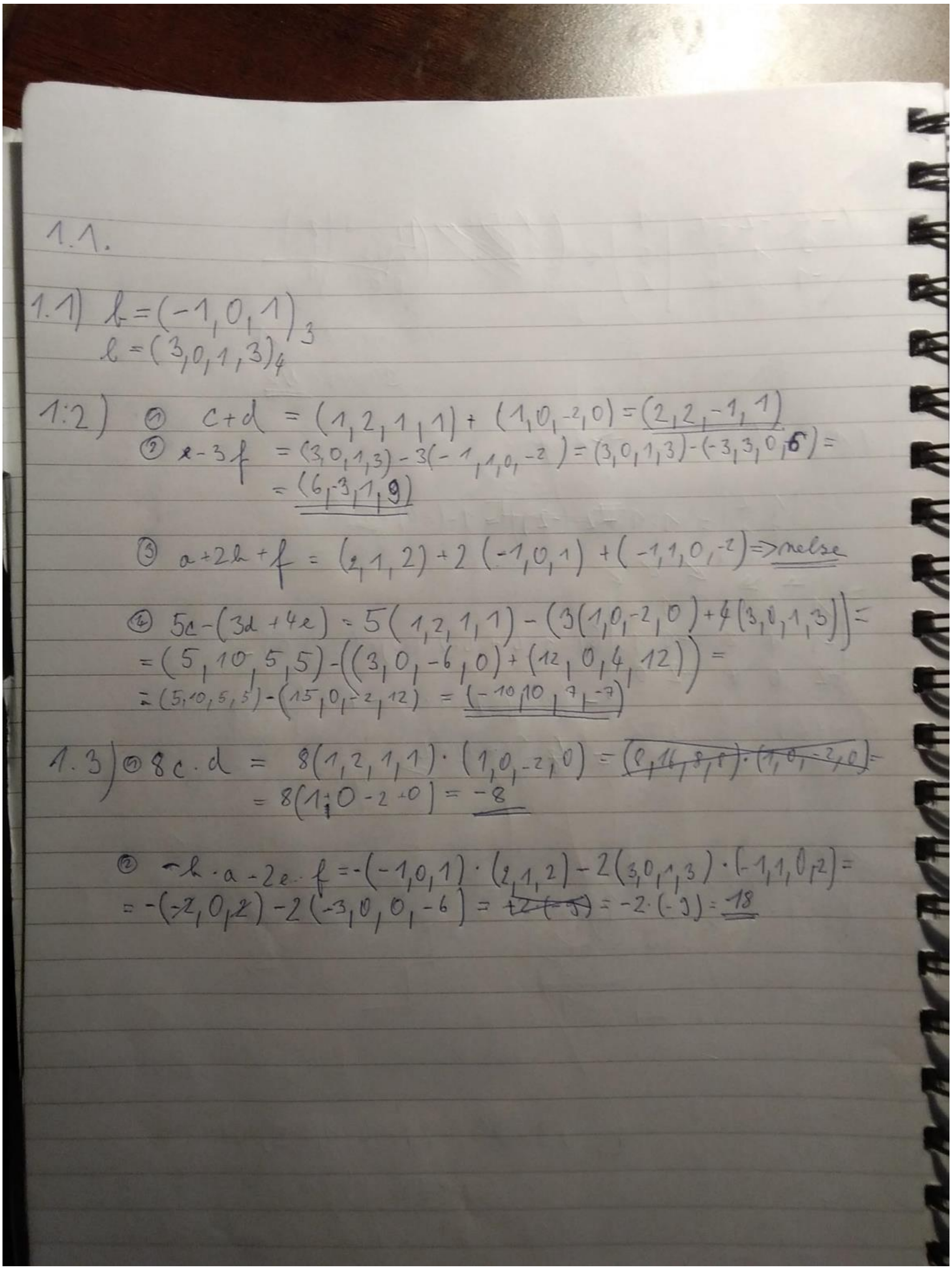
MATEMATIKA – 1. HODINA

SKUPINA A

1.1–1.3



1.1.

