SKUPINA C

1.1 Základní operace s vektory

$a = (2,1,2)$ $b = (-1,0,1)$ $c = (1,2,1,1)$ $d = (1,0,-2,0)$ $e = (3,0,1,3)$ $f = (-1,1,0,-2)$

1.1 Délka vektorů

1. $c = (1,2,1,1)_4$

2. $b = (-1,0,1)_3$

1.2 Sčítání vektorů, odčítání vektorů, násobení skalárem

1. $e + f = (3,0,1,3) + (-1,1,0,-2) = (3-1, 0+1, 1+0, 3-2) = (2,1,1,1)$

2. $2c - 2f = 2(1,2,1,1) - 2(-1,1,0,-2) = (2,4,2,2) - (-2,2,0,-4) = (4,2,2,6)$

3. $a + b - 4c = (2,1,2) + (-1,0,1) - 4(1,2,1,1)$ → nelze - vektory mají různé délky

4. $4d - 2(e-f) = 4(1,0,-2,0) - 2\big((3,0,1,3) - (-1,1,0,-2)\big) = (4,0,-8,0) - 2(4,-1,1,5) =$
$= (4,0,-8,0) - (8,-2,2,10) = (-4,2,-10,-10)$

1.3 Skalární součin vektorů

1. $7f \times c = 7(-1,1,0,-2) \times (1,2,1,1) = 7(-1\cdot1 + 1\cdot2 + 0\cdot1 + 1\cdot(-2)) = 7(-1+2+0-2) =$
$= (-7 + 14 + 0 - 14) = -7$

2. $b \times a - 6e \times d = (-1,0,1) \times (2,1,2) - 6(3,0,1,3) \times (1,0,-2,0) = (-2+0+2) -$
$-6(3+0-2+0) = 0 - (18+0-12+0) = 0 - 6 = -6$

1.2 Základní operace s maticemi

$A = (2\ 1\ 2)$ $B = \begin{pmatrix}-1\\0\\1\end{pmatrix}$ $C = \begin{pmatrix}1 & 3 & 0\\-1 & 0 & -2\end{pmatrix}$ $D = \begin{pmatrix}2 & -1\\0 & 2\\-1 & 3\end{pmatrix}$ $E = \begin{pmatrix}-2 & 0\\1 & -2\end{pmatrix}$ $F = \begin{pmatrix}1 & 3 & 2\\-1 & 2 & 0\\0 & 1 & 2\end{pmatrix}$

1.4 Transpozice matice

1. C^T $C = \begin{pmatrix}1 & 3 & 0\\-1 & 0 & -2\end{pmatrix}$ $C^T = \begin{pmatrix}1 & -1\\3 & 0\\0 & -2\end{pmatrix}$

2. E^T $E = \begin{pmatrix}-2 & 0\\1 & -2\end{pmatrix}$ $E^T = \begin{pmatrix}-2 & 1\\0 & -2\end{pmatrix}$

1.5 Dimenze matice

1. D $D = \begin{pmatrix}2 & -1\\0 & 2\\-1 & 3\end{pmatrix}$ $\dim(D) = 3 \times 2$

2. $D^T \times F^T = \dim(D^T_{2\times 3} \times F^T_{3\times 3}) = 2\times 3$

$\dim(D) = 3\times 2 \Rightarrow \dim(D^T) = 2\times 3$

$\dim(F) = 3\times 3 \Rightarrow \dim(F^T) = 3\times 3$

3. $C \times E = \dim(C_{2\times 3} \times E_{2\times 2})$ = nelze

$\dim(C) = 2\times 3$

$\dim(E) = 2\times 2$

1.6 Sčítání matic, odčítání matic, násobení skalárem

1. $3F - B^T = 3\begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix} - \begin{pmatrix} -1 & 0 & 1 \end{pmatrix}$ → nelze – různé dimenze matic

