$$Q(x) = \frac{2x^{14} - 5x^{13} + 3x^{12} - 10x^{11} - 22x^{10} + 11x^{9} + 116x^{8} + 298x^{7} + 504x^{6} + 975x^{5} + 1440x^{4} + 2220x^{3} + 1744x^{2} + 1280x + 128}{(x+1)\cdot(x-2)^2\cdot(x^2+2x+2)\cdot(x^2-x+1)\cdot(x^2+x+2)^3}$$

$$\frac{5 = G}{(x-2)^2} + \frac{H = 0}{x-2}$$

$$\frac{Ax+B}{(\quad)^3} + \frac{\overset{=0}{C}x+\overset{=0}{D}}{(\quad)^2} + \frac{\overset{=0}{E}x+\overset{=0}{F}}{(\quad)^1}$$

$$\int Q(x)\,dx = ???$$

$$Q(x) = \underbrace{2x + 3}_{} + \underbrace{\frac{-4}{x+1} + \frac{5}{(x-2)^2} + \frac{-7}{x^2+2x+2} + \frac{2x+5}{x^2-x+1} + \frac{2-3x}{(x^2+x+2)^3}}_{}$$

$$\int \varphi(x)\,dx = \underbrace{\int 2x+3\,dx}_{I_1} - 4\cdot\underbrace{\int \frac{1}{x+1}\,dx}_{I_2} + 5\cdot\underbrace{\int \frac{1}{(x-2)^2}\,dx}_{I_3} - 7\cdot\underbrace{\int \frac{1}{x^2+2x+2}\,dx}_{I_4} + \underbrace{\int \frac{2x+5}{x^2-x+1}\,dx}_{I_5}$$

$$+\underbrace{\int \frac{2-3x}{(x^2+x+2)^3}\,dx}_{I_6}$$

$$I_1 = 2\cdot\frac{x^2}{2} + 3x + C_1 = x^2 + 3x + C_1$$

$$\boxed{\int \frac{f'}{f}\,dx = \ln|f| + C}$$

$$I_2 = \int \frac{1}{x+1}\,dx = \ln|x+1| + C_2$$

$$I_2 = \left| \begin{matrix} t = x+1 \\ dt = dx \end{matrix} \right| = \int \frac{1}{t}\,dt = \ldots$$

$$I_3 = \int \frac{1}{(x-2)^2}\,dx = \left| \begin{array}{l} t = x-2 \\ 1\,dt = 1\,dx \end{array} \right| = \int \frac{1}{t^2}\,dt = \int t^{-2}\,dt = \frac{t^{-1}}{-1} + C_3 =$$

$$= \frac{-1}{x-2} + C_3$$

$$I_4 = \int \frac{1}{x^2+2x+2}\,dx = \left| D=4-8<0 \right| = \int \frac{1}{(x+1)^2-1+2}\,dx = \int \frac{1}{(x+1)^2+1}\,dx =$$

$$x^2+2x+1$$

$$= \left| \begin{matrix} t=x+1 \\ dt=dx \end{matrix} \right| = \int \frac{1}{t^2+1}\,dt = \operatorname{arctg} t + C_4 = \underline{\underline{\operatorname{arctg}(x+1) + C_4}}$$

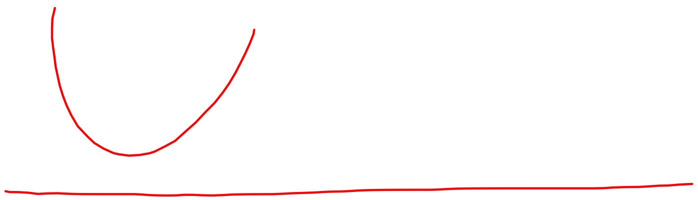
$$\boxed{\int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x + c \quad \Big\| \quad \int \frac{1}{A^2+x^2}\,dx = \frac{1}{A}\cdot\operatorname{arctg}\frac{x}{A} + c}$$

$$I_5 = \int \frac{2x+5}{x^2-x+1}\,dx = \int \frac{2x-1+1+5}{x^2-x+1}\,dx = \int \frac{2x-1}{x^2-x+1}\,dx + \int \frac{6}{x^2-x+1}\,dx =$$

$2x-1$

$$= \ln|x^2-x+1| + 6\cdot \boxed{\int \frac{1}{x^2-x+1}\,dx} = \ln(x^2-x+1) + 6\cdot\frac{2}{\sqrt{3}}\cdot \operatorname{arctg}\left(\frac{2x-1}{\sqrt{3}}\right) + C_5$$

