

Skupina B

1

1.1

$$a = (2, 1, 2)_3$$

$$f = (-1, 1, 0, -2)_4$$

2.2

$$d + e = (1, 0, -2, 0) + (3, 0, 1, 3) = (4, 0, -2, 3)$$

$$2c - d = 2(1, 2, 1, 1) - (1, 0, -2, 0) = (2, 4, 2, 2) - (1, 0, -2, 0) = (1, 4, 4, 2)$$

$$b - f + 3e = (-1, 0, 1) - (-1, 1, 0, -2) + 3(3, 0, 1, 3) = \text{r\u016fzne dimenzie,}$$

nem\u00e1 sa vyrie\u0161it

$$3f - (4d - c) = 3(-1, 1, 0, -2) - [(4, 0, -8, 0) - (1, 2, 1, 1)] = (-3, 3, 0, -6) - (3, -2, -9, -1) \\ = (-6, 5, 9, -5)$$

1.3

$$3e \cdot c = 3(3, 0, 1, 3) \cdot (1, 2, 1, 1) = (9, 0, 3, 9) \cdot (1, 2, 1, 1) = \\ = 9 + 0 + 3 + 9 = \underline{\underline{21}}$$

$$= (-6, 5, 9, -5)$$

1.3

$$\begin{aligned} 3e \cdot c &= 3(3, 0, 1, 3) \cdot (1, 2, 1, 1) = (9, 0, 3, 9) \cdot (1, 2, 1, 1) = \\ &= 9 + 0 + 3 + 9 = \underline{21} \end{aligned}$$

$$\begin{aligned} a \cdot b - 8d \cdot f &= (2, 1, 2) \cdot (-1, 0, 1) - 8(1, 0, -2, 0) \cdot (-1, 1, 0, -2) = (-2, 0, 2) - \\ &= (-2, 0, 2) - (-8, 0, 0, 0) \end{aligned}$$

1.4

$$D^T = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 2 & 3 \end{pmatrix}$$

$$F^T = \begin{pmatrix} 1 & -1 & 0 \\ 3 & 2 & 1 \\ 2 & 0 & 2 \end{pmatrix}$$

1.5

$$\dim C = \underline{2 \times 3}$$

$$\dim D^T \cdot C^T = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= D^T_{2 \times 3} \cdot C^T_{3 \times 2} \Rightarrow \dim D^T \cdot C^T = \underline{2 \times 2}$$

$$\dim E \times F =$$

1.6

$$B - 2E = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - 2 \begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix}$$

matrice nie s\u00e1 kompatibiln\u00e9 \Rightarrow
nejde od\u00fasat

$$3D^T + 2C = 3 \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix} + 2 \begin{pmatrix} 1 & 3 & 0 \\ -1 & 0 & -2 \end{pmatrix} = \begin{pmatrix} 6 & 0 & -3 \\ -3 & 6 & 9 \end{pmatrix} + \begin{pmatrix} 2 & 6 & 0 \\ -2 & 0 & -4 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 6 & -3 \\ -5 & 6 & 5 \end{pmatrix}$$

1.7

$$D^T \cdot C^T = \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 5 & -5 \end{pmatrix}$$

$$C \cdot F + 2D^T = \begin{pmatrix} 1 & 3 & 0 \\ -1 & 0 & -2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix} + 2 \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} -2 & 9 & 2 \\ -1 & -5 & -6 \end{pmatrix} +$$

$$+ \begin{pmatrix} 4 & 0 & -2 \\ -2 & 4 & 6 \end{pmatrix} = \begin{pmatrix} 2 & 9 & 0 \\ -3 & -1 & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} 4 & 0 & -2 \\ -2 & 4 & 6 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 2 & 9 & 0 \\ -3 & -1 & 0 \end{pmatrix}}}$$

1.8

$D^T \cdot C^T$

$$C \cdot F + 2D^T = \begin{pmatrix} 2 & 9 & 0 \\ -3 & -1 & 0 \end{pmatrix} \quad \text{diag} = \underline{\underline{(2 \ -1)}}$$

