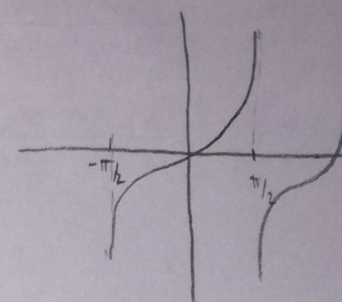
CVIČENÍ č. 2. - SKUPINA A

[2.1]

a) 1. $D_f = \mathbb{R} \setminus \{-\frac{\pi}{2}; +\frac{\pi}{2}\}$ $\quad f(x) = \tan(x)$

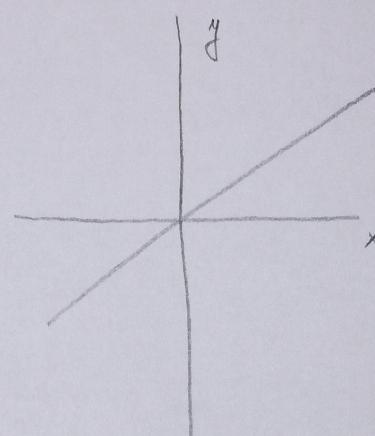
2. $R_f = \mathbb{R}$
3. Funkce není spojitá
4. není ohraničená
5. ano, je periodická (vždy o π)
6. paricita - lichá => symetrie podle počátku
7. rostoucí
8.



[2.2]

b) $f(x) = x^1$

1. $D_f = \mathbb{R}$
2. $H_f = \mathbb{R}$
3. funkce je spojitá
4. není ohraničená
5. není periodická
6. paricita - lichá
7. fce - rostoucí



[2.3]

1. $x^2 + x - 2 = (x+2)(x-1)$

2. $x^3 - 3x^2 - 6x + 8 = \pm 1 \;\; \pm 2 \;\; \pm 4$

	1	-3	-6	8
1	1	-2	-8	0 ✓

$(x-1)(x^2-2x-8) = (x-1)(x-4)(x+2)$

[2.5]

1. $\lim_{x \to -3} x^2 + 3x + 2 = (-3)^2 + 3(-3) + 2 = 9 - 9 + 2 = \underline{\underline{2}}$

2. $\lim_{x \to 2} \frac{3^x - 2^x}{5^x} = \frac{3^2 - 2^2}{5^2} = \frac{9-4}{25} = \frac{5}{25} = \underline{\underline{\frac{1}{5}}}$

3. $\lim_{x \to 1} \frac{4x^3 - x + 2}{x^4 - 6x^3 - 9x + 4} = \frac{4 - 1 + 2}{1 - 6 - 9 + 4} = \frac{5}{-10} = \underline{\underline{-\frac{1}{2}}}$

4. $\lim_{x \to 2} \frac{x^3 + 2x^2 - 5x - 6}{x^2 - 4} = \frac{-(x-3)\cancel{(x+2)}(x+1)}{\cancel{(x+2)}(x-2)} = \_\_\_\_\_\_$

	1	2	-5	-6	
1	1	3	-2	-8	×
-1	-1	1	6	0	✓

$\pm 1 \quad \pm 2 \quad \pm 3$

$(x+1)(-x^2 + x + 6) = (x+1)\cdot(x-3)(x+2)$

$\Rightarrow -(x-3)(x+2)(x+1)$

$\Rightarrow$ l'Hobitalovo pravidlo

$= \frac{2x^2 + 2x - 5}{2x} = \frac{2\cdot 4 + 2\cdot 2 - 5}{2\cdot 2} = \frac{8 + 4 - 5}{4} = \underline{\underline{\frac{7}{4}}}$

~~[2.4]~~ – [2.5]

1. $\lim\limits_{x \to \infty} 2 - \frac{3}{x^2} = 2 - \frac{3}{\infty^2} = 2 - 0 = \underline{\underline{2}}$

2. $\lim\limits_{x \to -\infty} \frac{2 + x^3 - x^4}{x^3 - 3x^5 - 2x^4 + 1} = \frac{\cancel{x^5}\left(\frac{2}{x^5} + \frac{1}{x^2} - \frac{1}{x}\right)}{\cancel{x^5}\left(\frac{1}{x^2} - \frac{3}{1} - \frac{2}{x} + \frac{1}{x^5}\right)} = \frac{0}{3} = \underline{\underline{0}}$

