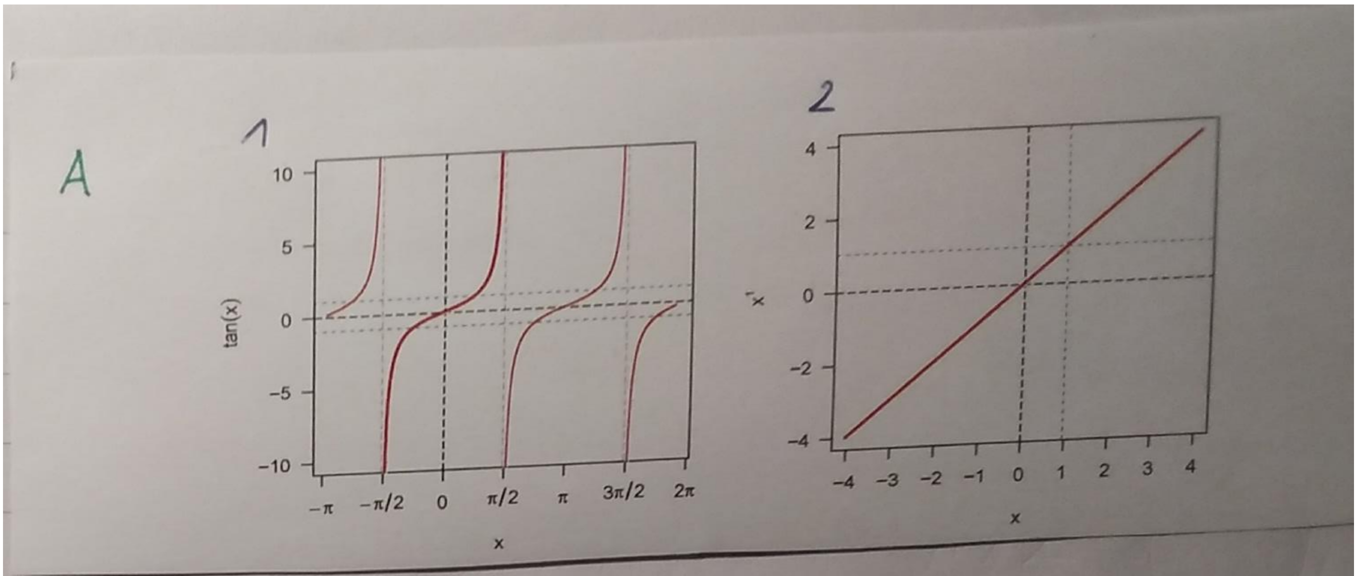
Matematika – 2. hodina

Skupina: A

Příklad 2.1. – 2.2: Základní vlastnosti funkce



1) $D(f)$ $R - (\frac{\pi}{2} + k\pi)$
2) $H(f)$ $(-\infty; \infty) \in R$
3) fce není spojitá
4) fce není omezená
5) je periodická vždy o π
6) lichá fce – symetrie podle počátku
7) rostoucí fce

2) 1) $D(f)$ $\langle -4; 4 \rangle$
2) $H(f)$ $\langle -4; 4 \rangle$
3) fce je spojitá
4) fce je shora i zdola oh
5) fce není periodická
6) lichá fce
7) fce je rostoucí

Příklad 2.2 – 2.4 : Výpočty limit

2.2 | 2.3

1) $x^2+x-2=(x+2)(x-1)$

2) $x^3-3x^2-6x+8=(x-1)(x+2)(x-4)$ $\quad \pm 1, \pm 2; \pm 4, \pm 8$

	1	-3	-6	8	
1	1	-2	-8	0	✓
~~1~~	~~1~~	~~-1~~	~~-9~~	~~-1~~	~~×~~
2	1	1	-7	6	×
-2	1	-4	0		

$x=1 \Rightarrow (x-1)(x+2)(x-4)$

2.2 | 2.4

① $\lim\limits_{x\to -3} x^2+3x+2 = \lim\limits_{x\to -3} -3^2+3\cdot(-3)+2 = \cancel{9}-9+2=\underline{\underline{2}}$

~~② $\lim\limits_{x\to 2} = \frac{3^2-2^2}{5^2} = \frac{9+4}{25} = \frac{13}{25}$~~

~~$\frac{3^x-2^x}{5^x}$~~

② $\lim\limits_{x\to 2} \frac{3^x-2^x}{5^x} = \frac{3^2-(2^2)}{5^2} = \frac{9-4}{25} = \underline{\underline{\frac{1}{5}}}$

③ $\lim\limits_{x\to 1} \frac{4x^3-x+2}{x^4-6x^3-9x+4} = \frac{5}{-10} = -\frac{1}{2}$