2. $D - 2C^T = \begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix} - 2\begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix} - \begin{pmatrix} 2 & -2 \\ 6 & 0 \\ 0 & -4 \end{pmatrix} = \begin{pmatrix} 2-2 & -1+2 \\ 0-6 & 2-0 \\ -1-0 & 3+4 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -6 & 2 \\ -1 & 7 \end{pmatrix}$

1.7 Násobení matic

1. $D^T \times F^T = \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & -1 & 0 \\ 3 & 2 & 1 \\ 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 2+0-2 & -2+0+0 & 0+0-2 \\ -1+6+6 & 1+4+0 & 0+2+6 \end{pmatrix} = \begin{pmatrix} 0 & -2 & -2 \\ 11 & 5 & 8 \end{pmatrix}$

2. $A \times D - 2B^T \times C^T = \begin{pmatrix} 2 & 1 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix} - 2\begin{pmatrix} -1 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 2 & 6 \end{pmatrix} - \begin{pmatrix} -2 & 0 & 2 \end{pmatrix} \times \begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix} =$

$= \begin{pmatrix} 2 & 6 \end{pmatrix} - \begin{pmatrix} -2 & -2 \end{pmatrix} = \begin{pmatrix} 4 & 8 \end{pmatrix}$

1.8 Diagonála matice

1. $D^T \times F^T$ $\quad \operatorname{diag}\begin{pmatrix} 0 & -2 & -2 \\ 11 & 5 & 8 \end{pmatrix} = \begin{pmatrix} 0 & 5 \end{pmatrix}$

2. $A \times D - 2B^T \times C^T$ $\quad \operatorname{diag}\begin{pmatrix} 4 & 8 \end{pmatrix} = 4$

.3 Gaussova eliminace, lineární (ne)závislost vektorů, soustavy lineárních rovnic

1.9 Lineární závislost a nezávislost vektorů

1. $(-2\ 3\ 0), (-2\ 1\ 5), (-1\ 0\ 4)$

$\begin{pmatrix} -2 & -2 & -1 \\ 3 & 1 & 0 \\ 0 & 5 & 4 \end{pmatrix} \begin{matrix} \\ \cdot 3 \\ \cdot 2 \end{matrix} \sim \begin{pmatrix} -2 & -2 & -1 \\ 0 & -4 & -3 \\ 0 & 5 & 4 \end{pmatrix} \begin{matrix} \\ \cdot 5 \\ \cdot 4 \end{matrix} \sim \begin{pmatrix} -2 & -2 & -1 \\ 0 & -4 & -3 \\ 0 & 0 & 1 \end{pmatrix}$ => lineárně nezávislé

2. $\begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$ $\quad \begin{pmatrix} 0 & 1 & -2 \\ 2 & 2 & 1 \\ 4 & 3 & 4 \end{pmatrix} \sim \begin{pmatrix} 2 & 2 & 1 \\ 0 & 1 & -2 \\ 4 & 3 & 4 \end{pmatrix} \cdot(-2) \sim \begin{pmatrix} 2 & 2 & 1 \\ 0 & 1 & -2 \\ 0 & -1 & 2 \end{pmatrix} \cdot(-1) \sim \begin{pmatrix} 2 & 2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix} \cdot(-2) \sim$

$\begin{pmatrix} 2 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix} /:2 \sim \begin{pmatrix} 1 & 0 & \frac{5}{2} \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$ => lineárně závislé $\quad \frac{5}{2}\begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} - 2\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$

.10 Hodnost matice

1. $\text{rank}\begin{pmatrix} -2 & -2 & -1 \\ 3 & 1 & 0 \\ 0 & 5 & 4 \end{pmatrix} \overset{\text{viz } 1.9/1}{\sim} \text{rank}\begin{pmatrix} -2 & -2 & -1 \\ 0 & -4 & -3 \\ 0 & 0 & 1 \end{pmatrix} = 3$

2. $\text{rank}\begin{pmatrix} 0 & 1 & -2 \\ 2 & 2 & 1 \\ 4 & 3 & 4 \end{pmatrix} \overset{\text{viz } 1.9/2}{\sim} \text{rank}\begin{pmatrix} 2 & 2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix} = 2$

