SKUPINA A

1.1

1. $b = (-1, 0, 1) \Rightarrow 3$
2. $e = (3, 0, 1, 3) \Rightarrow 4$

> pouze spočítam kolik je tam položek

1.2

1. $c + d = (1, 2, 1, 1) + (1, 0, -2, 0) = (2, 2, -1, 1)$
2. $e - 3f = (3, 0, 1, 3) + 3(-1, 1, 0, -2) = (3, 0, 1, 3) - (-3, 3, 0, -6) =$
   $= (6, -3, 1, 9)$
3. $a + 2b + f = (2, 1, 2) + 2(-1, 0, 1) + (-1, 1, 0, -2) \Rightarrow$ nejde
4. $5c - (3d + 4e) = 5(1, 2, 1, 1) - (3(1, 0, -2, 0) + 4(3, 0, 1, 3)) =$
   $= (5, 10, 5, 5) - ((3, 0, -6, 0) + (12, 0, 4, 12)) =$
   $= (5, 10, 5, 5) - (15, 0, -2, 12) = (-10, 10, 7, -7)$

1.3

1. $8c \times d = 8(1, 2, 1, 1) \cdot (1, 0, -2, 0) = 8(1 + 0 - 2 + 0) = -8$

2. $-b \times a - 2e \times f = -(-1, 0, 1) \cdot (2, 1, 2) - [2(3, 0, 1, 3) \cdot (-1, 1, 0, -2)] =$
   $= -(-2 + 0 + 2) - (2 \cdot (-3 + 0 + 0 - 6)) =$
   $= -(-2 + 0 + 2) - (-6 + 0 + 0 - 12) = 0 - (-18) = 18$

1.4

1. $C^T = \begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix}$ $\quad C = \begin{pmatrix} 1 & 3 & 0 \\ -1 & 0 & -2 \end{pmatrix}$

2. $F^T = \begin{pmatrix} 1 & -1 & 0 \\ 3 & 2 & 1 \\ 2 & 0 & 2 \end{pmatrix}$ $\quad F = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

1.5

1. $B \Rightarrow \dim(B) = 3 \times 1$
2. $E \times D^T \Rightarrow \dim(E \times D^T) =$ $\dim(E) = 2 \times 2$, $\dim(D^T) = 2 \times 3$ $\;>\; E_{2\times2} D^T_{2\times3} = 2 \times 3$
3. $F^T \times C \Rightarrow \dim(F^T \times C) =$ $\dim(F^T) = 3 \times 3$, $\dim(C) = 3 \times 2$

[1.6]

1. $E - F = \begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix} - \begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix} \Rightarrow$ HELZE

2. $2C^T + 4D = 2\begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix} + 4\begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 6 & 0 \\ 0 & -4 \end{pmatrix} + \begin{pmatrix} 8 & -4 \\ 0 & 8 \\ -4 & 12 \end{pmatrix} =$

$= \begin{pmatrix} 10 & -6 \\ 6 & 8 \\ -4 & 8 \end{pmatrix}$

[1.7]

1. $E \times D^T = \begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} -2\cdot2 + 0\cdot(-1) & -2\cdot0 + 0\cdot2 & -2\cdot(-1) + 0\cdot3 \\ 1\cdot2 + (-2)(-1) & 1\cdot0 + (-2)2 & 1(-1) + (-2)\cdot3 \end{pmatrix}$

$= \begin{pmatrix} -4 & 0 & 2 \\ 4 & -4 & -7 \end{pmatrix}$

2. $C \times C^T + 3E = \begin{pmatrix} 1 & 3 & 0 \\ -1 & 0 & -2 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix} + 3\begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix} =$

$= \begin{pmatrix} 1\cdot1 + 3\cdot3 + 0\cdot0 & 1\cdot(-1) + 3\cdot0 + (-2)\cdot0 \\ -1\cdot1 + 3\cdot0 + (-2)\cdot0 & (-1)\cdot(-1) + (-1)\cdot0 + (-2)(-2) \end{pmatrix} + \begin{pmatrix} -6 & 0 \\ 3 & -6 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 2 & -1 \end{pmatrix}$

[1.8]

1. $E \times D^T = \begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} -2\cdot2 + 0\cdot(-1) & -2\cdot0 + 0\cdot2 & -2\cdot(-1) + 0\cdot3 \\ 1\cdot2 + (-2)(-1) & 1\cdot0 + (-2)\cdot2 & 1\cdot(-1) + (-2)\cdot3 \end{pmatrix}$

$= \begin{pmatrix} -4 & 0 & 2 \\ 4 & -4 & -7 \end{pmatrix}$

$\mathrm{diag}(E \times D^T) = \begin{pmatrix} -4 & -4 \end{pmatrix}$

2. $C \times C^T + 3E = \begin{pmatrix} 4 & -1 \\ 2 & -1 \end{pmatrix} \Rightarrow \mathrm{diag}(C \times C^T + 3E) = \begin{pmatrix} 4 & -1 \end{pmatrix}$