3. $\lim\limits_{x \to \infty} \frac{2^x - 4^x}{5^x} = \underbrace{\left(\frac{2}{5}\right)}_{<1}^{x} - \underbrace{\left(\frac{4}{5}\right)}_{<1}^{x} = 0 - 0 = \underline{\underline{0}}$

4. $\lim\limits_{x \to \infty} \frac{4 + 2^x}{2 + 5^x} = \frac{4 + \left(\frac{1}{2}\right)^x_{<1}}{2 + \left(\frac{1}{5}\right)^x_{<1}} = \frac{4}{2} = \underline{\underline{2}}$

5. $\lim\limits_{x \to -\infty} \frac{6x^7 - 5x^3 + 4x^4 - 1}{6 + x^2 - 3x^5 + 4x^7} = \frac{\cancel{x^7}\left(6 - \frac{5}{x^4} + \frac{4}{x^3} - \frac{1}{x^7}\right)}{\cancel{x^7}\left(\frac{6}{x^7} + \frac{1}{x^5} - \frac{3}{x^2} + \frac{4}{1}\right)} = \frac{6}{4} = \underline{\underline{\frac{3}{2}}}$

6. $\lim\limits_{x \to \infty} \frac{8^x - 2^x}{4^x} = \underbrace{\left(\frac{8}{4}\right)}_{>1}^{x} - \underbrace{\left(\frac{2}{4}\right)}_{<1}^{x} = \infty - 0 = \underline{\underline{\infty}}$

7. $\lim\limits_{x \to \infty} \frac{3x^4 + 4x^8 - 3}{2x^6 - x^5 + 3x^4 - 5x} = \frac{\cancel{x^8}\left(\frac{3}{x^4} + 4 + \frac{3}{x^8}\right)}{\cancel{x^8}\left(\frac{2}{x^2} - \frac{1}{x^3} + \frac{3}{x^4} - \frac{5}{x^7}\right)} = \frac{4}{0} = \underline{\underline{\infty}}$

8. $\lim\limits_{x \to \infty} \frac{3^x - 6^x}{6^x} = \underbrace{\left(\frac{3}{6}\right)}_{<1}^{x} - \underbrace{\left(\frac{6}{6}\right)}_{=1}^{x} = 0 - 1 = \underline{\underline{-1}}$

~~$x e^{4x-2} = 1 e^{4x-2} + x e^{4x-2} + 4$~~

~~[1,3]~~ - [2,6]

1. $(x^8 + x^{-8} + x^0 - \cos(x) + e^x)' = 8x^7 + (-8x^{-7}) + \sin(x) + e^x$

2. $(3x^5 - 2x^3 - 4x + 4)' = 15x^4 - 6x^2 - 4$

3. $(x^3 \sin(x) + 4x \tan(x))' = 3x^2 \sin(x) + x^3 \cos(x) + 4\tan(x) + 4x \frac{1}{\cos(x)}$

4. $$\left(\frac{\cos(x)}{\sin(x)}\right)' = \frac{-\sin(x)\cdot\sin(x) - \cos(x)\cos(x)}{\sin^2(x)} = \frac{-1}{\sin^2(x)}$$

- derivace prvního x nederivace jmenovatele (druhého) $\ominus$ nederivace prvního x derivace druhého

5. $$\left(\frac{x^2-3x+2}{x-2}\right)' = \frac{(2x-3)\cdot(x-2) - (x^2-3x+2)\cdot(1)}{(x-2)^2} = \frac{2x^2-4x-3x+6-x^2+3x-2}{x^2-4x+4} =$$

$$= \frac{x^2-4x+4}{x^2-4x+4} = \underline{\underline{1}}$$

6. $$(\ln(2x^2-4x))' = \frac{1}{2x^2-4x}\cdot(4x-4) = \frac{4(x-1)}{2x(x-2)} = \frac{2(x-1)}{x(x-2)}$$

7. $(\cos(x^2) + \sin(2x))' = (-\sin x^2 + \cos 2x) + \cos 2x \cdot 2 = 2\cos(2x) - 2x\cdot\sin x^2$?