$(*)$



$$\circledast \quad \int \frac{1}{x^2-x+1}\,dx = \int \frac{1}{\left(x-\frac{1}{2}\right)^2-\frac{1}{4}+1}\,dx = \int \frac{1}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}\,dx =$$

$$x^2-2\cdot\frac{1}{2}\cdot x+\frac{1}{4} = x^2-x+\frac{1}{4}$$

$$\overset{a)}{=} \left| \begin{array}{l} t = x-\frac{1}{2} \\ dt = dx \end{array} \right| = \int \frac{1}{t^2+\frac{3}{4}}\,dt = \int \frac{1}{t^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dt = \frac{1}{\frac{\sqrt{3}}{2}}\cdot \operatorname{arctg}\frac{t}{\frac{\sqrt{3}}{2}} + \hat{C} =$$

$$= \frac{2}{\sqrt{3}}\cdot \operatorname{arctg}\frac{2\left(x-\frac{1}{2}\right)}{\sqrt{3}} + \hat{C}$$

$$\boxed{\int \frac{1}{t^2+A^2}\,dt = \frac{1}{A}\cdot \operatorname{arctg}\frac{t}{A} + C}$$

b) $= \int \frac{1}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}\,dx = \frac{1}{\frac{3}{4}} \cdot \int \frac{1}{\frac{\left(x-\frac{1}{2}\right)^2}{\frac{3}{4}}+1}\,dx = \frac{4}{3} \cdot \int \frac{1}{\left(\frac{x-\frac{1}{2}}{\sqrt{\frac{3}{4}}}\right)^2+1}\,dx =$

$$= \frac{4}{3} \cdot \int \frac{1}{\left(\frac{2x-1}{\sqrt{3}}\right)^2+1}\,dx = \left| \begin{array}{l} t = \frac{2x-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}x - \frac{1}{\sqrt{3}} \\ 1\,dt = \frac{2}{\sqrt{3}}\,dx \Rightarrow dx = \frac{\sqrt{3}}{2}\,dt \end{array} \right| = \frac{4}{3} \cdot \int \frac{1}{t^2+1} \cdot \frac{\sqrt{3}}{2} \cdot dt =$$

$$= \frac{4}{3} \cdot \frac{\sqrt{3}}{2} \cdot \int \frac{1}{t^2+1}\,dt = \frac{2\sqrt{3}}{3} \cdot \operatorname{arctg} t + \hat{C} = \underline{\underline{\frac{2\sqrt{3}}{3} \cdot \operatorname{arctg} \frac{2x-1}{\sqrt{3}} + \hat{C}}}$$

$$\int \frac{1}{1+x^2}\,dx = \underline{\underline{\operatorname{arctg} x + C}}$$

$$I_6 = \int \frac{2-3x}{(x^2+x+2)^3}\,dx = -3\cdot\int \frac{x-\frac{2}{3}}{(\ldots)^3}\,dx = -3\cdot\int \frac{2}{2}\cdot\frac{x-\frac{2}{3}}{(\ldots)^3}\,dx = -\frac{3}{2}\cdot\int \frac{2x-\frac{4}{3}}{(\ldots)^3}\,dx =$$

$$2x+1$$

$$\boxed{3\cdot(x^2+x+2)^2\cdot(2x+1)}$$

$$\int f(\varphi(x))\,\varphi'(x)\,dx = \int f(t)\,dt$$

$$t = \varphi(x)$$

$$1\,dt = \varphi'(x)\,dx$$

$$= -\frac{3}{2}\cdot\int \frac{2x+1-1-\frac{4}{3}}{(x^2+x+2)^3}\,dx = -\frac{3}{2}\cdot\int\frac{2x+1}{(\dots)^3}\,dx - \frac{3}{2}\cdot\int\frac{-\frac{7}{3}}{(x^2+x+2)^3}\,dx = \left| \begin{array}{l} t = x^2+x+2 \\ 1\,dt = (2x+1)\,dx \end{array} \right| =$$

$$= -\frac{3}{2}\cdot\int\frac{dt}{t^3} + \frac{7}{2}\cdot\int\frac{1}{(x^2+x+2)^3}\,dx = -\frac{3}{2}\cdot\int t^{-3}\,dt + \frac{7}{2}\cdot\int\frac{1}{(\dots)^3}\,dx =$$

$$= -\frac{3}{2}\cdot\frac{t^{-2}}{-2} + \frac{7}{2}\cdot\int\frac{1}{(\dots)^3}\,dx = \frac{3}{4\cdot(x^2+x+2)^2} + \frac{7}{2}\cdot\int\frac{1}{(\dots)^3}\,dx = \circledast$$

II

$$\mathrm{II} = \int \frac{1}{(x^2+x+2)^3}\,dx = \int \frac{1}{\left[\left(x+\frac{1}{2}\right)^2-\frac{1}{4}+2\right]^3}\,dx = \int \frac{1}{\left[\left(x+\frac{1}{2}\right)^2+\underline{\underline{\frac{7}{4}}}\right]^3}\,dx =$$

$$= \int \frac{1}{\left(\frac{7}{4}\right)^3\cdot\left[\frac{\left(x+\frac{1}{2}\right)^2}{\frac{7}{4}}+1\right]^3}\,dx = \left(\frac{4}{7}\right)^3\cdot\int \frac{1}{\left[\left(\frac{x+\frac{1}{2}}{\sqrt{\frac{7}{4}}}\right)^2+1\right]^3}\,dx =$$

$$= \left(\frac{4}{7}\right)^3\cdot\int \frac{1}{\left[\left(\frac{2x+1}{\sqrt{7}}\right)^2+1\right]^3}\,dx = \left|\begin{array}{l} t = \frac{2x+1}{\sqrt{7}} = \frac{2}{\sqrt{7}}x+\frac{1}{\sqrt{7}} \\ 1\,dt = \frac{2}{\sqrt{7}}\,dx \Rightarrow dx = \underline{\underline{\frac{\sqrt{7}}{2}\,dt}} \end{array}\right| =$$

