

Problem solving seminar IV

14. Let $f_n : \mathbb{R} \rightarrow \mathbb{R}$ be differentiable functions with $|f'_n(x)| \leq 1$ for all $x \in \mathbb{R}$ and $n \geq 1$. Assume

$$\lim_{n \rightarrow \infty} f_n(x) = g(x)$$

for all x . Prove that $g : \mathbb{R} \rightarrow \mathbb{R}$ is continuous.

15. Show that the interval $[0, 1]$ cannot be written as a countably infinite disjoint union of closed subintervals of $[0, 1]$.

16. Let N be a linear operator on an n -dimensional vector space, $n > 1$, such that $N^n = 0$, $N^{n-1} \neq 0$. Prove that there is no operator X with $X^2 = N$.