

Problem solving seminar III

10. Suppose that f_n is a sequence of nondecreasing functions which map the unit interval into itself. Suppose that

$$\lim_{n \rightarrow \infty} f_n(x) = f(x)$$

pointwise and that f is continuous function. Prove that $f_n(x) \rightarrow f(x)$ uniformly as $n \rightarrow \infty$, $0 \leq x \leq 1$. Note that the functions f_n are not necessarily continuous.

11. Let G be a group and H a subgroup of index $n < \infty$. Prove or disprove the following statements:

(A) If $a \in G$, then $a^n \in H$.

(B) If $a \in G$, then there is k , $1 \leq k \leq n$ such that $a^k \in H$.

12. Let A be an $n \times n$ matrix and A^t its transpose. Show that $A^t A$ and A^t have the same rank.

13. Let $X \subset \mathbb{R}^n$ be compact and let $f : X \rightarrow \mathbb{R}$ be continuous. Given $\varepsilon > 0$, show that there is M such that for all $x, y \in X$

$$|f(x) - f(y)| \leq M||x - y|| + \varepsilon.$$