1.9

$$\begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 & -3 & 1 \\ 2 & 2 & 0 \\ 5 & 7 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 3 & -1 \\ 0 & -4 & 2 \\ 5 & 7 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & -1 \\ 0 & -2 & 1 \\ 0 & 8 & 3 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 3 & -1 \\ 0 & -2 & 4 \\ 0 & 0 & 3 \end{pmatrix}$$

riviksi har \Rightarrow

linear. nezavisli

1.9.2

$$\begin{pmatrix} 1 & -1 & -3 \\ 0 & 2 & 2 \\ -1 & 5 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -3 \\ 0 & 0 & -4 \\ 0 & 4 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -3 \\ 0 & 0 & -4 \\ 0 & 0 & -8 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -3 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -1 & -3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{array}{l} \text{lineare} \\ \text{abhängig} \end{array}$$

$$-2 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + 1 \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ 7 \end{pmatrix}$$

1.10

$$\begin{pmatrix} -1 & -3 & 1 \\ 2 & 2 & 0 \\ 5 & 7 & -2 \end{pmatrix} \quad \text{rank} = 3$$

$$\begin{pmatrix} +1 & -1 & -3 \\ 0 & 2 & 2 \\ -1 & 5 & 7 \end{pmatrix} \quad \text{rank} = 2 ?$$

1.12

$$2x_1 - 3x_2 - x_3 = -7$$

$$3x_1 + x_2 + x_3 = 4$$

$$-x_1 + 4x_2 + 6x_3 = -3$$

$$\left(\begin{array}{ccc|c} 2 & -3 & -1 & -7 \\ 3 & 1 & 1 & 4 \\ -1 & 4 & 6 & -3 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 4 & 6 & -3 \\ 3 & 1 & 1 & 4 \\ 2 & -3 & 1 & 7 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & -4 & -6 & 3 \\ 0 & 13 & 19 & -5 \\ 0 & 5 & 11 & -13 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -4 & -6 & 3 \\ 0 & 8 & 8 & 8 \\ 0 & 5 & 11 & -13 \end{array} \right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & -4 & -6 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 8 & -18 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -4 & -6 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -3 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -4 & -6 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{array} \right) \sim$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -8 & 19 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{array} \right)$$

$$x_1 = 1$$

$$x_2 = 4$$

$$x_3 = -3$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -6 & 9 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{array} \right)$$

$$x_1 = 1$$

$$x_2 = 4$$

$$x_3 = -3$$

$$2x_1 - x_2 = 4$$

$$-3x_1 + x_2 + 4x_3 = -1$$

$$-x_1 + 4x_3 = 1$$

$$\left(\begin{array}{ccc|c} 2 & -1 & 0 & 4 \\ -3 & 1 & 4 & -1 \\ -1 & 0 & 4 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -4 & -1 \\ -3 & 1 & 4 & -1 \\ 2 & -1 & 0 & 4 \end{array} \right) \sim$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -4 & -1 \\ 0 & 0 & -8 & -4 \\ 0 & 0 & 0 & +4 \end{array} \right) \Rightarrow 0 \neq 4 \text{ nemá řešení}$$

1.12

$$\begin{vmatrix} 2 & -5 \\ 1 & 4 \end{vmatrix} = 2 \cdot 4 + 5 = 13$$

$$\begin{vmatrix} 2 & -1 & 0 \\ -3 & 1 & 4 \\ -1 & 1 & 5 \end{vmatrix} = 10 + 4 - 8 - 15 = -9$$

1.13

$$\begin{vmatrix} 0 & 2 & x \\ -1 & x & 0 \\ 3 & 2 & -1 \end{vmatrix} + \begin{vmatrix} 3 & x & 2 \\ -1 & 0 & 2 \\ x & -1 & x \end{vmatrix} = 4$$

$$0 - 0 - 2x + 0 + 2x^2 + 2 - 0 + 6 - x^2 = 4$$

$$x^2 - 2x + 4 = 0$$

$$\Delta = b^2 - 4ac$$

$$= 4 - 4$$

$$= 0$$

$$x_1 = \frac{-b}{2a} = \frac{2}{2} = 1$$