④ $\lim\limits_{x\to 2} \frac{x^3+2x^2-5x-6}{x^2-4} = \frac{8+8-10-6}{0} = -\frac{16}{0}$

Příklad: 2.5: Limity funkcí v nevlastním bodě

2.5(1)

$$\lim_{x\to\infty} 2-\frac{3}{x^2} = \lim_{x\to\infty} 2-\frac{3}{\text{"}\infty^2\text{"}} = 2+0=\underline{2}$$

2.5(2)

$$\lim_{x\to-\infty} \frac{2+x^3-x^4}{x^3-3x^5-2x^4+1} = \lim_{x\to-\infty} \frac{-x^4+x^3+2}{-3x^5-2x^4+x^3+1} =$$

$$\lim_{x\to-\infty} \frac{\cancel{x^5}(\overbrace{\tfrac{1}{x}}^{0}+\overbrace{\tfrac{1}{x^2}}^{0}+\overbrace{\tfrac{2}{x^5}}^{0})}{\cancel{x^5}(-3-\underbrace{\tfrac{1}{x}}_{0}+\underbrace{\tfrac{1}{x^2}}_{0}+\underbrace{\tfrac{1}{x^5}}_{0})} = \frac{0}{-3} = \underline{0}$$

2.5(3)

$$\lim_{x\to\infty} \frac{2^x-4^x}{5^x} = \frac{2^x}{5^x}-\frac{4^x}{5^x} = \lim_{x\to\infty}\left(\frac{2}{5}\right)^x-\left(\frac{4}{5}\right)^x = 0+0=\underline{0}$$

2.5(4)

$$\lim_{x\to-\infty} \frac{4+2^x}{2+5^x} = \lim_{x\to-\infty} \frac{4+2^{-\infty}}{2+5^{-\infty}} = \lim_{x\to-\infty} \frac{4+\left(\frac{1}{2}\right)^x}{2+\left(\frac{1}{5}\right)^x} =$$

$$\lim_{x\to-\infty} \frac{4+0}{2+0} = \frac{4}{2} = \underline{2}$$

2.5

5. $$\lim_{x\to\infty} \frac{6x^7-5x^3+4x^4-1}{6+x^2-3x^5+4x^7} = \lim_{x\to-\infty} \frac{6x^7-4x^4-5x^3-1}{4x^7-3x^5+x^2+6} =$$

$$= \lim_{x\to-\infty} \frac{x^7\left(6-\frac{4}{x^3}-\frac{5}{x^4}-\frac{1}{x^7}\right)}{x^7\left(4-\frac{3}{x^2}-\frac{1}{x^5}+\frac{6}{x^7}\right)} = \frac{6}{4} = \frac{3}{2}$$

6. $$\lim_{x\to\infty} \frac{8^x-2^x}{4^x} = \frac{\infty}{\infty}$$

$$\downarrow$$

$$\lim_{x\to\infty} \frac{8^x}{4^x} - \frac{2^x}{4^x} = 2^x - \left(\frac{1}{2}\right)^x = \infty - 0 = \infty$$

7. $$\lim_{x\to\infty} \frac{3x^4+4x^8-3}{2x^6-x^5+3x^4-5x} = \lim_{x\to\infty} \frac{4x^8+3x^4-3}{2x^6-x^5+3x^4-5x} =$$

$$= \lim_{x\to\infty} \frac{\cancel{x^8}\left(4+\frac{3}{x^4}-\frac{3}{x^8}\right)}{\cancel{x^8}\left(\frac{2}{x^2}-\frac{1}{x^3}+\frac{3}{x^4}-\frac{5}{x^7}\right)} = \frac{4}{0} = \infty$$

8. $$\lim_{x\to\infty} \frac{3^x-6^x}{6^x} = \frac{\infty}{\infty}$$

$$\downarrow$$

$$\lim_{x\to\infty} \frac{3^x}{6^x} - \frac{6^x}{6^x} = \boxed{\lim_{x\to\infty} \left(\frac{1}{2}\right)^x - 1^x = 0 - 1 = -1}$$

Příklad: 2.6: Derivace prvního řádu funkce

2.6

1. $(x^8 + x^{-8} + x^0 - \cos(x) + e^x)' = 8x^7 + 8x^{-9} - (-\sin(x)) + e^x = \underline{\underline{8x^7 + 8x^{-9} + \sin(x) + e^x}}$

2. $(3x^5 - 2x^3 - 4x + 4)' = 3\cdot 5x^4 - 2\cdot 3x^2 - 4\cdot 1 + 0 = \underline{\underline{15x^4 - 6x^2 - 4}}$

