

## NOTE

### A FORGOTTEN CHAPTER IN THE HISTORY OF FELIX KLEIN'S ERLANGER PROGRAMM

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Felix Klein's single most important mathematical accomplishment was unquestionably his "Erlanger Programm" of 1872 [1]. This work remained a *leitmotiv* not only for Klein but for much of the mathematical world throughout his lifetime [2]. The purpose of this note is to correct a widespread misconception concerning the original presentation of the "Erlanger Programm" and, at the same time, to point out how this error has obscured a significant event in Klein's early development as an educator and scientific organizer. Hopefully this will serve not only to reduce the possibility of future confusion, but also to give a clearer sense of the manner in which Klein's later pedagogical activities were related to the interests and ideas of his youth.

By now the error in question has been firmly entrenched in the secondary literature; in fact, it has become part of the standard lore in the history of mathematics. Within the last fifteen years three eminent historians of mathematics have written:

*In 1872 Klein became professor at Erlangen. In his inaugural address he explained the importance of the group conception for the classification of the different fields of mathematics. The address, which became known as the "Erlangen Program," declared every geometry to be the theory of invariants of a particular transformation group. [Struik 1967, 175]*

*Klein, in a celebrated inaugural address in 1872, when he became professor at Erlangen, showed how it [the group concept] could be applied as a convenient means of characterizing the various geometries that had appeared during the century. [Boyer 1968, 592]*

*He gave this characterization in a speech of 1872, "Vergleichende Betrachtungen ueber neuere geometrische Forschungen" (A Comparative Review of Recent Researches in Geometry), on the occasion of his admission to the University of Erlangen, and the views expressed in it are known as the Erlanger Programm. [Kline 1972, 917]*

Each of these accounts incorrectly identifies the content of Klein's speech, his Erlangen Antrittsrede of December 7, 1872,

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with the content of his much more famous *Eintrittsprogramm*, "Vergleichende Betrachtungen ueber neuere geometrische Forschungen." In fact, the two were devoted to wholly different subjects. Klein himself was perfectly clear on this point in his introductory remarks to the chapter "Zum Erlanger Programm" in [Klein 1921, 411-412], written some fifty years after the event.

According to his own account, the source of the "Erlanger Programm" can be traced back to ideas presented in his "Ueber die sogenannte Nicht-Euklidische Geometrie, Zweiter Aufsatz" [Klein 1921, 311-343], which was completed in early June of 1872 (it appeared the following year in Band 6 of *Mathematische Annalen*). As for the "Erlanger Programm" itself, Klein remarked that it was written under the influence of two very particular circumstances.

The first of these was the stimulation he received through numerous conversations with Sophus Lie, who had paid him a visit during the months of September and October, 1872. Lie had just developed the fundamental notions underlying his theory of first-order partial differential equations and, in particular, his theory of contact transformations. During this period they met daily, sharing the excitement of their respective discoveries. Klein was especially impressed by Lie's thoughts on the group-theoretic classification of geometries, a subject they both investigated with the greatest enthusiasm.

The second and more immediate factor leading to the publication of the "Erlanger Programm" was actually a matter of obligation connected with Klein's appointment at the University. As he himself noted:

*"in Erlangen the newly appointed professor, in addition to a speech through which he introduces himself to the circle of his colleagues, traditionally submitted a published program" [Klein 1921, 411].*

Thus, what today is referred to as the "Erlanger Programm" was, in fact, only the written portion of Klein's original inaugural presentation; his public lecture, naturally enough, was reserved for matters more accessible to a general university audience. Indeed, Klein made explicit mention of the lecture topic he had chosen on this occasion in the passage quoted above which continues:

*I regard this custom, despite all the discomfort for the participants, as very valuable, since it allows the new professor the unusual opportunity to address his colleagues on matters he feels are significant, while also compelling him to clarify his own thoughts regarding these concerns. In my case, the public lecture took place on December 7, at which time the published work was distributed to the listeners. I chose, it might*

*be noted in passing, as the theme for my lecture, the pedagogical principles and goals for my future academic activity.... [Klein 1921, 411-412]*

The most recent conflation of Klein's "Erlanger Programm" with his public lecture occurs in [Portnoy 1982], where the author makes two erroneous inferences, both due to this very misconception. She writes that the "Erlanger Programm" was first published in an Italian translation of 1890 (Fano's translation in *Annali di Matematica*, 2(17)) and cites the 1893 German version that appeared in *Mathematische Annalen* [Klein 1893]. But the latter was only a republication of the original [Klein 1872], to which Klein added a series of remarks. Obviously this mistake would have been avoided had the author known that the "Vergleichende Betrachtungen ueber neuere geometrische Forschungen" was a "published work... distributed to the listeners" at the time of Klein's inaugural lecture.

