Př: 3. řádu, MACLAURIN ($x_0 = 0$)

a) $\frac{1}{\cos x}$, $\cos x \approx 1 - \frac{x^2}{2}$

$$\frac{1}{\cos x} \approx a + bx + cx^2 + dx^3$$

Teď:

$$\frac{1}{1 - \frac{x^2}{2}} = a + bx + cx^2 + dx^3 \quad / \cdot \left(1 - \frac{x^2}{2}\right)$$

$$1 = (a + bx + cx^2 + dx^3) \cdot \left(1 - \frac{x^2}{2}\right)$$

$$1 = a + bx + cx^2 + dx^3 - \frac{a}{2}x^2 - \frac{b}{2}x^3 - \frac{c}{2}x^4 - \frac{d}{2}x^5$$

x^0: $1 = a$

x^1: $0 = b$

x^2: $0 = c - \frac{a}{2} = c - \frac{1}{2} \Rightarrow c = \frac{1}{2}$

x^3: $0 = d - \frac{b}{2} = d \Rightarrow d = 0$

$$\Rightarrow \frac{1}{\cos x} \approx 1 + \frac{1}{2}x^2$$

b) $e^{-\frac{x^2}{2}}$, $e^t \approx 1 + t + \frac{t^2}{2} + \frac{t^3}{3!} + \dots$

$$t \to -\frac{x^2}{2} \Rightarrow e^{-\frac{x^2}{2}} \approx 1 + \left(-\frac{x^2}{2}\right) + \frac{1}{2}\cdot\left(-\frac{x^2}{2}\right)^2 + \frac{1}{6}\cdot\left(-\frac{x^2}{2}\right)^3 + \dots$$

$$\approx 1 - \frac{x^2}{2}$$

c) $\sin(\sin x)$

$$\sin t \approx t - \frac{t^3}{3!}$$

$$t = \sin x = x - \frac{x^3}{6}$$

$$\sin(\sin x) \approx \left(x - \frac{x^3}{6}\right) - \frac{1}{6} \cdot \left(x - \frac{x^3}{6}\right)^3$$

$$\Rightarrow \sin(\sin x) \approx x - \frac{x^3}{6} - \frac{x^3}{6} = x - \frac{x^3}{3}$$

d) $\operatorname{tg} x = \dfrac{\sin x}{\cos x}$, $\sin x \approx x - \dfrac{x^3}{6}$, $\cos x \approx 1 - \dfrac{x^2}{2}$

$$\operatorname{tg} x \approx \left(x - \frac{x^3}{6}\right) : \left(1 - \frac{x^2}{2}\right) = \boxed{x + \frac{x^3}{3}}$$

$$\frac{-\left(x - \frac{x^3}{2}\right)}{}$$

$$\frac{x^3}{3}$$

$$-\left(\frac{x^3}{3} - \frac{x^5}{6}\right)$$

e) $e^x \cdot \sin x$ $\quad$ $e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$, $\sin x \approx x - \frac{x^3}{6}$

$$e^x \cdot \sin x \approx \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right) \cdot \left(x - \frac{x^3}{6}\right) \approx$$

$$\approx x - \frac{x^3}{6} + x^2 + \frac{x^3}{2} + \cancel{\frac{x^4}{6}} = \underline{\underline{x + x^2 + \frac{x^3}{3}}}$$

$f(x) = \underbrace{x^3 - 2x + 5}_{=4}$ DO PROG. $u = x - 1$

$f'(x) = 3x^2 - 2$	$= 1$
$f''(x) = 6x$	$= 6$
$f'''(x) = 6$	$= 6$
$f^{(4)}(x) = 0$	$x = 1$
$\vdots$	

