SKUPINA C

1.1
1. c(1 2 1 1) 4
2. b(-1 0 1) 3

1.2 1. e=(3 0 1 3)
f=(-1 1 0 -2)
e+f=(3 0 1 3) + (-1 1 0 -2) = (2 1 1 1)

2. c=(1 2 1 1)
f=(-1 1 0 -2)
2c - 2f = 2·(1 2 1 1) - 2(-1 1 0 -2) = (2 4 2 2) - (-2 2 0 -4)
= (4 2 2 6)

3. a + b = -4c
a=(2 1 2)
b=(-1 0 1)
c=(1 2 1 1)
NEJDE

4. $d=(1\ 0\ -2\ 0)$

$e=(3\ 0\ 1\ 3)$

$f=(-1\ 1\ 0\ -2)$

$(4\ -1\ 1\ 5)$

$4d-2(e-f)=4.(1\ 0\ -2\ 0)-2.((3\ 0\ 1\ 3)-(-1\ 1\ 0\ -2))$

$=(4\ 0\ -8\ 0)-(8\ -2\ 2\ 10)=(-4\ 2\ -10\ -10)$

1.3 1. $f=(-1\ 1\ 0\ -2)$

$c=(1\ 2\ 1\ 1)$

$f\times c=7.(-1\ 1\ 0\ -2)\times(1\ 2\ 1\ 1)=(-7\ 7\ 0\ -1$

$\times(1\ 2\ 1\ 1)=(-7+14+0-14)=-7$

2. $b=(-1\ 0\ 1\ 1)$

$a=(2\ 1\ 2)$

$e=(3\ 0\ 1\ 3)$

~~$d=(1\ 1\ 0\ -2)$~~ $d=(1\ 0\ -2\ 0)$ $(18\ 0\ 6\ 18)$

$b\times a-6e\times d=(-1\ 0\ 1)\cdot(2\ 1\ 2)-6.(3\ 0\ 1\ 3)\cdot(1\ 0\ -2\ 0)$

$(-2+0+2)-(18+0-12+0)=0-(6)=-6$

1.4 1. $C = \begin{pmatrix} 1 & 3 & 0 \\ -1 & 0 & -2 \end{pmatrix}$

$C^T = \begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix}$

2. $E = \begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix}$

$E^T = \begin{pmatrix} -2 & 1 \\ 0 & -2 \end{pmatrix}$

1.5 1. $D = \begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix}$ $\dim(A) = 3 \times 2$

2. $D = \begin{pmatrix} 2 & -1 \\ 0 & 2 \\ 1 & 3 \end{pmatrix}$ $F \begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

$\dim(A) = 3 \times 2$ $\dim(F) = 3 \times 3$

$\dim(A^T) = 2 \times 3$ $\dim(F^T) = 3 \times 3$

$\dim(D^T \times F^T) = 2 \times 3$

3. $C = \begin{pmatrix} 1 & 3 & 0 \\ -1 & 0 & -2 \end{pmatrix}$ $E = \begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix}$ $\dim C \times E =$ NEJDE

$\dim(C) = 2 \times 3$ $\dim^{(E)} = 2 \times 2$

1.6 1. $F = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$ $B = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ $B^T = (-1 \ 0 \ 1)$

$3F - B^T =$ IMPOSSIBLE

2. $D = \begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 3 & 0 \\ -1 & 0 & -2 \end{pmatrix}$ $C^T = \begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix}$

$D - 2 \cdot C^T = \begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix} - 2 \cdot \begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -6 & 2 \\ -1 & 7 \end{pmatrix}$

$\begin{pmatrix} 2 & -2 \\ 6 & 0 \\ 0 & -4 \end{pmatrix}$

1.7 1) $D = \begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix}$ $F = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

$D^T = \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix}$ $F^T \begin{pmatrix} 1 & -1 & 0 \\ 3 & 2 & 1 \\ 2 & 0 & 2 \end{pmatrix}$

$D^T \times F^T = \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 0 \\ 3 & 2 & 1 \\ 2 & 0 & 2 \end{pmatrix} = \begin{matrix} 2+0-2 & -2+0+0 & 0+0+2 \\ -1+6+6 & 1+4+0 & 0+2+6 \end{matrix}$

$= \begin{pmatrix} 0 & -2 & -2 \\ 11 & 5 & 8 \end{pmatrix}$

2) $A \times D - 2B^T \times C^T$

$(2\ 1\ 2) \cdot \begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix} - 2 \cdot (-1\ 0\ 1) \cdot \begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix} = (4+0-2)(-2+2+6)$

