Problem solving seminar V

17. Let $f:\ [0,1] \to \mathbb{R}$ be continuously differentiable with $f(0) = 0$. Prove that

$$\sup_{0 \le x \le 1} |f(x)| \le \sqrt{\int_0^1 (f'(x))^2 dx}.$$

18. Prove or supply a counterexample: If f is a nondecreasing real valued function on $[0,1]$, then there is a sequence $\{f_n\}$ of continuous functions on $[0,1]$ such that for each $x \in [0,1]$

$$\lim_{n \to \infty} f_n(x) = f(x).$$

19. Let G be a group of order 10 which has a normal subgroup of order 2. Prove that G is abelian.

20. Let G be a group and H and K subgroups such that H has a finite index in G. Prove that $K \cap H$ has a finite index in K.