1.1. DÉLKA VEKTORŮ

1. $a = (2, 1, 2)_3$
2. $f = (-1, 1, 0, -2)_4$

1.2. SČÍTÁNÍ VEKTORŮ, ODČÍTÁNÍ VEKTORŮ, NÁSOBENÍ SKALÁREM

1. $d + e = (1, 0, -2, 0) + (3, 0, 1, 3) = (4, 0, -1, 3)$
2. $2c - d = 2\,(1, 2, 1, 1) - (1, 0, -2, 0) = (2, 4, 2, 2) - (1, 0, -2, 0) = (1, 4, 4, 2)$
3. $b - f + 3e = (-1, 0, 1) - (-1, 1, 0, -2) + 3\,(3, 0, 1, 3) =$ nelze
4. $3f - (4d - c) = 3\,(-1, 1, 0, -2) - [4\,(1, 0, -2, 0) - (1, 2, 1, 1)] =$
   $= (-3, 3, 0, -6) - [(4, 0, -8, 0) - (1, 2, 1, 1)] =$
   $= (-3, 3, 0, -6) - (3, -2, -9, -1) = (-6, 5, 9, -5)$

1.3. SKALÁRNÍ SOUČIN VEKTORŮ

1. $3e \times c = 3\,(3, 0, 1, 3) \times (1, 2, 1, 1) = (9, 0, 3, 9) \times (1, 2, 1, 1) = (9 + 0 + 3 + 9) = 21$
2. $a \times b - 8d \times f = (2, 1, 2) \times (-1, 0, 1) - 8\,(1, 0, -2, 0) \times (-1, 1, 0, -2) =$
   $= (-2 + 0 + 2) + (-8, 0, +16, 0) \times (-1, 1, 0, -2) = (8 + 0 + 0 + 0) = 8$

1.4. TRANSPOZICE MATIC

1. $D^T = \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix}$
2. $F^T = \begin{pmatrix} 1 & -1 & 0 \\ 3 & 2 & 1 \\ 2 & 0 & 2 \end{pmatrix}$

$D = \begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix}$

$F = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

1.5. DIMENZE MATIC

1. C

   $\dim C = 2 \times 3$
2. $D^T \times C^T$

   $\dim (\underset{2\times3}{D^T} \times \underset{3\times2}{C^T}) = 2 \times 2$
3. $E \times F$

   $\dim (E \times F) =$ nelze

1.6. SČÍTÁNÍ MATIC, ODČÍTÁNÍ MATIC, NÁSOBENÍ SKALÁREM

1. $B - 2E = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - 2 \begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix} =$ nelze
2. $3D^T + 2C = 3 \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix} + 2 \begin{pmatrix} 1 & 3 & 0 \\ -1 & 0 & -2 \end{pmatrix} = \begin{pmatrix} 6 & 0 & -3 \\ -3 & 6 & 9 \end{pmatrix} + \begin{pmatrix} 2 & 6 & 0 \\ -2 & 0 & -4 \end{pmatrix} = \begin{pmatrix} 8 & 6 & -3 \\ -5 & 6 & 5 \end{pmatrix}$

$$D = \begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix}$$

$$D^T = \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 3 & 0 \\ -1 & 0 & -2 \end{pmatrix}$$

$$C^T = \begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix}$$

$$F = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$$

1.7. NÁSOBENÍ MATIC

1. $$D^T \times C^T = \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix} =$$

$$= \begin{pmatrix} 2+0+0 \; ; & -2+0+2 \\ -1+6+0 \; ; & +1+0-6 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 5 & -5 \end{pmatrix}$$

2. $$C \times F + 2D^T = \begin{pmatrix} 1 & 3 & 0 \\ -1 & 0 & -2 \end{pmatrix} \times \begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix} + 2\begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix} =$$

$$= \begin{pmatrix} +1-3+0 \; ; & 3+6+0 \; ; & 2+0+0 \\ -1+0+0 \; ; & -3+0-2 \; ; & -2+0-4 \end{pmatrix} = \begin{pmatrix} -2 & 9 & 2 \\ -1 & -5 & -6 \end{pmatrix} + \begin{pmatrix} 4 & 0 & -2 \\ -2 & 4 & 6 \end{pmatrix} =$$

$$= \begin{pmatrix} 2 & 9 & 0 \\ -3 & -1 & 0 \end{pmatrix}$$

1.8. DIAGONÁLA MATICE

$$D^T \times C^T = \begin{pmatrix} 2 & 0 \\ 5 & -5 \end{pmatrix} = \begin{pmatrix} 2 & -5 \end{pmatrix}$$

$$C \times F + 2D^T = \begin{pmatrix} 2 & 9 & 0 \\ -3 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & -1 \end{pmatrix}$$

1.9. LINEÁRNÍ ZÁVISLOST A NEZÁVISLOST VEKTORŮ

1. $(-1\;2\;5)$; $(-3\;2\;7)$; $(1\;0\;-2)$

$$\begin{pmatrix} -1 & -3 & 1 \\ 2 & 2 & 0 \\ 5 & 7 & -2 \end{pmatrix} \sim \begin{pmatrix} -1 & -3 & 1 \\ -4 & -4 & 0 \\ 5 & 7 & -2 \end{pmatrix} \sim \begin{pmatrix} -1 & -3 & 1 \\ -5 & -7 & 1 \\ 5 & 7 & -2 \end{pmatrix} \sim \begin{pmatrix} -1 & -3 & 1 \\ -5 & -7 & 1 \\ 0 & 0 & -2 \end{pmatrix} \sim \begin{pmatrix} 5 & 15 & -5 \\ -5 & -7 & 1 \\ 0 & 0 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & -2 \end{pmatrix}$$