Cíl:

$$\frac{1}{(t^2+1)^3}$$

$$= \left(\frac{4}{7}\right)^3 \cdot \int \frac{1}{\left[t^2+1\right]^3} \frac{\sqrt{7}}{2}\, dt = \left(\frac{4}{7}\right)^3 \cdot \frac{\sqrt{7}}{2} \cdot \int \frac{1}{(t^2+1)^3}\, dt = \left| \begin{array}{c} \text{VZOREC} \\ \text{ř. 46} \\ \hline n=3 \end{array} \right|$$

$$= \left(\frac{4}{7}\right)^3 \cdot \frac{\sqrt{7}}{2} \cdot \left[\frac{t}{2\cdot 2\cdot (1+t^2)^2} + \frac{3}{4} \cdot \boxed{\int \frac{1}{(t^2+1)^2}\, dt} \right] = \left| \text{VZOREC, ř. 46, } n=2 \right|$$

$$= \left(\frac{4}{7}\right)^3 \cdot \frac{\sqrt{7}}{2} \cdot \frac{t}{4\cdot (1+t^2)^2} + \left(\frac{4}{7}\right)^3 \cdot \frac{\sqrt{7}}{2} \cdot \frac{3}{4} \cdot \left[\frac{t}{2\cdot 1\cdot (1+t^2)^1} + \frac{1}{2} \cdot \int \frac{1}{(t^2+1)^1}\, dt \right] =$$

$$= \left(\frac{4}{7}\right)^3 \cdot \frac{\sqrt{7}}{2} \cdot \frac{t}{4\cdot (1+t^2)^2} + \left(\frac{4}{7}\right)^3 \cdot \frac{\sqrt{7}}{2} \cdot \frac{3}{4} \cdot \left[\frac{t}{2\cdot (1+t^2)} + \frac{1}{2} \cdot \operatorname{arctg} t \right] + C =$$

$$= \underline{\underline{\text{VRÁTíME SUBST.}}} \,!$$

POZOR (BREZ VZORCE)

$$\int \frac{1}{(1+t^2)^3}\,dt = \int \frac{1+t^2-t^2}{(1+t^2)^3}\,dt =$$

$$= \int \frac{1+t^2}{(1+t^2)^3}\,dt - \int \frac{t^2}{(1+t^2)^3}\,dt = \int \frac{1}{(1+t^2)^2}\,dt - \int t\cdot\frac{t}{(1+t^2)^3}\,dt =$$

$$= \int \frac{1}{(\ )^2}\,dt - \left[t\cdot\frac{-1}{4\cdot(1+t^2)^2} - \int \frac{-1}{4\cdot(1+t^2)^2}\,dt \right] = \int \frac{1}{(\ )^2}\,dt + \frac{t}{4\cdot(1+t^2)^2} - \frac{1}{4}\cdot\int\frac{1}{(\ )^2}\,dt =$$

$$u = t \qquad u' = 1$$

$$v' = \frac{t}{(1+t^2)^3} \qquad v = \int \frac{t}{(1+t^2)^3}\,dt = \left| \begin{matrix} z = 1+t^2 \\ dz = 2t\,dt \end{matrix} \right| = \int \frac{\frac{1}{2}\,dz}{z^3} = \frac{1}{2}\cdot\int z^{-3}\,dz = \frac{1}{2}\cdot\frac{z^{-2}}{-2} = \frac{1}{-4\cdot(1+t^2)^2}$$

$$= \frac{t}{4\cdot(1+t^2)^2} + \left(1-\frac{1}{4}\right)\cdot\int\frac{1}{(1+t^2)^2}\,dt = \frac{t}{4\cdot(1+t^2)^2} + \frac{3}{4}\cdot\int\frac{1}{(1+t^2)^2}\,dt =$$

$$= \frac{t}{4\cdot(1+t^2)^2} + \frac{3}{4}\cdot\int\frac{1+t^2-t^2}{(1+t^2)^2}\,dt = \dots$$

$$\int \frac{3\cdot\cos^4 x - 7\cdot\sin^2 x + 8}{\cdots\cdots\cdots} \,dx$$

$$\sin^8 x = (\sin^2 x)^4 = (1-\cos^2 x)^4 =$$

$$= (1-t^2)^4$$

$$\frac{1}{1+3\cdot\cos^2 x} = \frac{1}{1+3\cdot\boxed{(-\cos x)}^{2}}$$