+\log x}$ $\left[10^{-5}; 10^3\right]$