1.1) $b = (-1, 0, 1)_3$

$e = (3, 0, 1, 3)_4$

1.2) ① $c + d = (1, 2, 1, 1) + (1, 0, -2, 0) = \underline{(2, 2, -1, 1)}$

② $e - 3f = (3, 0, 1, 3) - 3(-1, 1, 0, -2) = (3, 0, 1, 3) - (-3, 3, 0, 6) =$

$= \underline{\underline{(6, -3, 1, 9)}}$

③ $a + 2b + f = (2, 1, 2) + 2(-1, 0, 1) + (-1, 1, 0, -2) \Rightarrow$ nelze

④ $5c - (3d + 4e) = 5(1, 2, 1, 1) - (3(1, 0, -2, 0) + 4(3, 0, 1, 3)) =$

$= (5, 10, 5, 5) - ((3, 0, -6, 0) + (12, 0, 4, 12)) =$

$= (5, 10, 5, 5) - (15, 0, -2, 12) = \underline{\underline{(-10, 10, 7, -7)}}$

1.3) ① $8c \cdot d = 8(1, 2, 1, 1) \cdot (1, 0, -2, 0) = \cancel{(8, 16, 8, 8) \cdot (1, 0, -2, 0)} =$

$= 8(1 + 0 - 2 + 0) = \underline{-8}$

② $-b \cdot a - 2e \cdot f = -(-1, 0, 1) \cdot (2, 1, 2) - 2(3, 0, 1, 3) \cdot (-1, 1, 0, 2) =$

$= -(-2, 0, 2) - 2(-3, 0, 0, -6) = \cancel{+2(-9)} = -2 \cdot (-9) = \underline{\underline{18}}$

1.4–1.7 (1)

$\boxed{1.2}$

$A = (2\,1\,2)$ $B = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 3 & 0 \\ -1 & 0 & -2 \end{pmatrix}$ $D = \begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix}$ $E = \begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix}$ $F = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

1.4

1. $C^T = \begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix}$

2. $F^T = \begin{pmatrix} 1 & -1 & 0 \\ 3 & 2 & 1 \\ 2 & 0 & 2 \end{pmatrix}$

1.5

1. B $\quad dim(B) = 3 \times 1$

2. $E \times D^T$ $\quad \left.\begin{array}{l} dim(E) = 2 \times 2 \\ dim(D) = 3 \times 2 \\ dim(D^T) = 2 \times 3 \end{array}\right\}$ $dim(E \times D^T) = dim(E_{2\times\cancel{2}} \times D^T_{\cancel{2}\times 3}) = 2 \times 3$

3. $F^T . C$ $\quad \left.\begin{array}{l} dim(F) = 3 \times 3 \\ dim(F^T) = 3 \times 3 \\ dim(C) = 2 \times 3 \end{array}\right\}$ $dim(F^T \times C) = dim(F^T_{3\times 3} \times C_{2\times 3})$

$\Rightarrow$ nejde

1.6

1. $E - F = \begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix} - \begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix} \Rightarrow$ nelze

2. $2C^T + 4D = 2.\begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & 2 \end{pmatrix} + 4\begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 6 & 0 \\ 0 & -4 \end{pmatrix} + \begin{pmatrix} 8 & -4 \\ 0 & 8 \\ -4 & 12 \end{pmatrix} = \begin{pmatrix} 10 & -6 \\ 6 & 8 \\ -4 & 8 \end{pmatrix}$

1.7

1. $E \times D^T = \begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix} \times \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix} = ((-2).2 + 0.(-1)\ ;\ (-2).0 + 0.2\ ;\ (-2).(-1) + 0.3)$

$= (-4+0\ ;\ 0+0\ ;\ 2+0) = -4;0;2$

$[(1.2) + (-2.)(-1)\ ;\ 1.0 + (-2).2\ ;\ 1.(-1) + (-2).3] = (2+2\ ;\ 0+(-4)\ ;\ (-1)+(-6))$

$= 4;-4;-7$

$\Rightarrow \begin{pmatrix} -4 & 0 & 2 \\ 4 & -4 & -7 \end{pmatrix}$

1.7 (2)

Příklad: 1.7

Násobení matic $C\begin{pmatrix}1 & 3 & 0\\ -1 & 0 & -2\end{pmatrix}$ $E\begin{pmatrix}-2 & 0\\ 1 & -2\end{pmatrix}$

2. $C \times C^T + 3E$ $\quad C^T\begin{pmatrix}1 & -1\\ 3 & 0\\ 0 & -2\end{pmatrix}$

$$\begin{pmatrix}1 & 3 & 0\\ -1 & 0 & -2\end{pmatrix}\cdot\begin{pmatrix}1 & -1\\ 3 & 0\\ 0 & -2\end{pmatrix} + 3\cdot\begin{pmatrix}-2 & 0\\ 1 & -2\end{pmatrix} =$$

$$= \begin{pmatrix}(1\cdot 1)+(3\cdot 3)+(0\cdot 0)\ ; & (-1\cdot 1)+(3\cdot 0)+(0\cdot -2)\\ (-1\cdot 1)+(0\cdot 3)+(-2\cdot 0)\ ; & (-1\cdot(-1))+(0\cdot 0)+(-2\cdot(-2))\end{pmatrix} + \begin{pmatrix}-6 & 0\\ 3 & -6\end{pmatrix} =$$

$$= \begin{pmatrix}1+9+0\ ; & -1+0+0\\ -1+0+0\ ; & 1+0+4\end{pmatrix} + \begin{pmatrix}-6 & 0\\ 3 & -6\end{pmatrix} = \begin{pmatrix}10 & -1\\ -1 & 5\end{pmatrix} + \begin{pmatrix}-6 & 0\\ 3 & -6\end{pmatrix} =$$

$$= \begin{pmatrix}4 & -1\\ 2 & -1\end{pmatrix}$$

1.8 (1)