$$T(x) = \sum_{i=0}^{n} \frac{f^{(i)}(1)}{i!} \cdot (x-1)^i =$$

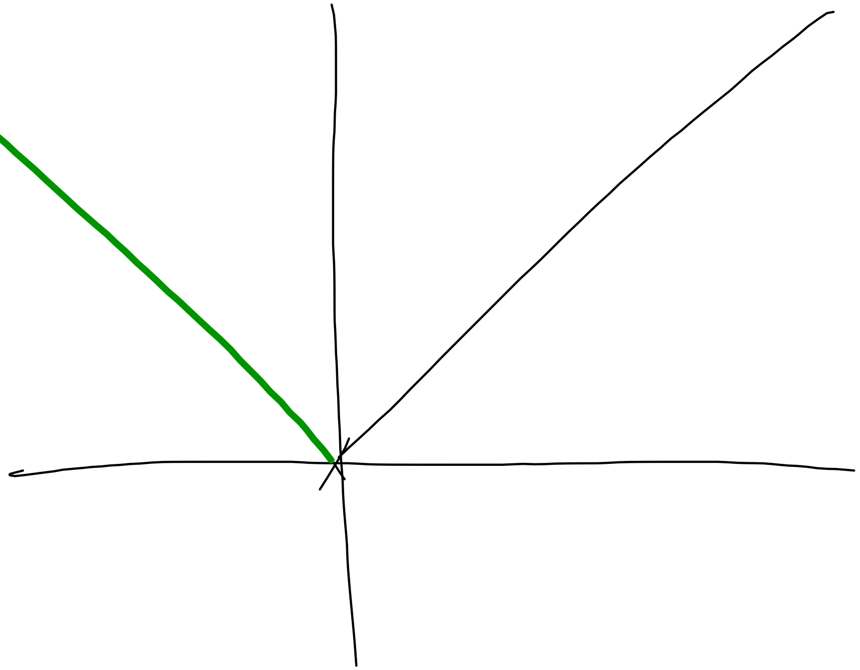
$$= \frac{4}{1} \cdot 1 + \frac{1}{1} \cdot (x-1) + \frac{6}{2} \cdot (x-1)^2 + \frac{6}{6} \cdot (x-1)^3 = \underline{\underline{4 + u + 3u^2 + u^3}}$$