. $$(\tan(x)\cos(x) - 3\ln(x)\cos(x))' = \frac{1}{\cos^2 x}\cdot\cos x + \cancel{\tan}\ \frac{\sin x}{\cos x}\cdot -\sin x -$$

$$\left(3\cdot\frac{1}{x}\cdot\cos(x) - 3\ln(x) - \sin x\right) = \frac{1}{\cos(x)} + \frac{\sin x}{\cos x}\cdot -\sin(x) - \left(3\cdot\frac{1}{x}\cdot\cos x -\right.$$

$$\left.3\ln x\cdot -\sin x\right) = \frac{1}{\cos x} + \frac{-\sin^2 x}{\cos x} - \left(\frac{3\cos x}{x} - 3\ln x\cdot -\sin x\right) =$$

$$= \frac{1-\sin^2 x}{\cos x} - \frac{3\cos x}{x} + 3\ln x\cdot\sin(x)$$

1. $(2x^5 - x^3 - 4x + 4)'' = (10x^4 - 3x^2 - 4)' = \underline{40x^3 - 6x}$

2. $((x^4-1)e^x)''$ = derivace 1. · nederivace 2. ⊕ nederivace 1. · derivace 2.

$= (4x^3 \cdot e^x + (x^4-1)\cdot e^x)' = 12x^2 \cdot e^x + 4x^3 \cdot e^x + 4x^3 \cdot e^x + (x^4-1)\cdot e^x =$

$= \underline{e^x (x^4 + 8x^3 + 12x^2 - 1)}$

3. $(3e^x \sin(x))'' = (3e^x \sin(x) + 3e^x \cos(x))' = 3e^x \sin(x) + 3e^x \cos(x) + 3e^x \cos(x) - 3e^x \sin(x) = \underline{6e^x \cos(x)}$

4. $\left(\frac{x\cdot e^{4x} - 2}{2x}\right)'' = \left(\frac{(e^{4x} + xe^{4x}\cdot 4)\cdot 2x - (xe^{4x}-2)\cdot 2}{4x^2}\right)' \Rightarrow \frac{\cancel{2xe^{4x}} + 8x^2e^{4x} - \cancel{2xe^{4x}} + 4}{4x^2} =$

$= \frac{\cancel{4}(2x^2e^{4x}+1)}{\cancel{4}x^2} = \frac{2x^2e^{4x}+1}{x^2}$

$\Rightarrow \left(\frac{2x^2e^{4x}+1}{x^2}\right)' = \frac{(4xe^{4x} + 2x^2e^{4x}\cdot 4)\cdot x^2\,(2x^2e^{4x}+1)\cdot 2x}{x^4} = \frac{(4xe^{4x}+8x^2e^{4x})\cdot x^2 - (2x^2e^{4x}+1)2x}{x^4} =$

$= \frac{\cancel{x}(4x^2e^{4x} + 8x^2e^{4x})\cdot \cancel{x} - (2x^2e^{4x}+1)\cdot 2}{x^{\cancel{4}\,3}} = \frac{4x^2e^{4x}+8x^3e^{4x} - 4x^2e^{4x} - 2}{x^3} = \frac{8x^3e^{4x}-2}{x^3} =$

$= \underline{8e^{4x} - \frac{2}{x^3}}$

[2.8]

1. $$\lim_{x \to 2} \frac{x^3 + 2x^2 - 5x - 6}{x^2 - 4} = \frac{8 + 2\cdot 4 - 5\cdot 2 - 6}{4 - 4} = \frac{'0'}{0} \Rightarrow \checkmark$$ l'Hobitalovo pravidlo

$$\lim_{x \to 2} \frac{3x^2 + 4x - 5}{2x} = \lim_{x \to 2} \frac{3\cdot 4 + 4\cdot 2 - 5}{2\cdot 2} = \frac{15}{4}$$

2. $$\lim_{x \to 2} \frac{x^3 + 2x^2 + 5x - 6}{x^2 - 4} = \frac{8 + 2\cdot 4 + 10 - 6}{0} = \frac{20}{0} \Rightarrow$$ nelze použít l'Hobitalovo pravidlo

3. $$\lim_{x \to -2} \frac{3x^3 + 10x^2 + 9x + 2}{x^2 - 3x - 10} = \frac{\overset{-24}{3\cdot(-8)} \overset{+40}{+ 10\cdot 4} \overset{-18}{+ 9\cdot(-2)} \overset{+2}{+ 2}}{4 - 3(-2) - 10} = \frac{'0'}{0} = \checkmark$$ platí l'Hobitalovo pravidlo

$$\lim_{x \to -2} \frac{9x^2 + 20x + 9}{2x - 3} = \frac{9\cdot 4 + 20\cdot(-2) + 9}{2\cdot(-2) - 3} = \frac{36 - 40 + 9}{-4 - 3} = -\frac{5}{7}$$