D 1.1 $a=(2;1;2)$ $b=(-1;0;1)$ $c=(1;2;1;1)$ $d=(1;0;-2;0)$ $e=(3;0;1;3)$ $f=(-1;1;0;-2)$

1, $d=(1,0,-2,0)_4$

3, $a=(2;1;2)_3$

1.2

1, $f+c=(-1,1,0,-2)+(1,2,1,1)=\underline{(0;3;1;-1)}$

3, $-d+3e=(-1;0;2;0)+3(3;0;1;3)=\underline{(8;0;5;9)}$

3, $4a-e+d=$ nejde

4, $3\cdot(2c-e)-4f=3\left[(2;4;2;2)-(3;0;1;3)\right]-4\cdot(-1;1;0;-2)=$

$=3\cdot(-1;4;1;-1)-(-4;4;0;-8)=\underline{(1;8;3;5)}$

1.3

1, $6f\times d=6\cdot(-1;1;0;2)\cdot(1;0;-2;0)=6\cdot(-1;0;0;0)=\underline{-6}$

2, $-a\times b+2c\times e=(-2;-1;-2)\cdot(-1;0;1)+2(1;2;1;1)\cdot(3;0;1;3)=$

$=(2;0;-2)+(6;0;2;6)=\underline{14}$

1.4

1, $D^T=\begin{pmatrix}2 & -1\\ 0 & 2\\ -1 & 3\end{pmatrix}^T=\underline{\begin{pmatrix}2 & 0 & -1\\ -1 & 2 & 3\end{pmatrix}}$

2, $E^T=\begin{pmatrix}-2 & 0\\ 1 & -2\end{pmatrix}^T=\underline{\begin{pmatrix}-2 & 1\\ 0 & -2\end{pmatrix}}$

1.5

1, $E=\underline{2\times 2}$

2, $C^T\times E=\underset{3\times 2}{\begin{pmatrix}1 & -1\\ 3 & 0\\ 0 & -2\end{pmatrix}}\times\underset{2\times 2}{\begin{pmatrix}-2 & 0\\ 1 & -2\end{pmatrix}}=\underline{3\times 2}$

3, $F\times D^T=\underset{3\times 3}{\begin{pmatrix}1 & 3 & 2\\ -1 & 2 & 0\\ 0 & 1 & 2\end{pmatrix}}\cdot\underset{2\times 3}{\begin{pmatrix}2 & 0 & -1\\ -1 & 2 & 3\end{pmatrix}}=$

= nejde

1.6. 1. $E^T + 4A$ = nelze

2. $3C - D^T$

$$3\begin{pmatrix}1&3&0\\-1&0&-2\end{pmatrix} - \begin{pmatrix}2&0&-1\\-1&2&3\end{pmatrix} = \begin{pmatrix}3&9&0\\-3&0&-6\end{pmatrix} - \begin{pmatrix}2&0&-1\\-1&2&3\end{pmatrix} =$$

$$= \underline{\begin{pmatrix}1&9&1\\-2&-2&-9\end{pmatrix}}$$

1.7. 1. $C^T \times E$

$$\begin{pmatrix}1&-1\\3&0\\0&-2\end{pmatrix} \times \begin{pmatrix}-2&0\\1&-2\end{pmatrix} = \begin{pmatrix}1\cdot(-2)+(-1\cdot1), & 1\cdot0-1\cdot(-2)\\ 3\cdot-2+0\cdot1, & 3\cdot0+0\cdot(-2)\\ 0\cdot(-2)+(-2)\cdot1, & 0\cdot0+(-2)\cdot(-2)\end{pmatrix} =$$

$$= \underline{\begin{pmatrix}-3&2\\-6&0\\-2&4\end{pmatrix}}$$

2. $$(-1\ 0\ 1) \cdot \begin{pmatrix}1&3&2\\-1&2&0\\0&1&2\end{pmatrix} + (2\ 1\ 2) \cdot \begin{pmatrix}1&-1&0\\3&2&1\\2&0&2\end{pmatrix} =$$

$$(-1+0+0, -3+0+1, -2+0+2) + (2+3+4, -2+2+0, 0+1+4) =$$
$$= (-1\ -2\ 0) + (9\ 0\ 5) = \underline{(8\ -2\ 5)}$$