3. $(x^3\sin(x) + 4x\tan(x))' = 3x^2\cdot\sin(x) + x^3\cdot\cos(x) + 4\cdot\tan(x) + 4x\cdot\frac{1}{\cos^2(x)} = 3x^2\sin(x) + x^3\cos(x) + 4\tan(x) + \frac{4x}{\cos^2(x)}$

4. $\left(\frac{\cos(x)}{\sin(x)}\right)' = \cancel{\frac{-\sin(x)\cdot\sin x - \cos(x)\cdot\cos(x)}{\sin^2(x)}} = \cancel{-\sin(x) - \cos^2}$

$= \frac{-\sin(x)\cdot\sin(x) - \cos(x)\cdot\cos(x)}{\sin^2(x)} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \underline{\underline{\frac{-1}{\sin^2 x}}}$

5. $\left(\frac{x^2 - 3x + 2}{x-2}\right)' = \frac{(x^2-3x+2)'\cdot(x-2) - (x^2-3x+2)\cdot(x-2)'}{(x-2)^2} = \frac{(2x-3)\cdot(x-2) - (x^2-3x+2)\cdot 1}{(x-2)^2} =$

$= \frac{2x^2 - 4x - 3x + 6 - x^2 + 3x - 2}{(x-2)^2} = \frac{x^2 - 4x + 4}{x^2 - 4x + 4} = \underline{\underline{1}}$

6. $(\ln(2x^2 - 4x))' = \frac{1}{2x(x-2)}\cdot(2x^2-4x)' = \frac{1}{2x(x-2)}\cdot(4x-4) = \frac{4x-4}{2x(x-2)} = \frac{4(x-1)}{2x(x-2)} = \underline{\underline{\frac{2(x-1)}{x(x-2)}}}$

6. $(\ln(2x^2 - 4x))' = \frac{1}{2x(x-2)}\cdot(2x^2-4x)' = \frac{1}{2x(x-2)}\cdot(4x-4) = \frac{4x-4}{2x(x-2)} = \frac{4(x-1)}{2x(x-2)} = \underline{\underline{\frac{2(x-1)}{x(x-2)}}}$

7. $(\cos(x^2) + \sin(2x))' = -\sin(x^2)\cdot 2x + \cos(2x)\cdot 2 = \underline{\underline{2\cos(2x) - 2x\cdot\sin(x^2)}}$

8. $(\tan(x)\cos(x) - 3\ln(x)\cos(x))' = \frac{1}{\cos^2(x)}\cdot\cos(x) + \frac{\sin x}{\cos x}\cdot -\sin x - \left(3\cdot\frac{1}{x}\cdot\cos(x) + 3\ln(x)\cdot -\sin x\right)$

$= \frac{1}{\cos(x)} + \frac{\sin x}{\cos x}\cdot -\sin(x) - \left(3\cdot\frac{1}{x}\cdot\cos(x) + 3\ln(x)\cdot -\sin(x)\right) = \frac{1}{\cos x} + \frac{-\sin^2 x}{\cos x} - \left(\frac{3\cos(x)}{x} + 3\ln(x)\cdot -\sin(x)\right)$

$= \underline{\underline{\frac{1 - \sin^2 x}{\cos x} - \frac{3\cos(x)}{x} + 3\ln(x)\cdot\sin(x)}}$

Příklad: 2.7: Derivace druhého řádu funkce

Příklad: 2.7. – Derivace druhého řádu funkce

1) $(2x^5 - x^3 - 4x + 4)'' = (10x^4 - 3x^2 - 4)' = \underline{\underline{(40x^3 - 6x)''}}$

Příklad: 2.7. – Derivace druhého řádu funkce

1) $(2x^5 - x^3 - 4x + 4)'' = (10x^4 - 3x^2 - 4)' = \underline{\underline{40x^3 - 6x}}$

2) $((x^4-1)e^x)'' = ((4x^3 \cdot e^x) + ((x^4-1) \cdot e^x))' = 12x^2 \cdot e^x + 4x^3 \cdot e^x + 4x^3 \cdot e^x + (x^4-1) \cdot e^x =$
$= \underline{\underline{e^x \cdot (x^4 + 8x^3 + 12x^2 - 1)}}$

3) $(3e^x \cdot \sin(x))'' = (3 \cdot (e^x \sin x) + (e^x \cos x))' = 3((e^x \sin x) + (e^x \cos x) + (e^x \cos x) \cdot (e^x \cdot (-\sin x)))'' =$
$= 3 \cdot (e^x \sin x + e^x \cdot \cos x + e^x \cos x - e^x \cdot \sin x) = 3e^x \cdot (\sin x + \cos x + \cos x - \sin x) =$
$= 3e^x \cdot 2\cos x = \underline{\underline{6e^x \cos x}}$