Portnoy's second erroneous inference occurs when she invites comparison between Klein's "Erlanger Programm" and Riemann's equally famous *Probevorlesung* of 1854, "Ueber die Hypothesen welche der Geometrie zu Grunde liegen." The latter was, of course, the written version of Riemann's oral presentation delivered before the faculty in Göttingen. Portnoy suggests that, by comparison with Klein's "Erlanger Programm," Riemann's classic was a "poor candidate for publication, especially while its author was relatively unknown" [Portnoy 1982, 13]. Her conclusion here may be perfectly correct, but since Klein's paper was neither delivered orally nor written as a publishable version of a public presentation, it is hard to see how comparing the two can lend any support to her statement. The circumstances surrounding both works were in fact so dissimilar that it is difficult to imagine any meaningful basis for comparison between them.

As for Klein's *Antrittsrede*, its theme, as already noted, was mathematics education. Pedagogy, it might be recalled, was one of Klein's central interests during his later career at Göttingen (1886-1913), especially after 1892, when H. A. Schwarz was called from Göttingen to Berlin [3]. Klein's lectures for *Lehramtskandidaten*, published under the title *Elementarmathematik vom höheren Standpunkte aus* [Klein 1924-1928], became exceptionally popular, were translated into several foreign languages, and were repeatedly issued in numerous editions. Moreover, in 1908 Klein was elected president of the *Internationale Mathematische Unterrichtskommission*, and during his tenure he oversaw publication of several large tomes devoted to the history of mathematics education prior to World War I. All told, he published over thirty books and articles dealing with educational matters.

Considering Klein's intense interest and active participation in pedagogical problems later in his career, as well as his many accomplishments within this sphere, it certainly is surprising

that historians of mathematics have generally overlooked the fact that the "Erlanger Programm" was accompanied by an educational program as well. This omission is especially curious considering Klein's own, very extensive discussion of his Erlangen Antrittsrede in his autobiographical sketch:

*In my inaugural speech in December, I set forth a detailed program for my teaching plans while asserting that the unity of all knowledge and the ideal of a complete education should not be neglected because of specialized studies. Accordingly, humanistic and mathematical-scientific education belong together, and should not be placed in opposition to one another. On the other hand, it is necessary to cultivate applied mathematics as well as pure, in order to preserve the connection between neighboring disciplines such as physics and technology. Furthermore, along with the development of logical capability, equal account must be taken of the need to develop intuition and, more generally, mathematical imagination, out of which creative originality can arise. Finally the universities should pay heed to the preparatory teaching in the schools, and thus place particular emphasis on the education of school teachers. Here the organizations in the technical high schools could be examined, and in many respects taken as a model.*

*On the basis of these considerations, the following practical steps were suggested. Regularly repeated elementary lectures should be held along with the special lectures intended for a small number of serious students of science, both backed up by means of exercises and seminar activity. Moreover, courses in descriptive geometry should be offered that stress drawing capability. Furthermore a reading room and library with open-stacks should be established, which would make it possible for students to study the published literature, while extensive model collections would aid them in developing their mathematical intuition.*

*These points of view and practical suggestions grew out of the numerous observations I had made during my years of travel. They have also remained the essential guiding principles for my later activity; only in practice, due to the extent of the demands placed on the students, have I modified this somewhat by separating the elementary lectures into beginning and intermediate courses. [4] [Klein 1923, 18]*

It is important to see this speech in its proper perspective. Klein had just been called to Erlangen as an *Ordinarius* (i.e.,

full professor) at the unusually young age of twenty-three. His *Antrittsrede* was given before the faculty senate, a body to which only the *Ordinarien* were allowed membership. It would be interesting to know how the esteemed senate responded to this address, but one can well imagine that many probably dismissed it as nothing more than the wild-eyed idealism of a mere youth.