[1.9] + [1.10]

1. $\begin{pmatrix}-1\\2\\5\end{pmatrix};\ \begin{pmatrix}1\\0\\-2\end{pmatrix};\ \begin{pmatrix}-2\\2\\7\end{pmatrix}$

$$\begin{pmatrix}-1 & 1 & -2\\ 2 & 0 & 2\\ 5 & -2 & 7\end{pmatrix}\begin{matrix}\cdot 2\\ \\ \\ \end{matrix} \sim \begin{pmatrix}-2 & 2 & -4\\ 2 & 0 & 2\\ 5 & -2 & 7\end{pmatrix}\begin{matrix}+\\ \\ \\ \end{matrix} \sim \begin{pmatrix}-2 & 2 & -4\\ 0 & 2 & -2\\ 5 & -2 & 7\end{pmatrix}\begin{matrix}:2,\cdot 5\\ \\ \\ \end{matrix} \sim$$

$$\sim \begin{pmatrix}-5 & 5 & -10\\ 0 & 2 & -2\\ 5 & -2 & 7\end{pmatrix}\begin{matrix}+\\ \\ \\ \end{matrix} \sim \begin{pmatrix}-5 & 5 & -10\\ 0 & 2 & -2\\ 0 & 3 & -3\end{pmatrix}\begin{matrix}:5\\ :2\\ :3\end{matrix} \sim \begin{pmatrix}-1 & 1 & -2\\ 0 & 1 & -1\\ 0 & 1 & -1\end{pmatrix}\begin{matrix}\\ \\ -\end{matrix} \sim$$

$$\sim \begin{pmatrix}-1 & 1 & -2\\ 0 & 1 & -1\\ 0 & 0 & 0\end{pmatrix}$$

- lineárně závislé
- hodnota matice 2

2. $(2\ -1\ 3);\ (-1, 0, -2);\ (5\ 1\ 4)$

$$\begin{pmatrix}2 & -1 & 5\\ -1 & 0 & 1\\ 3 & -2 & 4\end{pmatrix}\begin{matrix}\\ \cdot 2\\ \\ \end{matrix} \sim \begin{pmatrix}2 & -1 & 5\\ -2 & 0 & 2\\ 3 & -2 & 4\end{pmatrix}\begin{matrix}+\\ \\ \\ \end{matrix} \sim \begin{pmatrix}2 & -1 & 5\\ 0 & -1 & 7\\ 3 & -2 & 4\end{pmatrix}\begin{matrix}\cdot 3\\ \\ \cdot 2\end{matrix} \sim$$

$$\sim \begin{pmatrix}6 & -3 & 15\\ 0 & -1 & 7\\ 6 & -4 & 8\end{pmatrix}\begin{matrix}\\ -\\ \\ \end{matrix} \sim \begin{pmatrix}6 & -3 & 15\\ 0 & -1 & 7\\ 0 & -1 & -7\end{pmatrix}\begin{matrix}:3\\ \\ -\end{matrix} \sim \begin{pmatrix}2 & -1 & 5\\ 0 & -1 & 7\\ 0 & 0 & +14\end{pmatrix}$$

- hodnota matice 3, lineárně nezávislá

1.11

1.
$$
\begin{aligned}
x_1 + 2x_2 + 3x_3 &= -2 \\
2x_2 + 4x_3 &= 1 \\
-2x_1 + x_2 + 4x_3 &= 0
\end{aligned}
$$

$$
\left(\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 2 & 4 & 1 \\ -2 & 1 & 4 & 0 \end{array}\right) \cdot 2
\sim
\left(\begin{array}{ccc|c} 2 & 4 & 6 & -4 \\ 0 & 2 & 4 & 2 \\ -2 & 1 & 4 & 0 \end{array}\right) +
\sim
$$

$$
\sim
\left(\begin{array}{ccc|c} 2 & 4 & 6 & -4 \\ 0 & 2 & 4 & 1 \\ 0 & 5 & 10 & -4 \end{array}\right)
\begin{array}{l} :2 \\ \cdot 5 \\ \cdot 2 \end{array}
\sim
\left(\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 10 & 20 & 5 \\ 0 & 10 & 20 & -8 \end{array}\right) -
\sim
\left(\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 10 & 20 & 5 \\ 0 & 0 & 0 & -13 \end{array}\right)
$$