$- (2\ 0\ 2) \cdot \begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix} = (2\ 6) - (-2+0+0) \cdot (2+0-4) =$

$(2\ 6) - (-2\ -2) =$ (4 8)

1.8 1. $D^T \times F^T = \begin{pmatrix} 0 & -2 & -2 \\ 11 & 5 & 8 \end{pmatrix}$ → (0 5)

2. $A \times D - 2B^T \times C^T = (4\ \ 8)$ → 4

1.9 1. $(-2\ 3\ 0)$ $(-2\ 1\ 5)$ $(-1\ 0\ 4)$

$$\begin{pmatrix} -2 & -2 & -1 \\ 3 & 1 & 0 \\ 0 & 5 & 4 \end{pmatrix} \sim \begin{pmatrix} -6 & -6 & -3 \\ 6 & 2 & 0 \\ 0 & 5 & 4 \end{pmatrix} \sim \begin{pmatrix} 4 & 4 & 2 \\ 0 & -4 & -3 \\ 0 & 5 & 4 \end{pmatrix} \sim$$

$$\begin{pmatrix} 4 & 0 & -1 \\ 0 & -4 & -3 \\ 0 & 5 & 4 \end{pmatrix} \sim \begin{pmatrix} 4 & 0 & 1 \\ 0 & -4 & -3 \\ 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 4 & 0 & 1 \\ 0 & -4 & 3 \\ 0 & 4 & 4 \end{pmatrix} \sim \begin{pmatrix} 4 & 0 & 1 \\ 0 & -4 & -3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$1\begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} - 3\begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} -2+6 \\ 3-3 \\ 0-15 \end{pmatrix} \neq \begin{pmatrix} 4 \\ 0 \\ 15 \end{pmatrix}$$

→ LINEÁRNĚ NEZÁVISLÉ

2. $\begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$

$$\begin{pmatrix} 0 & 1 & -2 \\ 2 & 2 & 1 \\ 4 & 3 & 4 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & -2 \\ -4 & -4 & -2 \\ 4 & 3 & 4 \end{pmatrix} \sim \begin{pmatrix} 4 & 3 & 4 \\ -4 & -4 & -2 \\ 0 & 1 & -2 \end{pmatrix} \sim \begin{pmatrix} 4 & 3 & 4 \\ 0 & -1 & 2 \\ 0 & 1 & -2 \end{pmatrix} \sim \begin{pmatrix} 4 & 3 & 4 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 4 & 3 & 4 \\ 0 & -3 & 6 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 4 & 0 & 10 \\ 0 & -3 & 6 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 2 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & \frac{5}{2} \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$$

→ LIN. ZÁVISLÉ

$$\frac{5}{2}\begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} - 2\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 5 & -4 \\ 10 & -6 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

1.10

1. $\begin{pmatrix} -2 & -2 & -1 \\ 3 & 1 & 0 \\ 0 & 5 & 4 \end{pmatrix} \sim \begin{pmatrix} -2 & -2 & 1 \\ 3 & 1 & 0 \\ 0 & 5 & 4 \end{pmatrix} \sim \begin{pmatrix} -2 & -2 & -1 \\ 0 & -4 & -3 \\ 0 & 5 & 4 \end{pmatrix}$

$\sim \begin{pmatrix} -2 & -2 & -1 \\ 0 & -4 & -3 \\ 0 & 0 & 1 \end{pmatrix} \begin{matrix} \cdot 2 \\ \cdot 1 \end{matrix} \sim \begin{pmatrix} -4 & -4 & 2 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} -4 & 0 & 1 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{pmatrix}$

ran $k = 3$

2. $\begin{pmatrix} 0 & 1 & -2 \\ 2 & 2 & 1 \\ 4 & 3 & 4 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & -2 \\ 0 & -1 & 2 \\ 4 & 3 & 4 \end{pmatrix} \sim \begin{pmatrix} 4 & 3 & 4 \\ 0 & 1 & -2 \\ 0 & -1 & 2 \end{pmatrix} \sim \begin{pmatrix} 4 & 3 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} -4 & 0 & -10 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$

ran $k = 2$

1. 11. 1. $2x_1 - x_2 + 3x_3 = 6$

$x_1 + 2x_3 = 5$

$6x_1 + 3x_2 + 4x_3 = -2$

$$\left(\begin{array}{ccc|c} 2 & -1 & 3 & 6 \\ 1 & 0 & 2 & 5 \\ 6 & 3 & 4 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & -1 & 3 & 6 \\ 2 & 0 & 4 & 10 \\ 6 & 3 & 4 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & -1 & 3 & 6 \\ 0 & 1 & 1 & 4 \\ 6 & 3 & 4 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 6 & -3 & 9 & 18 \\ 6 & 3 & 4 & -2 \\ 0 & 1 & 1 & 4 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 2 & -1 & 3 & 6 \\ 0 & 6 & -5 & -20 \\ 0 & 1 & 1 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & -1 & 3 & 6 \\ 0 & 1 & 1 & 4 \\ 0 & 6 & 5 & 20 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & 4 & 10 \\ 0 & 6 & 6 & 24 \\ 0 & 6 & -5 & -20 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 0 & 1 & 1 & 4 \\ 0 & 6 & -11 & 44 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

