Test from Discrete mathematics 20/10/2016

Name and surname	1	2	3	4	5	Sum

Two points for every task. Use a space below the tasks for answers.

1. a) Decide whether formula $\varphi = (\forall x)(x = 0 \to x = x + 1)$ is valid in $\mathbb{R}, \mathbb{Z}$, resp. $\mathbb{N}$, and explain why.

b) Write a negation of φ and modify it to a form in which the negation operation may appear just with the subformulas without logical connectives (atomic subformulas).

2. Write as a formula:

a) One can express b from equation $a = b + 5$ for any a, b.

b) 0 is not a smallest number.

3. Express the set A provided that $\emptyset \in A, A \subseteq \{\emptyset, \{\emptyset\}\}, A \neq \{\{\emptyset\}\}$.

4. For any sets A, B, C, prove

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

5. Express the sets as lists of elements:
   a) $\mathcal{P}(\mathcal{P}(\{\emptyset\}))$,

   b) $\{\{\emptyset\}, \emptyset\} \times \{\{\emptyset\}, \emptyset\}$.