LN

2. $$\begin{pmatrix} 1 & -1 & -3 \\ 0 & 2 & 2 \\ -1 & 5 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -3 \\ 0 & 2 & 2 \\ 0 & 4 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -3 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -3 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 2 & -2 & -6 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix} \sim$$

$$\begin{pmatrix} 2 & 0 & -4 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$ Lin. závislé

$$-2\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + 1\begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ 7 \end{pmatrix}$$

1.10. HODNOST MATICE

1. $\operatorname{rank}\begin{pmatrix} -1 & -3 & 1 \\ 2 & 2 & 0 \\ 5 & 7 & -2 \end{pmatrix} \underset{\operatorname{rank}}{\sim} \begin{pmatrix} 1 & 3 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & -2 \end{pmatrix} = 3$

2. $\operatorname{rank}\begin{pmatrix} 1 & -1 & -3 \\ 0 & 2 & 2 \\ -1 & 5 & 7 \end{pmatrix} \underset{\operatorname{rank}}{\sim} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} = 2$

1.11. ŘEŠENÍ SOUSTAVY LIN. ROVNIC

1. $\left(\begin{array}{ccc|c} 2 & -3 & -1 & -7 \\ 3 & 1 & 1 & 4 \\ -1 & 4 & 6 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -4 & -6 & 3 \\ 2 & -3 & -1 & -7 \\ 3 & 1 & 1 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -4 & -6 & 3 \\ 0 & 5 & 11 & 13 \\ 0 & 13 & 19 & -5 \end{array}\right) \sim$

$\sim \left(\begin{array}{ccc|c} 1 & -4 & -6 & 3 \\ 0 & 5 & 11 & -13 \\ 0 & 3 & -3 & 21 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -4 & -6 & 3 \\ 0 & 5 & 11 & -13 \\ 0 & 1 & -1 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -4 & -6 & 3 \\ 0 & 5 & 11 & -13 \\ 0 & 0 & 16 & -48 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -4 & -6 & 3 \\ 0 & 5 & 11 & -13 \\ 0 & 0 & 1 & -3 \end{array}\right) \sim$

$\sim \left(\begin{array}{ccc|c} 1 & -4 & -6 & 3 \\ 0 & 5 & 0 & 20 \\ 0 & 0 & 1 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -6 & 19 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{array}\right)$

$x_1 = 1$

$x_2 = 4$

$x_3 = -3$

2. $\left(\begin{array}{ccc|c} 2 & -1 & 0 & 4 \\ -3 & 1 & 4 & -1 \\ -1 & 0 & 4 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -4 & -1 \\ 2 & -1 & 0 & 4 \\ -3 & 1 & 4 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -4 & -1 \\ 0 & -1 & 8 & 6 \\ 0 & 1 & -8 & -4 \end{array}\right) \sim$

$\sim \left(\begin{array}{ccc|c} 1 & 0 & -4 & -1 \\ 0 & -1 & 8 & 6 \\ 0 & 0 & 0 & 2 \end{array}\right)$ nemá řešení

1.12. DETERMINANT MATICE

1. $\begin{vmatrix} 2 & -5 \\ 1 & 4 \end{vmatrix} = (2 \cdot 4) - (-5) \cdot 1 = 8 + 5 = 13$

2. $\begin{vmatrix} 2 & -1 & 0 \\ -3 & 1 & 4 \\ -1 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 2 & -1 & 0 \\ -3 & 1 & 4 \\ -1 & 1 & 5 \end{vmatrix} = 10 + 4 + 0 - (15 + 8 + 0) = 14 - 23 = -9$

1.13. ROVNICE S DETERMINANTY

1. $\begin{vmatrix} 0 & 2 & x \\ -1 & x & 0 \\ 3 & 2 & -1 \end{vmatrix} + \begin{vmatrix} 3 & x & 2 \\ -1 & 0 & 2 \\ x & -1 & x \end{vmatrix} = 4$

$$\begin{vmatrix} 0 & 2 & x \\ -1 & x & 0 \\ 3 & 2 & -1 \end{vmatrix} + \begin{vmatrix} 3 & x & 2 \\ -1 & 0 & 2 \\ x & -1 & x \end{vmatrix} = 4$$

$$-2x - (2 + 3x^2) + [(2x^2 + 2) - (-x^2 - 6)] = 4$$

$$-2x - 2 - 3x^2 + (2x^2 - 2 + x^2 + 6) = 4$$

$$-2x - 2 - 3x^2 + 2x^2 - 2 + x^2 + 6 = 4$$

$$0x^2 - 2x - 2 = 0$$

$$2x - 2 = 0$$

$$2x = 2$$

$$x = 1$$

DOSAZENÍ:

$$\begin{vmatrix} 0 & 2 & 1 \\ -1 & 1 & 0 \\ 3 & 2 & -1 \end{vmatrix} + \begin{vmatrix} 3 & 1 & 2 \\ -1 & 0 & 2 \\ 1 & -1 & 1 \end{vmatrix} = 4$$

$$\begin{vmatrix} 0 & 2 & 1 \\ -1 & 1 & 0 \\ 3 & 2 & -1 \end{vmatrix} + \begin{vmatrix} 3 & 1 & 2 \\ -1 & 0 & 2 \\ 1 & -1 & 1 \end{vmatrix}$$

$$-2 - (3 + 2) + [(2 + 2) - (-1 - 6) = 4$$

$$2 - 5 + (4 + 7) = 4$$

$$-7 + 11 = 4$$

$$4 = 4$$