$$\left\{ \underline{\underline{[\cos t, \sin t]}}, \ t \in \ldots \right\}$$

$t=0 \Rightarrow [0,0]$

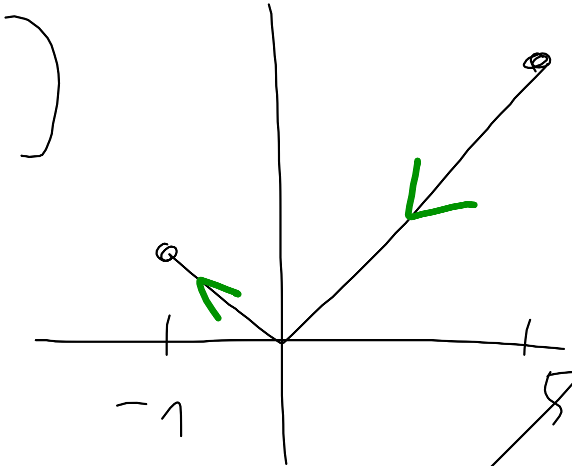
$t=-1 \Rightarrow [-1,1]$

$t=2 \Rightarrow [-4,4]$



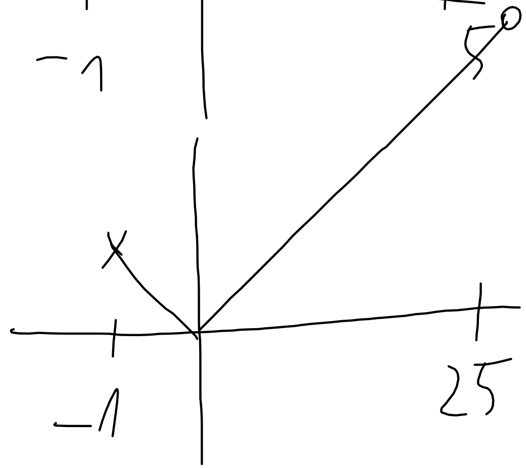
$$x = t \qquad t \in [-1, 5)$$

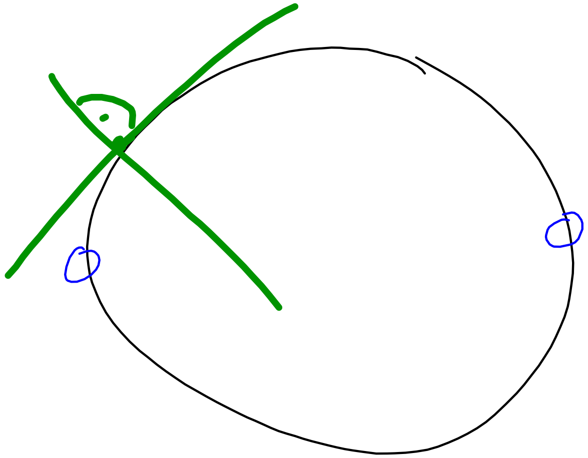
$$y = |t|$$

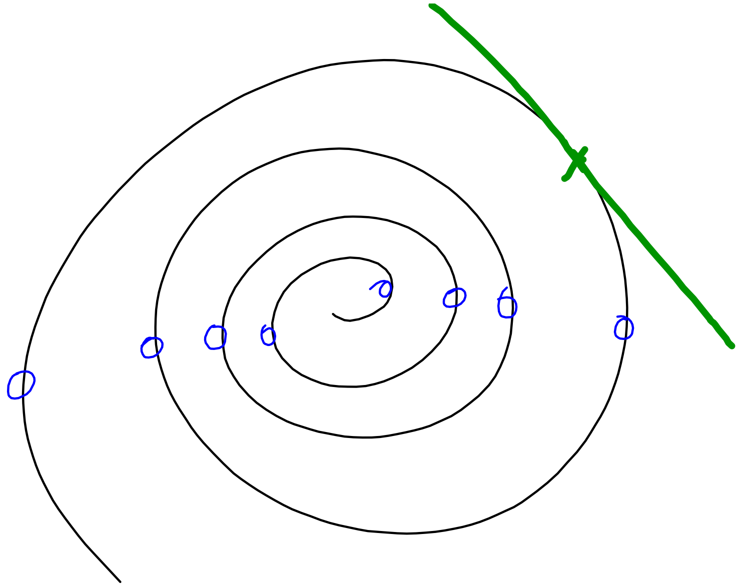


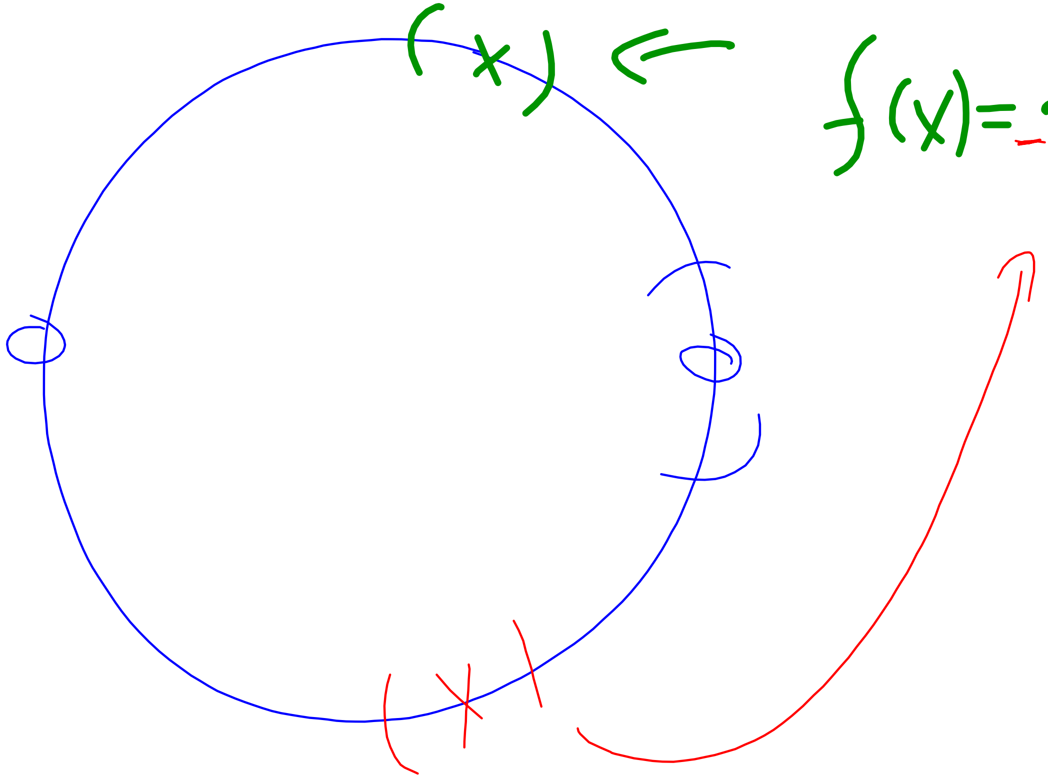
$$x = \begin{cases} -t^2, & t < 0 \\ t^2, & \text{jinak} \end{cases}, \quad t \in [-1, 5)$$