Diagonální matice

$E \times D^T$ $\quad D^T$

E

$$\begin{pmatrix}-2 & 0\\ 1 & -2\end{pmatrix} \quad \begin{pmatrix}2 & 0 & -1\\ -1 & 2 & 3\end{pmatrix}$$

$E \times D^T$

$$\begin{pmatrix}(-2\cdot 2)+(0\cdot(-1))\ ; & (-2\cdot 0)+(0\cdot 2)\ ; & -2\cdot(-1)+(0\cdot 3)\\ (1\cdot 2)+(-2\cdot 2)\ ; & (1\cdot 0)+(-2\cdot 2)\ ; & 1\cdot(-1)+(-2\cdot 3)\end{pmatrix} = \begin{pmatrix}-4+0\ ; & 0+0\ ; & 2+0\\ 2-4\ ; & 0-4\ ; & -1-6\end{pmatrix}$$

$$= \begin{pmatrix}-4\ ; & 0\ ; & 2\\ -2\ ; & -4\ ; & -7\end{pmatrix} - (-4 - 4)$$

1.8 (2)

$2)\ C \times C^T + 3E = \begin{pmatrix} 4 & -1 \\ 2 & -1 \end{pmatrix} = (4; -1)$

$= (4\ \ -1)$

1.9 (1)

$(x+1)(x-3)=0$

$\downarrow \quad \downarrow$

$-1 \quad 3$

MATERIÁLY

1.3 PP: 1.9

$$1.\ \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \\ 7 \end{pmatrix} \Rightarrow \begin{pmatrix} -1 & 1 & -2 \\ 2 & 0 & 2 \\ 5 & -2 & 7 \end{pmatrix} \sim \begin{pmatrix} -1 & 1 & -2 \\ 0 & 2 & -2 \\ 0 & 3 & -3 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} -1 & 1 & -2 \\ 0 & 2 & -2 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} -1 & 1 & -2 \\ 6 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow 1 \cdot \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} - 1 \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 7 \end{pmatrix}$$

sú lineárne závislé

1.9 (2)

2. $(2\ -1\ 3),\ (-1\ 0\ -2),\ (5\ 1\ 4)$

$$\begin{pmatrix} 2 & -1 & 5 \\ -1 & 0 & 1 \\ 3 & -2 & 4 \end{pmatrix} \sim \begin{pmatrix} -1 & 0 & 1 \\ 2 & -1 & 5 \\ 3 & -2 & 4 \end{pmatrix} \sim \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 7 \\ 0 & -2 & 7 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 7 \\ 0 & 0 & 7 \end{pmatrix} \sim \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 7 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\downarrow$$

$$1 \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 7 \begin{pmatrix} -1 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 2-7 \\ -1+0 \\ 3-14 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \\ -11 \end{pmatrix}$$

$\begin{pmatrix} 5 \\ 1 \\ 4 \end{pmatrix} \neq \begin{pmatrix} -5 \\ -1 \\ -11 \end{pmatrix} \Rightarrow$ vektory sú lineárne nezávislé

1.10

HODNOST MATICE **1.10**

1. $$\begin{vmatrix} -1 & 1 & -2 \\ 2 & 0 & 2 \\ 5 & -2 & 7 \end{vmatrix} \sim \begin{vmatrix} -1 & 1 & -2 \\ 0 & 2 & -2 \\ 5 & -2 & 7 \end{vmatrix} \sim \begin{vmatrix} -1 & 1 & -2 \\ 0 & 2 & -2 \\ 0 & 3 & -3 \end{vmatrix} \sim \begin{vmatrix} -1 & 1 & -2 \\ 0 & 2 & -2 \\ 0 & 0 & 0 \end{vmatrix} = 2$$

2. $$\begin{vmatrix} 2 & -1 & 5 \\ -1 & 0 & 1 \\ 3 & -2 & 4 \end{vmatrix} \sim \begin{vmatrix} 2^{(\cdot)} & -1 & 5 \\ 0 & -1 & 7 \\ 3^{(\cdot -2)} & -2 & 4 \end{vmatrix} \sim \begin{vmatrix} 2 & -1 & 5 \\ 0 & -1 & 7 \\ 0 & 7 & 7 \end{vmatrix} \sim \begin{vmatrix} 2 & -1 & 5 \\ 0 & -1 & 7 \\ 0 & 0 & 56 \end{vmatrix} = 3$$

