Skupina B - derivácie & limity

1. $D(f) =]-\infty, \infty[$

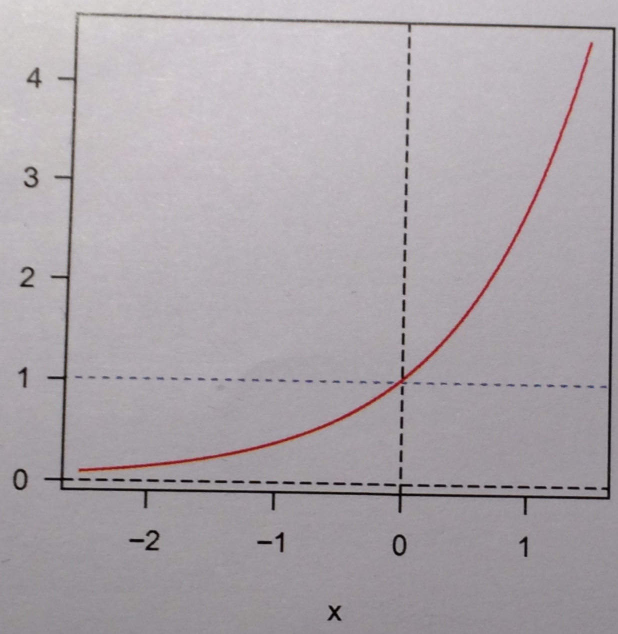
$H(f) =]0, \infty[$

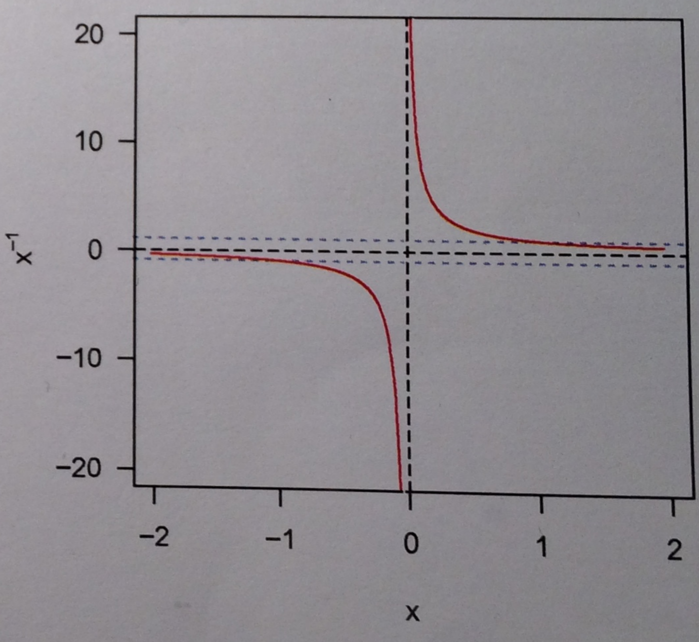
- spojitá
- zdola ohraničená (do 0)
- nie je periodická
- ani sudá ani lichá
- rastúca

2. $D(f) =]-\infty, 0[\cup]0, \infty[$

$H(f) =]-\infty, \infty[- \{0\}$

- nie je spojitá
- nie je ohraničená zdola, zhora ani celkovo
- nie je periodická
- lichá (nepárna)
- $]-\infty, 0[$ klesajúca
  $]0, \infty[$ rastúca





2.3. - trošku iný postup, nie vždy je ideálny

2. $x^3 - 7x - 6 = (x+1)(x+2)(x-3)$

$(-1): (-1)^3 + 7 - 6 = 0$

$(x^3 - 7x - 6):(x+1) = x^2 - x - 6$

$\underline{\overset{-}{x^3} \overset{-}{+} x^2}$

$-x^2 - 7x - 6$

$\underline{\overset{-}{x^2} \mp x}$

$-6x - 6$

$\underline{\pm 6x \pm 6}$

0

$(x^2 - x - 6)(x+1)$

$x^2 - x - 6 = 0$

$\Delta = b^2 - 4ac$

$= 1 + 4 \cdot 6$

$= 25$

$x_1 x_2 = \frac{-b \pm \sqrt{\Delta}}{2a} <>$

$x_1 = \frac{+1-5}{2} = -2$

$x_2 = \frac{+1+5}{2} = 3$

$(x+1)(x+2)(x-3)$

1. $x^2 - 5x + 4 = (x-1)(x-4)$

$x^2 - 5x + 4 = 0$

$\Delta = b^2 - 4ac$

$= 25 - 4 \cdot 4$

$= 9$

$x_1 x_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{5 \pm 3}{2} <>$

$x_1 = 4$

$x_2 = 1$

$=> (x-4)(x-1)$

2.4.