$$y = t^2$$

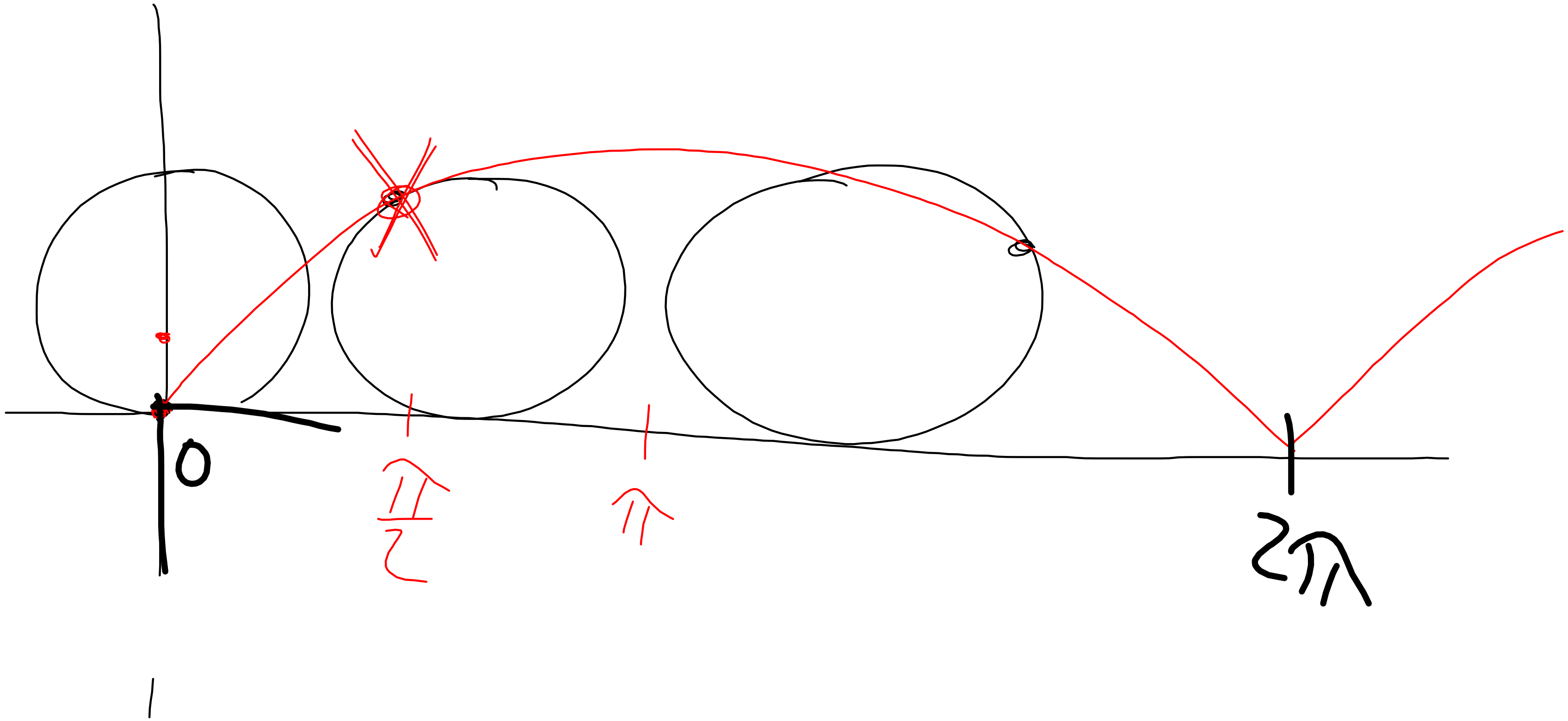


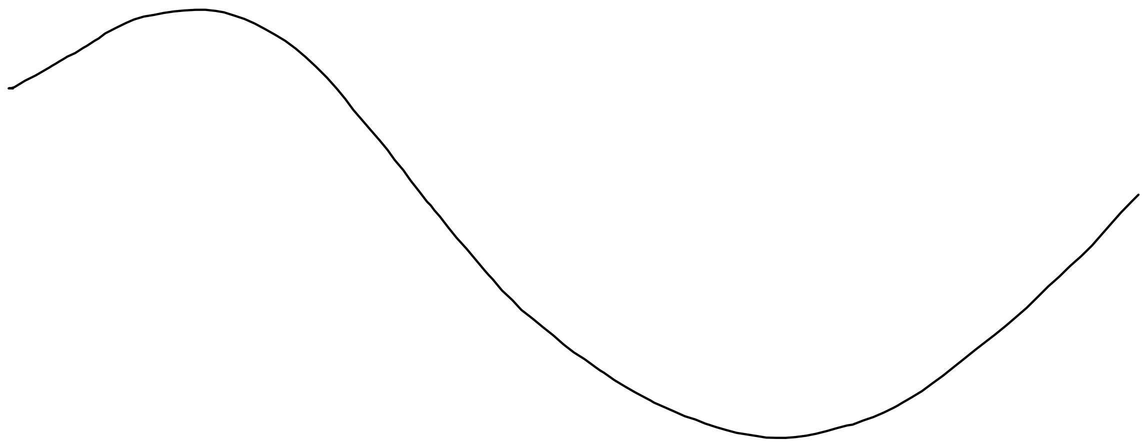


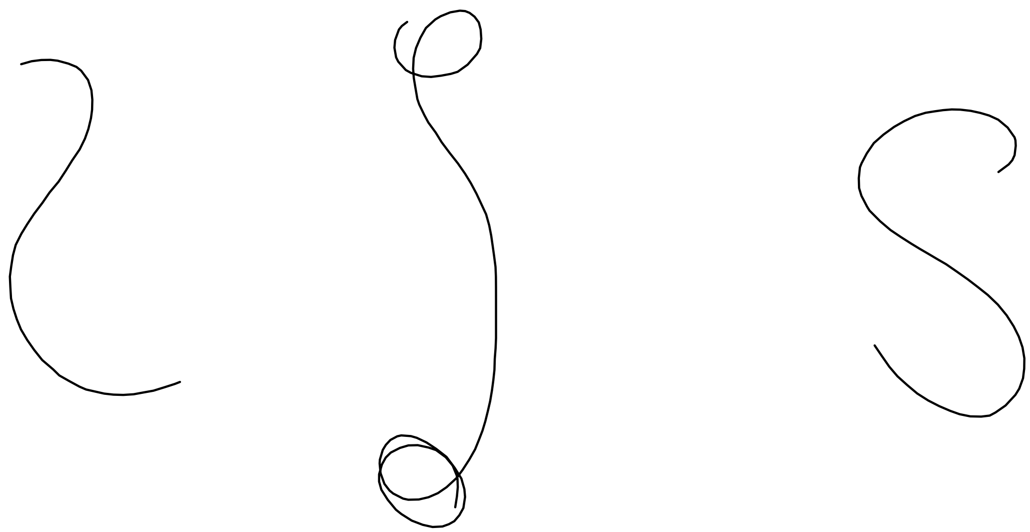




$$f(x) = \sqrt{r^2 - x^2}$$


0
$\frac{\pi}{2}$
π
2π





$$\sum_{n=1}^{5} n^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

$$\int_{0}^{5} x^2 \, dx =$$

$$(x^2)' = 2x$$

$$(x^2+5)' = 2x$$

$$(x^2 - 183{,}15)' = 2x$$

$$(x^2 + c)' = 2x$$

$$c \in \mathbb{R}$$

$$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$$