.11 Řešení soustavy lineárních rovnic

1. $2x_1 - x_2 + 3x_3 = 6$
$x_1 + 2x_3 = 5$
$6x_1 + 3x_2 + 4x_3 = -2$

$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 6 \\ 1 & 0 & 2 & 5 \\ 6 & 3 & 4 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 2 & -1 & 3 & 6 \\ 6 & 3 & 4 & -2 \end{array}\right) /\cdot(-2) /\cdot(-6) \sim \left(\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 0 & -1 & -1 & -4 \\ 0 & 3 & -8 & -32 \end{array}\right) /\cdot(-1) \sim$

$\sim \left(\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 0 & 1 & 1 & 4 \\ 0 & 3 & -8 & -32 \end{array}\right) /\cdot(-3) \sim \left(\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & -11 & -44 \end{array}\right) /:(-11) \sim \left(\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 4 \end{array}\right) /\cdot(-1) /\cdot(-2) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{array}\right) \Rightarrow \begin{array}{l} x_1 = -3 \\ x_2 = 0 \\ x_3 = 4 \end{array}$

2. $-x_1 + 2x_2 + 5x_3 = 3$
$-2x_1 + 2x_2 + 7x_3 = 0$
$x_1 - 2x_3 = -2$

$\left(\begin{array}{ccc|c} -1 & 2 & 5 & 3 \\ -2 & 2 & 7 & 0 \\ 1 & 0 & -2 & -2 \end{array}\right) /\cdot(-2) /\cdot 1 \sim \left(\begin{array}{ccc|c} -1 & 2 & 5 & 3 \\ 0 & -2 & -3 & -6 \\ 0 & 2 & 3 & 1 \end{array}\right) /\cdot 1 \sim \left(\begin{array}{ccc|c} -1 & 2 & 5 & 3 \\ 0 & -2 & -3 & -6 \\ 0 & 0 & 0 & -5 \end{array}\right) \Rightarrow$ nemá řešení

.4 Determinant matice

.12 Determinant matice

1. $\begin{vmatrix} -3 & 2 \\ 2 & -1 \end{vmatrix} = -3\cdot(-1) - 2\cdot 2 = 3 - 4 = -1$

2. $\begin{vmatrix} 2 & -1 & 3 \\ -1 & 0 & -2 \\ 5 & 1 & 4 \end{vmatrix}\begin{matrix} 2 & -1 \\ -1 & 0 \\ 5 & 1 \end{matrix} = 2\cdot0\cdot4 + (-1)\cdot(-2)\cdot5 + 3\cdot(-1)\cdot1 - (5\cdot0\cdot3 + 1\cdot(-2)\cdot2 + 4\cdot(-1)\cdot(-1)) = 0 + 10 - 3 - (0 - 4 + 4) =$

$= 7 - 0 = 7$

.13 Rovnice s determinanty

1. $2\begin{vmatrix} 1 & -1 & 0 \\ x & 3 & x \\ 0 & x & 1 \end{vmatrix} + \begin{vmatrix} x & 0 & 1 \\ -x & 0 & 1 \\ -1 & 3 & x \end{vmatrix} = 6$

$2(1\cdot3\cdot1 + (-1)\cdot x\cdot0 + 0\cdot x\cdot x - (0\cdot3\cdot0 + x\cdot x\cdot1 + 1\cdot x\cdot(-1))) + x\cdot0\cdot x + 0\cdot1\cdot(-1) + 1\cdot(-x)\cdot3 - (-1\cdot0\cdot1 + 3\cdot x\cdot1 + x\cdot(-x)\cdot0) = 6$

$2(3 + 0 + 0 - (0 + x^2 - x)) + 0 + 0 - 3x - (0 + 3x + 0) = 6$

$2(3 - x^2 + x) - 3x - 3x = 6$

$6 - 2x^2 + 2x - 3x - 3x = 6$

$-2x^2 - 4x = 0$

$-2x(x + 2) = 0$

$x_1 = 0 \quad x_2 = -2$