1. $\lim\limits_{x\to -2} \frac{x^3-3x^2-4x+12}{x^5-2} = \frac{(-2)^3-3(-2)^2+8+12}{(-2)^5-2} = \frac{-8+8-12+12}{-32-2} = \frac{0}{-34} = 0$

2. $\lim\limits_{x\to 0} \frac{2^x-6^x-3^x}{2^x+4^x} = \frac{2^0-6^0-3^0}{2^0+4^0} = \frac{1-1-1}{1+1} = -\frac{1}{2}$

3. $\lim\limits_{x\to 4} x^2-3x-4 = 4^2-3\cdot 4-4 = 16-12-4 = 0$

4. $\lim\limits_{x\to -2} \frac{x^3-x^2-4x+4}{x^2-3x+10} = \frac{(x+1)(x-1)(x+2)(x-2)}{(x+2)(x-5)} = \frac{(-2+1)(-2-1)(-2-2)}{(-2-5)} =$

$= \frac{-1\cdot(-3)\cdot(-4)}{-7} = -\frac{12}{7}$

2.5

1. $\lim\limits_{x\to\infty} \frac{2}{x^2+x} x = \frac{x\cdot 2}{x^2(1+\frac{1}{x})} = \frac{2}{\infty} = 0$

2. $\lim\limits_{x\to\infty} = \frac{3^x+5^x}{8^x} = \lim\limits_{x\to\infty} \left(\frac{3}{8}\right)^x + \left(\frac{5}{8}\right)^x = 0$

3. $\lim\limits_{x\to -\infty} \frac{6x^5-2x^3-8x+2}{3+x^2+4x^4} = \lim\limits_{x\to -\infty} \frac{6x^5-2x^3-8x+2}{4x^4+x^2+3} = \frac{x^5(6-\frac{2}{x^2}-\frac{8}{x^4}+\frac{2}{x^5})}{x^4(4+\frac{1}{x^2}+\frac{3}{x^4})}$

$= \frac{-\infty(6-0-0+0)}{4+0+0} = \frac{-\infty}{4} = -\infty$

4. $\lim\limits_{x\to\infty} \frac{\frac{1}{4^x}+1}{\frac{1}{3^x}-3} = \frac{0+1}{0-3} = -\frac{1}{3}$

5. $\lim\limits_{x \to -\infty} \frac{2x^6 - x^5 + 3x^4 - 5x}{3x^4 + 4x^8 - 3} = \lim\limits_{x \to -\infty} \frac{x^6(2 - \frac{1}{x} + \frac{3}{x^2} - \frac{5}{x^5})}{x^8(4 + \frac{3}{x^4} - \frac{3}{x^8})} = \frac{2}{x^2 4} = \frac{2}{-\infty} = 0$

6. $\lim\limits_{x \to -\infty} \frac{2^x + 4^x}{5^{-x}} = \left(\frac{2}{5}\right)^x + \left(\frac{4}{5}\right)^x = \left(\frac{5}{2}\right)^x + \left(\frac{5}{4}\right)^x = 0 + \infty = \infty$

7. $\lim\limits_{x \to \infty} \frac{3^x - 2^x}{3^x} = \left(\frac{3}{3}\right)^x - \left(\frac{2}{3}\right)^x = 1 - 0 = 1$

8. $\lim\limits_{x \to \infty} \frac{-4x^4 - x - 3}{x^3 - x + 9x^4 - 5} = \lim\limits_{x \to \infty} \frac{x^4(-4 - \frac{1}{x^3} - \frac{3}{x^4})}{x(\frac{1}{x} - \frac{1}{x^3} + 9 - \frac{5}{x^3})} = \frac{-4}{9}$