$$\operatorname{arctg} x + c = \int \frac{1}{1+x^2}\,dx$$

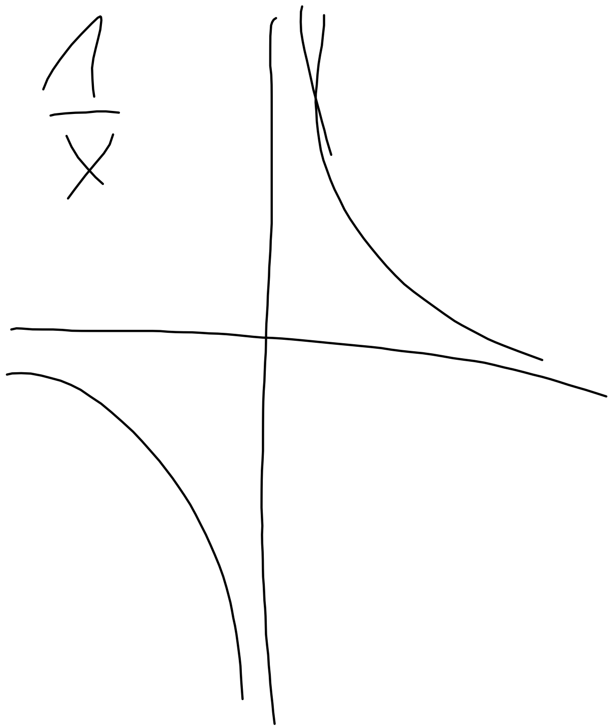
$$\frac{1}{4-x^2} = \frac{A}{2-x} + \frac{B}{2+x}$$

$$\left(x^{\alpha}\right)' = \alpha \cdot x^{\alpha-1}$$

$$x^{\alpha} = \int \alpha \cdot x^{\alpha-1}\,dx$$

$$\frac{x^{\alpha}}{\alpha} = \int x^{\alpha-1}\,dx$$

$\frac{1}{x}$



$x > 0 \qquad (\ln x)' = \frac{1}{x}$

$x < 0 \qquad (\ln|x|)' = (\ln(-x))' =$

$= \frac{1}{-x} \cdot (-1) = \frac{1}{x}$