1. derivace

4) $\left(\dfrac{xe^{4x} - 2}{2x}\right)'' = \left(\dfrac{(e^{4x} + xe^{4x} \cdot 4) \cdot 2x - (xe^{4x} - 2) \cdot 2}{(2x)^2}\right)' = \dfrac{\cancel{2xe^{4x}} + 8x^2 \cdot e^{4x} - \cancel{2xe^{4x}} + 4}{4x^2} = \dfrac{4 \cdot (2x^2e^{4x} + 1)}{4x^2} = \left(\dfrac{2x^2e^{4x} + 1}{x^2}\right)'$

$= \left(\dfrac{(2 \cdot 2xe^{4x} + 2x^2e^{4x} \cdot 4) \cdot x^2 - (2x^2e^{4x} + 1) \cdot 2x}{(x^2)^2}\right)' = \dfrac{(4xe^{4x} + 8x^2e^{4x}) \cdot x^2 - (2x^2e^{4x} + 1) \cdot 2x}{x^4} = \dfrac{\cancel{x} \cdot (4xe^{4x} + 8x^2e^{4x}) \cdot x - (2x^2e^{4x} + 1) \cdot 2}{x^{3\cancel{4}}}$

$= \dfrac{\cancel{4x^2e^{4x}} + 8x^3e^{4x} - \cancel{4x^2e^{4x}} - 2}{x^3} = \dfrac{8x^3e^{4x} - 2}{x^3} = \dfrac{8\cancel{x^3}e^{4x}}{\cancel{x^3}} - \dfrac{2}{x^3} = \underline{\underline{8e^{4x} - \dfrac{2}{x^3}}}$

2.4: l'Hospitalovo pravidlo

Příklad: 2.8: l´Hospitalovo pravidlo

2.4: l'Hospitalovo pravidlo

Příklad: 2.8. – l'Hospitalovo pravidlo

1) $\lim\limits_{x\to 2} \dfrac{x^3 + 2x^2 - 5x - 6}{x^2 - 4} \lim\limits_{x\to 2} \dfrac{2^3 + 2 \cdot 2^2 - (5 \cdot 2) - 6}{2^2 - 4} \underset{x\to 2}{\lim} = \dfrac{8 + 8 - 10 - 6}{4 - 4} \underset{x\to 2}{\lim} = \underline{\underline{\dfrac{0}{0}}}$ ✓ platí l'Hospitalovo pravidlo

$\lim\limits_{x\to 2} \dfrac{(x^3 + 2x^2 - 5x - 6)'}{(x^2 - 4)'} \lim\limits_{x\to 2} \dfrac{3x^2 + 4x - 5}{2x} \lim\limits_{x\to 2} \dfrac{3 \cdot (2)^2 + 4 \cdot 2 - 5}{2 \cdot (2)} \underset{x\to 2}{\lim} = \dfrac{12 + 8 - 5}{4} = \underline{\underline{\dfrac{15}{4}}}$

2) $\lim\limits_{x\to 2} \dfrac{x^3 + 2x^2 + 5x - 6}{x^2 - 4} \lim\limits_{x\to 2} \dfrac{2^3 + 2 \cdot (2)^2 + 5 \cdot 2 - 6}{2^2 - 4} \underset{x\to 2}{\lim} = \dfrac{8 + 8 + 10 - 6}{4 - 4} = \underline{\underline{\dfrac{20}{0}}}$ ✗ neplatí l'Hospitalovo pravidlo

3) $\lim\limits_{x\to -2} \dfrac{3x^3 + 10x^2 + 9x + 2}{x^2 - 3x - 10} \lim\limits_{x\to -2} \dfrac{3 \cdot (-2)^3 + 10 \cdot (-2)^2 + 9 \cdot (-2) + 2}{(-2)^2 - 3 \cdot (-2) - 10} \lim\limits_{x\to -2} \dfrac{3 \cdot (-8) + 40 - 18 + 2}{4 + 6 - 10} = \underline{\underline{\dfrac{0}{0}}}$ ✓ platí l'Hospitalovo pravidlo

$\lim\limits_{x\to -2} \dfrac{(3x^3 + 10x^2 + 9x + 2)'}{(x^2 - 3x - 10)'} \lim\limits_{x\to -2} \dfrac{9x^2 + 20x + 9}{2x - 3} \lim\limits_{x\to -2} \dfrac{9 \cdot (-2)^2 + 20 \cdot (-2) + 9}{2(-2) - 3} \underset{x\to -2}{\lim} = \dfrac{36 - 40 + 9}{-7} = \underline{\underline{-\dfrac{5}{7}}}$