2.6

1. $(x^5 - 4x^4 - 5x^2 - x - 3)' = 5x^4 - 16x^3 - 10x - 1)$

2. $(\cos^4 x + \tan(3x))' = 4\cos^3(-\sin x) + 3 \cdot \frac{1}{\cos^2 3x}) = \cancel{\ldots} - 4\cos^3 x \cdot \sin x + \frac{3}{\cos^2(3x)}$

3. $\left(\frac{4 - \cos(x)}{e^x}\right)' = \frac{-\sin x \cdot e^x - 4e^x + \cos x \cdot e^x}{(e^x)^2} =$

$= \frac{\cancel{e^x}(-\sin - 4 + \cos x)}{(e^x)^{\cancel{2}}} = \frac{-\sin - 4 + \cos(x)}{e^x}$

4. $(e^x \sin(x) - 4\ln(x)\cos(x))' = -2x\sin x + (2 + x^2)\cos x - (3x^2\cos x - x^3 \sin x)$

$= -2x\sin x + 2\cos x + x^2\cos x - 3x^2\cos x + x^3 \sin x$

$= x \sin x(-2 + x^2) + 2\cos(1 - 2x^2)$

5. $\left(\frac{x^2 + x - 6}{x + 3}\right)' = \frac{(2x + 1)(x + 3) - (x^2 + x - 6)}{(x + 3)^2} = \frac{2x^2 + 6x + x + 3 - x^2 - x + 6}{x^2 + 6x + 9}$

$= \frac{x^2 + 6x + 9}{x^2 + 6x + 9} = 1$

6. $(x^7 - x^7 - x^0 - \ln(x) + \tan(x))' = 7x^6 - 7x^6 + \frac{1}{x} + \frac{1}{\cos^2 x}$

?

7. $((2-x^2)\sin x - x^3 \cos x)' = -2x\sin x + (2-x^2)\cos x - (3x^2\cos x - x^3\sin x)$

$= -2x\sin x + 2\cos x - 4x^2\cos x + x^3\sin x$

$= x\sin(x^2-2) + 2\cos(1-2x^2)$

8. $\left(\frac{xe^{4x}-2}{2x}\right)' = \frac{xe^{4x}}{2x} - \frac{2}{2x} = \frac{2e^{4x}\cdot 4 - e^{4x}\cdot 0}{4} - \frac{-2\cdot 2}{4x^2} = 2e^{4x} + \frac{1}{x^2}$

2.7.

1. $(\cos(x)\ln(x))'' =$

2. $(x^5 - x^4 - 5x^2 + x - 3)'' = (5x^4 - 4x^3 - 10x)' = (20x^3 - 12x^2 - 10) = 2(10x^3 - 6x^2 - 5)$

3. $\left(\frac{\ln(x^2)}{x}\right)'' = \frac{\frac{1}{x^2}\cdot 2x\cdot x - \ln(x^2)}{x^2} = \frac{\frac{2x^2}{x^2} - \ln(x^2)}{x^2} = \left(\frac{2-\ln(x^2)}{x^2}\right)' = \frac{-\frac{1}{x^2}2x\cdot x^2 - (2-\ln(x^2))2x}{x^4}$

$= \frac{-2x - 2x(2-\ln(x^2))}{x^4} = \frac{2x(1-2-\ln x^2)}{x^4} = \frac{2(\ln(x^2-3)}{x^3}$

4. $(x\cos(x))'' = (\cos x - x\sin x)' = -\sin x - \sin x + x\cos x = -2\sin x + x\cos x$

2.8

1. $\lim_{x \to -2} \frac{x^3 - x^2 - 4x + 4}{x^2 - 3x + 10} = \frac{(x+1)(x-1)\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x-5)} = \frac{(-2+1)(-2-1)(-2-2)}{(-2-5)}$

$= \frac{-1 \cdot (-3) \cdot (-4)}{-7} = -\frac{12}{7}$

2. $\lim_{x \to -2} \frac{x^3 - x^2 + 4x + 4}{x^2 - 3x - 10} =$

3. $\lim_{x \to 1} \frac{2x^3 - 3x^2 - 2x + 3}{5x^2 - 8x + 3} = \frac{2 - 3 - 2 + 3}{5 - 8 + 3} = "\frac{0}{0}"$

$\frac{6x^2 - 6x - 2}{10x - 8} = \frac{6 - 6 - 2}{10 - 8} = \frac{-2}{-2} = -1$