SOUSTAVY ROVNIC **1.11**

1. $$\begin{aligned} x_1 + 2x_2 + 3x_3 &= -2 \\ +2x_2 + 4x_3 &= 1 \\ -2x_1 + x_2 + 4x_3 &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 2 & 4 & 1 \\ -2 & 1 & 4 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 2^{(\cdot 5)} & 4 & 1 \\ 0 & 5^{(\cdot -2)} & 10 & -4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & 0 & 13 \end{array}\right] =$$

= Nemá řešení

$$= \left[\begin{array}{ccc|c} -1 & 3 & 5 & 5 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 0 & -2 & 4 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 0 & -2 & 4 \\ 0 & 3 & 3 & 9 \end{array}\right] (:3)$$

1.11

SOUSTAVY ROVNIC **1.11**

1. $$\begin{aligned} x_1 + 2x_2 + 3x_3 &= -2 \\ +2x_2 + 4x_3 &= 1 \\ -2x_1 + x_2 + 4x_3 &= 0 \end{aligned} \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 2 & 4 & 1 \\ -2 & 1 & 4 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & \overset{(-1)}{2} & 4 & 1 \\ 0 & \overset{(-2)}{5} & 10 & -4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & 0 & 13 \end{array}\right] =$$

= Nemá řešení

2. $$\begin{aligned} -x_1 + 3x_2 + 5x_3 &= 5 \\ -2x_1 + 2x_2 + 7x_3 &= -3 \\ x_1 \quad - 2x_3 &= 4 \end{aligned} \quad \left[\begin{array}{ccc|c} -1 & 3 & 5 & 5 \\ -2 & 2 & 7 & -3 \\ 1 & 0 & -2 & 4 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & -2 & 4 \\ -1 & 3 & 5 & 5 \\ -2 & 2 & 7 & -3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & 4 \\ 0 & 3 & 3 & 9 \\ -2 & 2 & 7 & -3 \end{array}\right] (:3)$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 2 & 3 & 5 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & 4 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right] = \boxed{\begin{aligned} x_1 &= 2 \\ x_2 &= 4 \\ x_3 &= -1 \end{aligned}}$$

1,12

1.4

1.12 1. $$\begin{vmatrix} -2 & -1 \\ 2 & 4 \end{vmatrix} = \left((-2)\cdot 4 - 2\cdot(-1)\right) = -6$$

2. $$\left|\begin{array}{ccc:cc} -2 & 3 & 0 & -2 & 3 \\ -2 & 1 & 5 & -2 & 1 \\ -1 & 0 & 4 & -1 & 0 \end{array}\right| =$$

$$= \left((-2)\cdot 1\cdot 4 + 3\cdot 5\cdot(-1) + 0\cdot(-2)\cdot 0\right) - \left(0\cdot 1\cdot(-1) + (-2)\cdot 5\cdot 0 + 3\cdot(-2)\cdot 4\right) =$$

$$= \left(\cancel{-8} -8 - 15\right) - \left(-24\right) = 1$$

1.13

Rovnice s determinanty:

1.

$$3\cdot\begin{vmatrix} 1 & x & 1 \\ 0 & -1 & 0 \\ -1 & x & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & x \\ 1 & -1 & 0 \\ x & 0 & 1 \end{vmatrix} = 5$$

$$3\cdot\left|\begin{array}{ccc|cc} 1 & x & 1 & 1 & x \\ 0 & -1 & 0 & 0 & -1 \\ -1 & x & 0 & -1 & x \end{array}\right| + \left|\begin{array}{ccc|cc} 0 & 1 & x & 0 & 1 \\ 1 & -1 & 0 & 1 & -1 \\ x & 0 & 1 & x & 0 \end{array}\right| = 5$$

$$3\cdot\left|[1\cdot(-1)\cdot 0] + [x\cdot 0\cdot(-1)] + (1\cdot 0\cdot x) - [-1\cdot(-1)\cdot 1] - (1\cdot 0\cdot x) - (0\cdot 0\cdot x)\right| +$$

$$+\left|[(0\cdot(-1)\cdot 1] + (1\cdot 0\cdot x) + (x\cdot 1\cdot 0) - [x\cdot x\cdot(-1)] - (0\cdot 0\cdot 0) - (1\cdot 1\cdot 1)\right| = 5$$

$$3\cdot(0+0+0-1-0-0) + (0+0+0-x^2-1) = 5$$

$$3\cdot(1) + (-x^2-1) = 5$$

$$3 - x^2 - 1 = 5$$

$$-x^2 = 9$$

$$\underline{\underline{x = \pm 3}}$$