1.8. 1. $$C^T \times X = \begin{pmatrix}-3&2\\-6&0\\-2&4\end{pmatrix} \quad diag(C^T \times X) = (-3\ 0)$$

2. $B^T \times F + A \times F^T = \underline{\underline{(8)}}$

1.9

2. LN? $(-2\ {-1}\ 0),\ (-3\ 1\ 4),\ (-1\ 1\ 5)$

$$\begin{pmatrix}-2 & -3 & -1\\ -1 & 1 & 1\\ 0 & 4 & 5\end{pmatrix} \sim \begin{pmatrix}1 & -1 & -1\\ -2 & -3 & -1\\ 0 & 4 & 5\end{pmatrix} \sim \begin{pmatrix}1 & -1 & -1\\ 0 & -5 & -3\\ 0 & 4 & 5\end{pmatrix} \sim \begin{pmatrix}1 & -1 & -1\\ 0 & -20 & -12\\ 0 & 20 & 25\end{pmatrix} \sim$$

$$\sim \begin{pmatrix}1 & -1 & -1\\ 0 & 20 & 25\\ 0 & -20 & -12\end{pmatrix} \sim \begin{pmatrix}1 & -1 & -1\\ 0 & 20 & 25\\ 0 & 0 & 13\end{pmatrix} \rightarrow$$ sú lineárne nezávislé

LN? $\begin{pmatrix}-1\\0\\4\end{pmatrix}\ \begin{pmatrix}-3\\1\\4\end{pmatrix}\ \begin{pmatrix}2\\-1\\0\end{pmatrix}$

1. $$\begin{pmatrix}-1 & -3 & 2\\ 0 & 1 & -1\\ 4 & 4 & 0\end{pmatrix} \sim \begin{pmatrix}1 & 3 & -2\\ 4 & 4 & 0\\ 0 & 1 & -1\end{pmatrix} \sim \begin{pmatrix}1 & 3 & -2\\ 0 & 8 & -8\\ 0 & 1 & -1\end{pmatrix} \sim \begin{pmatrix}1 & 3 & -2\\ 0 & 1 & -1\\ 0 & 8 & -8\end{pmatrix} \sim \begin{pmatrix}1 & 3 & -2\\ 0 & 1 & -1\\ 0 & 0 & 0\end{pmatrix} \sim$$

$$\sim \begin{pmatrix}1 & 0 & 1\\ 0 & 1 & -1\\ 0 & 0 & 0\end{pmatrix}$$ sú lineárne závislé $\quad 1\cdot\begin{pmatrix}-1\\0\\4\end{pmatrix} + (-1)\cdot\begin{pmatrix}-3\\1\\4\end{pmatrix} = \begin{pmatrix}2\\-1\\0\end{pmatrix}$

1.10 Hodnosť matice

1. $$\begin{pmatrix}-1 & -3 & -2\\ 0 & 1 & -1\\ 4 & 4 & 0\end{pmatrix} \sim \begin{pmatrix}1 & 0 & 1\\ 0 & 1 & -1\\ 0 & 0 & 0\end{pmatrix}$$ výpočet

hodnosť = 2

2. $$\begin{pmatrix}-2 & -3 & 1\\ -1 & 1 & 1\\ 0 & 4 & 5\end{pmatrix} \sim \begin{pmatrix}-1 & 1 & 1\\ -2 & -3 & 1\\ 0 & 4 & 5\end{pmatrix} \sim \begin{pmatrix}1 & -1 & -1\\ -2 & -3 & 1\\ 0 & 4 & 5\end{pmatrix} \sim \begin{pmatrix}1 & -1 & -1\\ 0 & -5 & -1\\ 0 & 4 & 5\end{pmatrix} \sim \begin{pmatrix}1 & -1 & -1\\ 0 & -20 & -4\\ 0 & 20 & 25\end{pmatrix} \sim$$

$$\sim \begin{pmatrix}1 & -1 & -1\\ 0 & 20 & 25\\ 0 & -20 & -4\end{pmatrix} \sim \begin{pmatrix}1 & -1 & -1\\ 0 & 20 & 25\\ 0 & 0 & 21\end{pmatrix}$$ hodnosť = 3