On the other hand, it must not be overlooked that by 1923 Klein's recollection of the speech he gave over fifty years earlier was apparently rather vague. This becomes quite obvious when comparing his remarks with the actual content of the Erlangen *Antrittsrede* (a transcription of Klein's *Antrittsrede* together with commentary will be presented in a forthcoming issue) [5]. Nevertheless, it cannot be denied that Felix Klein was very much interested in educational issues from the very beginning of his career. The fact that he had to overcome several decades of steadfast resistance to his efforts on behalf of educational reform may, quite possibly, serve to obscure rather than illuminate his activities in this arena [6]. In light of this and the obvious significance Klein himself attached to his Erlangen *Antrittsrede*, it is important to establish the actual facts and circumstances surrounding this event and its relation to the "Erlanger Programm" by giving Klein's speech the attention it deserves as an early statement of his pedagogical objectives—ones that would develop and mature over the remainder of his life.

#### NOTES

1. According to Richard Courant, the "Erlanger Programm" was perhaps the best read and most influential mathematical work of the sixty-year period 1865-1925 [Courant 1925, 766]. One finds it mentioned even in such unlikely places as Spengler's *Decline of the West* (New York: Modern Library, p. 68).

2. Klein returned to it often, but nowhere in more striking fashion than in his last work on relativity theory, where group invariants play an important role. See [Klein 1921, 533-612] and especially [Klein 1967, Vol. II]. For a general assessment of the significance of the "Erlanger Programm," see [Carathéodory 1919].

3. Schwarz succeeded Weierstrass who retired in 1892. This gave Klein the freedom to develop his scientific and educational reforms at Göttingen [Klein 1923, 23]. For a discussion of the behind the scenes rivalry between Klein and the Berlin school (including Schwarz), see [Biermann 1973, 129-131].

4. Because this work is fairly inaccessible (the only library in this country known to the author that holds it being Harvard University), the original German is made available here:

*In meiner Antrittsrede im Dezember legte ich ein ausführliches Programm meiner geplanten Unterrichtstätigkeit vor. Ueber den Spezialstudien darf die*

*Einheit aller Wissenschaft und das Ideal einer Gesamtbildung nicht vergessen werden. Daher gehören auch humanistische und mathematisch-naturwissenschaftliche Bildung zusammen und dürfen nicht in Gegensatz gebracht werden. Andererseits ist neben der reinen auch die angewandte Mathematik zu pflegen, um den Zusammenhang mit den angrenzenden Wissenschaftsgebieten wie Physik und Technik zu wahren. Ferner muss in der Mathematik neben den logischen Fähigkeiten die Anschauung als gleichberechtigter Faktor und überhaupt die mathematische Phantasie und die aus ihr entspringende Selbsttätigkeit entwickelt werden. Schliesslich hat die Universität auch den vorbereitenden Unterricht in den Schulen zu beachten und daher besonderes Gewicht auf die Ausbildung der Lehramtskandidaten zu legen, wobei die Einrichtungen der technischen Hochschulen in mancher Beziehung als vorbildlich betrachtet werden können. Aus diesen Gesichtspunkten ergaben sich folgende praktische Forderungen: Es sind regelmässig wiederholte Elementarvorlesungen und daneben Spezialvorlesungen für eine kleinere Zahl wissenschaftlich Interessierter abzuhalten, die sich beide auf Uebungen und Seminarbetrieb stützen. Daneben müssen Kurse in darstellender Geometrie mit Betonung des zeichnerischen Könnens stattfinden. Ferner ist ein Lesezimmer mit Präsenzbibliothek einzurichten, welches den Studierenden das Studium der einschlägigen Litteratur möglich macht, während ausgedehnte Modellsammlungen für die Ausbildung der mathematischen Anschauung zu sorgen haben. Diese Gesichtspunkte und praktischen Vorschläge hatte ich mir während der Jahre des Wanderns in mannigfacher Beobachtung gesammelt. Sie sind auch die wesentlichen Richtlinien für meine spätere Wirksamkeit geblieben; nur bin ich unter dem Druck der Praxis von der Höhe der an die Zuhörer gestellten Forderungen etwas abgegangen und habe später die Elementar-Vorlesungen in Anfangs- und Kursusvorlesungen getrennt.*

5. The unpublished manuscript of the Erlangen Antrittsrede is part of the Klein Nachlass housed at the Niedersächsische Staats- und Universitätsbibliothek Göttingen.

6. [Manegold 1970] has a good deal on the opposition to Klein's educational ideas (especially with regard to applied mathematics) and how it was eventually overcome. He also discusses Klein's Erlangen Antrittsrede (pp. 92-93).

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