$0 \neq 13 \Rightarrow$ NEMÁ ŘEŠENÍ

2.
$$
\begin{aligned}
-x_1 + 3x_2 + 5x_3 &= 5 \\
-2x_1 + 2x_2 + 7x_3 &= -3 \\
x_1 \qquad - 2x_3 &= 4
\end{aligned}
$$

$$
\left(\begin{array}{ccc|c} -1 & 3 & 5 & 5 \\ -2 & 2 & 7 & -3 \\ 1 & 0 & -2 & 4 \end{array}\right) \cdot 2
\sim
\left(\begin{array}{ccc|c} -2 & 6 & 10 & 10 \\ -2 & 2 & 7 & -3 \\ 1 & 0 & -2 & 4 \end{array}\right) -
\sim
\left(\begin{array}{ccc|c} -2 & 6 & 10 & 10 \\ 0 & -4 & -3 & -13 \\ 1 & 0 & -2 & 4 \end{array}\right) :2
\sim
$$

$$
\sim
\left(\begin{array}{ccc|c} -1 & 3 & 5 & 5 \\ 0 & -4 & -3 & -13 \\ 1 & 0 & -2 & 4 \end{array}\right) +
\sim
\left(\begin{array}{ccc|c} -1 & 3 & 5 & 5 \\ 0 & -4 & -3 & -13 \\ 0 & 3 & 3 & 9 \end{array}\right) :3
\sim
\left(\begin{array}{ccc|c} -1 & 3 & 5 & 5 \\ 0 & -4 & -3 & -13 \\ 0 & 1 & 1 & 3 \end{array}\right) \cdot 4
\sim
$$

$$
\sim
\left(\begin{array}{ccc|c} -1 & 3 & 5 & 5 \\ 0 & -4 & -3 & -13 \\ 0 & 4 & 4 & 12 \end{array}\right) +
\sim
\left(\begin{array}{ccc|c} -1 & 3 & 5 & 5 \\ 0 & -4 & -3 & -13 \\ 0 & 0 & 1 & -1 \end{array}\right) \cdot 3
\sim
\left(\begin{array}{ccc|c} -1 & 3 & 5 & 5 \\ 0 & -4 & -3 & -13 \\ 0 & 0 & 3 & -3 \end{array}\right) +
\sim
$$

$$
\sim
\left(\begin{array}{ccc|c} -1 & 3 & 5 & 5 \\ 0 & -4 & 0 & -16 \\ 0 & 0 & +3 & -3 \end{array}\right) :3;\ \cdot 5
\sim
\left(\begin{array}{ccc|c} -1 & 3 & 5 & 5 \\ 0 & -4 & 0 & -16 \\ 0 & 0 & 5 & -5 \end{array}\right) -
\sim
\left(\begin{array}{ccc|c} -1 & 3 & 0 & 10 \\ 0 & -4 & 0 & -16 \\ 0 & 0 & 5 & -5 \end{array}\right)
\begin{array}{l} \cdot 4 \\ \cdot 3 \\ :5 \end{array}
\sim
$$

$$
\sim
\left(\begin{array}{ccc|c} -4 & 12 & 0 & 40 \\ 0 & -12 & 0 & -48 \\ 0 & 0 & 1 & -1 \end{array}\right) +
\sim
\left(\begin{array}{ccc|c} -4 & 0 & 0 & -8 \\ 0 & -12 & 0 & -48 \\ 0 & 0 & 1 & -1 \end{array}\right)
\Rightarrow
\begin{array}{l} x_1 = 2 \\ x_2 = 4 \end{array}
\quad x_3 = -1
$$

1.12

1. $\begin{vmatrix} -2 & -1 \\ 2 & 4 \end{vmatrix} = -2 \cdot 4 - 2 \cdot (-1) = \underline{\underline{-6}}$

2. $\left|\begin{array}{ccc|cc} -2 & 3 & 0 & -2 & 3 \\ -2 & 1 & 5 & -2 & 1 \\ -1 & 0 & 4 & -1 & 0 \end{array}\right| = \left(-2 \cdot 1 \cdot 4 + 3 \cdot 5 \cdot (-1) + 0 \cdot -2 \cdot 0\right) - \left(-1 \cdot 1 \cdot 0 + 0 \cdot 5 \cdot -2 + 4 \cdot -2 \cdot 3\right)$

$= \left(-8 + (-15) + 0\right) - \left(0 + 0 + (-24)\right) = \underline{\underline{1}}$

1.13

1. $3 \underbrace{\begin{vmatrix} 1 & x & 1 \\ 0 & -1 & 0 \\ -1 & x & 0 \end{vmatrix}}_{a} + \underbrace{\begin{vmatrix} 0 & 1 & x \\ 1 & -1 & 0 \\ x & 0 & 1 \end{vmatrix}}_{b} = 5$

a, $3 \cdot \left[\left(1 \cdot (-1) \cdot 0 + x \cdot 0 \cdot -1 + 1 \cdot 0 \cdot x\right) - \left(-1 \cdot (-1) \cdot 1 + x \cdot 0 \cdot 1 + 0 \cdot 0 \cdot x\right)\right] =$
$= 3\,(0 - 1) = -3$

b, $\left(0 \cdot -1 \cdot 1 + 1 \cdot 0 \cdot x + x \cdot 1 \cdot 0\right) - \left(x \cdot -1 \cdot x + 0 \cdot 0 \cdot 0 + 1 \cdot 1 \cdot 1\right) = x^2 - 1$

$\Rightarrow$ $-3 + x^2 - 1 = 5$

$x^2 = 9$

$\underline{\underline{x = \pm 3}}$