

Problem solving seminar VIII

28. For $0 \leq x \leq \pi/2$ and $0 < p < 1$ prove that

$$(\cos x)^p \leq \cos(px).$$

29. Define a sequence of positive numbers as follows: let $x_0 > 0$ be any positive number and $x_{n+1} = (1 + x_n)^{-1}$. Prove that this sequence converges and find its limit.

30. Prove:

(1) If B is a real symmetric positive definite matrix, there is an invertible matrix C such that $B = C^T C$.

(2) If A is a real symmetric $n \times n$ matrix find a maximum of the function

$$F(x) = \frac{\langle Ax, x \rangle}{\langle x, x \rangle}$$

defined for $x \in \mathbb{R}^n - \{0\}$. In which x is this maximum attained?

(3) If A and B are real symmetric $n \times n$ matrices, B positive definite, then the function

$$G(x) = \frac{\langle Ax, x \rangle}{\langle Bx, x \rangle}$$

defined for $x \in \mathbb{R}^n - \{0\}$ takes its maximum only in an eigenvector for a certain matrix related to A and B . Show which matrix.