1.11 Nájdite riešenie sústavy lineárnych rovníc

1.
$$\begin{aligned} x_1 - 2x_2 - x_3 &= 1 \\ 3x_1 + 5x_2 + 4x_3 &= -5 \\ 2x_1 + x_2 + 3x_3 &= 2 \end{aligned} \quad \left(\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 3 & 5 & 4 & -5 \\ 2 & 1 & 3 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 11 & 7 & -8 \\ 0 & 5 & 5 & 0 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 11 & 7 & -8 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -4 & -8 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right) \longrightarrow \begin{aligned} x_1 &= -1 \\ x_2 &= -2 \\ x_3 &= 2 \end{aligned}$$

2.
$$\begin{aligned} -x_1 + 2x_2 + 5x_3 &= 0 \\ -3x_1 + 2x_2 + 7x_3 &= 3 \\ 2x_1 \quad - 2x_3 &= 4 \end{aligned} \rightarrow \left(\begin{array}{ccc|c} -1 & 2 & 5 & 0 \\ -3 & 2 & 7 & 3 \\ 2 & 0 & -2 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ -1 & 2 & 5 & 0 \\ -3 & 2 & 7 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 2 & 4 & 2 \\ 0 & 2 & 4 & 9 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 2 & 4 & 2 \\ 0 & 0 & 0 & 7 \end{array}\right) \quad 0 = 7 \rightarrow \text{nemá riešenie}$$

1.12

1. $\begin{vmatrix} 4 & -1 \\ 2 & -3 \end{vmatrix} = 4 \cdot (-3) - (-1) \cdot 2 = -12 + 2 = -10$

2. $\left|\begin{array}{ccc|cc} -1 & 2 & 5 & -1 & 2 \\ -3 & 2 & 7 & -3 & 2 \\ 1 & 0 & -2 & 1 & 0 \end{array}\right| = (-1) \cdot 2 \cdot (-2) + 2 \cdot 7 \cdot 1 + 5 \cdot (-3) \cdot 0$

$- 5 \cdot 2 \cdot 1 - (-1) \cdot 7 \cdot 0 - 2 \cdot (-3) \cdot (-2) =$

$4 + 14 + 0 - 10 + 0 - 12 = -4$

1.13

1. $\begin{vmatrix} x & 1 & 0 \\ 1 & 0 & x \\ x & 3 & 0 \end{vmatrix} - \begin{vmatrix} 1 & 0 & x \\ 0 & x & 0 \\ 1 & x & 1 \end{vmatrix} = -6$

$$\begin{vmatrix} x & 1 & 0 \\ 1 & 0 & x \\ x & 3 & 0 \end{vmatrix} = x \cdot 0 \cdot 0 + 1 \cdot x \cdot x + 0 \cdot 1 \cdot 3 - 0 \cdot 0 \cdot x - 1 \cdot 1 \cdot 0 - x \cdot x \cdot 3$$

$$= 0 + x^2 + 0 - 0 - 0 - 3x^2 = \underline{-2x^2}$$

$$\begin{vmatrix} 1 & 0 & x \\ 0 & x & 0 \\ 1 & x & 1 \end{vmatrix} = 1 \cdot x \cdot 1 + 0 \cdot 0 \cdot 1 + x \cdot 0 \cdot x - x \cdot x \cdot 1 - 0 \cdot 0 \cdot 1 - 1 \cdot 0 \cdot x$$

$$= x + 0 + 0 - x^2 + 0 - 0 = \underline{x - x^2}$$

$$-2x^2 - (x - x^2) = -6$$

$$-2x^2 - x + x^2 = -6$$

$$-x^2 - x = -6$$

$$x^2 + x - 6 = 0 \rightarrow x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-6)}}{2 \cdot 1}$$

$$= \frac{-1 \pm \sqrt{25}}{2} = \frac{-1 \pm 5}{2} \rightarrow \underline{x_1 = 2}, \quad \underline{x_2 = -3}$$