

GONIOMETRICKÉ ROVNICE

(Seminář z matematiky I - M1130/02 2016)

(1) Řešte v \mathbb{R} rovnice:

$$(a) \quad 2 \sin^2 x - \sin x = 0 \quad \left[K = \bigcup_{k \in \mathbb{Z}} \left\{ k\pi, \frac{\pi}{6} + 2k\pi, \frac{5}{6}\pi + 2k\pi \right\} \right]$$

$$(b) \quad \sin^2 x + 2 \sin x - 3 = 0 \quad \left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{\pi}{2} + 2k\pi \right\} \right]$$

$$(c) \quad \cos x (2 \cos x + 1) = 1 \quad \left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \pi + 2k\pi, \frac{\pi}{3} + 2k\pi, \frac{5}{3}\pi + 2k\pi \right\} \right]$$

$$(d) \quad \sqrt{3} \operatorname{tg}^2 x + 2 \operatorname{tg} x - \sqrt{3} = 0 \quad \left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{\pi}{6} + k\pi, \frac{2}{3}\pi + k\pi \right\} \right]$$

$$(e) \quad 3 \sin x = 2 \cos^2 x \quad \left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{\pi}{6} + 2k\pi, \frac{5}{6}\pi + 2k\pi \right\} \right]$$

$$(f) \quad \sin x + \cos^2 x = \frac{1}{4} \quad \left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{7}{6}\pi + 2k\pi, \frac{11}{6}\pi + 2k\pi \right\} \right]$$

$$(g) \quad 3 \cos^2 x - 4 \cos x - \sin^2 x - 2 = 0 \quad \left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{2}{3}\pi + 2k\pi, \frac{4}{3}\pi + 2k\pi \right\} \right]$$

$$(h) \quad \cos 2x + \sin x = 0 \quad \left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{\pi}{2} + 2k\pi, \frac{7}{6}\pi + 2k\pi, \frac{11}{6}\pi + 2k\pi \right\} \right]$$

$$(i) \quad \sin x = \cos x \quad \left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{\pi}{4} + k\pi \right\} \right]$$

$$(j) \quad \sin x + \sqrt{3} \cos x = 0 \quad \left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{2}{3}\pi + k\pi \right\} \right]$$

$$(k) \quad \frac{\sin x}{\sqrt{3}} + \cos x = 1 \quad \left[K = \bigcup_{k \in \mathbb{Z}} \left\{ 2k\pi, \frac{\pi}{3} + 2k\pi \right\} \right]$$

$$(l) \quad \sqrt{3} \sin x + \cos x = \sqrt{3} \quad \left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{\pi}{6} + 2k\pi, \frac{\pi}{2} + 2k\pi \right\} \right]$$

(2) Řešte v \mathbb{R} následující rovnice (nezapomeňte stanovit podmínky, je-li to třeba):

$$(a) \quad \sin 2x + \cos 2x = 1 + \operatorname{tg} x \quad \left[K = \bigcup_{k \in \mathbb{Z}} \left\{ k\pi, \frac{\pi}{8} + k\frac{\pi}{2} \right\} \right]$$

$$(b) \quad \sin \left(x + \frac{\pi}{6} \right) + \cos \left(x + \frac{\pi}{3} \right) = 1 + \cos 2x \quad \left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{\pi}{2} + k\pi, \frac{\pi}{3} + 2k\pi, \frac{5}{3}\pi + 2k\pi \right\} \right]$$

$$(c) \quad \sin^2 2x - \cos^2 2x = \cos 2x \quad \left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{\pi}{6} + k\pi, \frac{5}{6}\pi + k\pi, \frac{\pi}{2} + k\pi \right\} \right]$$

$$(d) \cos x + \sin x = \frac{\cos 2x}{1 - \sin 2x}$$

$$\left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{3}{4}\pi + k\pi, 2k\pi, \frac{3}{2}\pi + 2k\pi \right\} \right]$$

$$(e) \cos^4 x - \sin^4 x = \sin 2x$$

$$\left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{\pi}{8} + k\frac{\pi}{2} \right\} \right]$$

$$(f) \frac{\sin x + \sin 3x}{\cos x - \cos 3x} = \sqrt{3}$$

$$\left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{\pi}{6} + k\pi \right\} \right]$$

$$(g) \cos x \cos 2x = \cos 4x \cos 5x$$

$$\left[K = \bigcup_{k \in \mathbb{Z}} \left\{ k\frac{\pi}{6} \right\} \right]$$

$$(h) \sin 3x = 2 \sin x$$

$$\left[K = \bigcup_{k \in \mathbb{Z}} \left\{ k\pi, \frac{\pi}{6} + k\pi, \frac{5}{6}\pi + k\pi \right\} \right]$$

$$(i) \operatorname{tg} x - \sin x + \cos x = 1$$

$$\left[K = \bigcup_{k \in \mathbb{Z}} \left\{ 2k\pi, \frac{\pi}{4} + k\pi \right\} \right]$$

$$(j) 2 \sin^2 x + \cos x = 2 \sin^2 x \cos x + 1$$

$$\left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{\pi}{4} + k\frac{\pi}{2}, 2k\pi \right\} \right]$$

$$(k) \frac{\sqrt{3} + \operatorname{tg} x}{1 - \sqrt{3} \operatorname{tg} x} = 1$$

$$\left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{11}{12}\pi + k\pi \right\} \right]$$

$$(l) \sin^2 2x + \sin^2 4x = \frac{3}{2}$$

$$\left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{\pi}{6} + k\frac{\pi}{2}, \frac{\pi}{3} + k\frac{\pi}{2}, \frac{\pi}{8} + k\frac{\pi}{4} \right\} \right]$$

$$(m) |\sin x| = \sin x + 2$$

$$\left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{3}{2}\pi + 2k\pi \right\} \right]$$

$$(n) |\operatorname{tg} x + \operatorname{cotg} x| = \frac{4}{\sqrt{3}}$$

$$\left[K = \bigcup_{k \in \mathbb{Z}} \left\{ \frac{\pi}{6} + k\pi, \frac{\pi}{3} + k\pi, \frac{2}{3}\pi + k\pi, \frac{5}{6}\pi + k\pi \right\} \right]$$