$x_1 + 2x_3 = 5$

$x_2 + x_3 = 4$

$x_1 = -3 \quad x_2 = 0 \quad x_3 = 4$

2. $-x_1 + 2x_2 + 5x_3 = 3$

$-2x + 2x_2 + 7x_3 = 0$

$x_1 - 2x_3 = -2$

$$\left(\begin{array}{ccc|c} -1 & 2 & 5 & 3 \\ -2 & 2 & 7 & 0 \\ 1 & 0 & -2 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & -4 & -10 & -3 \\ -2 & 2 & 7 & 0 \\ 1 & 0 & -2 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & -4 & -10 & -3 \\ 0 & -2 & -3 & -3 \\ 1 & 0 & -2 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & -4 & -10 & -3 \\ 0 & 4 & 6 & 6 \\ 1 & 0 & -2 & -2 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 2 & 0 & -4 & 3 \\ 0 & 4 & 6 & 6 \\ 1 & 0 & -2 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & -4 & 3 \\ 0 & 4 & 6 & 6 \\ 2 & 0 & -4 & -4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & -4 & 3 \\ 0 & 4 & 6 & 6 \\ 0 & 0 & 0 & -7 \end{array}\right)$$

$0 \neq 7$ → NEMÁ ŘEŠENÍ

1.12 1. $\begin{vmatrix} -3 & 2 \\ 2 & -1 \end{vmatrix} = ((1-3\cdot(-1)) - (2\cdot 2)) = 3-4 = -1$

2. $\begin{pmatrix} 2 & -1 & 3 \\ -1 & 0 & -2 \\ 5 & 1 & 4 \end{pmatrix} \begin{matrix} 2 & -1 \\ -1 & 0 \\ 5 & 1 \end{matrix} = \begin{matrix} ((2\cdot 0\cdot 4)+(-1\cdot(-2)\cdot 5)+(3\cdot(-1)\cdot 1) \\ -((3\cdot 0\cdot 5)+(2\cdot(-2)\cdot 1)+(-1\cdot(-1)\cdot 4)) \end{matrix}$

$= (0+10-3) - (0-4+4) = 7$

1.13 $2\begin{pmatrix} 1 & -1 & 0 \\ x & 3 & x \\ 0 & x & 1 \end{pmatrix} + \begin{pmatrix} x & 0 & 1 \\ -x & 0 & 1 \\ -1 & 3 & x \end{pmatrix} = 6$

$2(3-x^2+x) - 6x = 6$

$\begin{matrix} 1 & -1 & 0 \\ x & 3 & x \\ 0 & x & 1 \end{matrix} \begin{matrix} 1 & -1 \\ x & 3 \\ 0 & x \end{matrix}$ $= [(1\cdot 3\cdot 1)+(-1\cdot x\cdot 0)+(0\cdot x\cdot x)] -$

$[(0\,3\,0)+(x\cdot x\cdot 1)+(1x(-1)]$

$= [3+0+0] - (0+x^2-x) = 3-x^2+x$

$\begin{matrix} x & 0 & 1 \\ -x & 0 & 1 \\ -1 & 3 & x \end{matrix} \begin{matrix} x & 0 \\ -x & 0 \\ -1 & 3 \end{matrix}$ $- [(x\cdot 0\cdot x)+(0\cdot 1\cdot(-1)+(1\cdot(-x)\cdot 3] -$

$[((-1)0\cdot 1)+(3x)+(x\cdot(-x)\cdot 0) =$

$0+0+(-3x) - [(0)+3x-0)$

$= -3x \quad -3x = -6x$

$6-2x^2+2x-6x=6$

$-2x^2-4x=0$

$-2x^2=4x$

$-x-2$

$x=-2$

$D=b^2-4ac \qquad x_{1,2} = \frac{4\pm 4}{2} \begin{matrix} / & -2 \\ \backslash & 0 \end{matrix}$

$D=16$

$\sqrt{D}=4$