SKUP. C

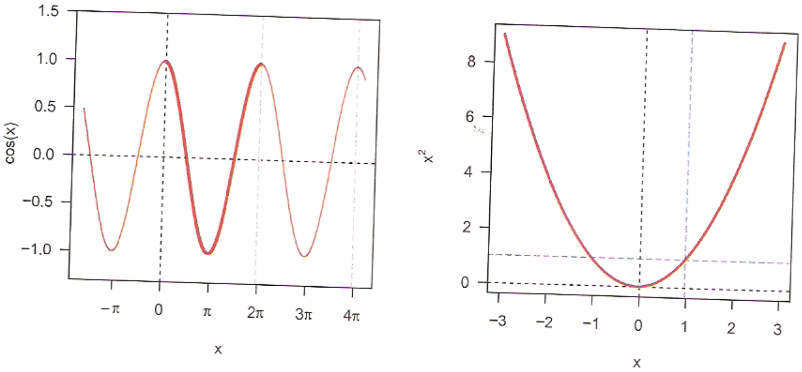
2.1. $f(x) = \cos(x)$

1. $D(f) = (-\infty, \infty)$
2. $H(f) = \langle -1, 1 \rangle$
3. spojitá
4. shora ohr. 1
   zdola ohr. -1
5. fce je periodická s periodou $2k\pi$
6. fce je sudá
7. fce není monotónní
8. $\lim\limits_{x \to \pi} f(x) = -1$

2.2. $f(x) = x^2$

1. $D(f) = (-\infty, \infty)$
2. $H(f) = \langle 0, \infty)$
3. spojitá
4. shora ohr. není
   zdola ohr. 0
5. fce není periodická
6. fce je sudá
7. fce není monotónní
8. $\lim\limits_{x \to 0} f(x) = 0$

C



2.3.

1. $x^2 + x - 6$ $\qquad \dfrac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-6)}}{2 \cdot 1} = \dfrac{-1 \pm \sqrt{25}}{2} = \dfrac{-1 \pm 5}{2} \begin{cases} -3 \\ 2 \end{cases}$

$(x+3)(x-2)$

2. $x^3 + 7x^2 + 11x + 5$

	1	7	11	5
1	1	8	19	24 ✗
−1	1	6	5	0
−1	1	5	0	

$x_1 = -1$ $\qquad (x+1)(x^2+6x+5)$

$x_2 = -1$

$x_3 = -5$ $\qquad (x+1)(x+1)(x+5)$

2.4.

1. $\lim\limits_{x \to 5} \dfrac{x^2 - 5x + 4}{x - 1} = \dfrac{5^2 - 5 \cdot 5 + 4}{5 - 1} = \dfrac{25 - 25 + 4}{4} = \underline{\underline{1}}$

2. $\lim\limits_{x\to 2} \frac{3^x+3}{2^x \pm 4^x} = \frac{3^2+3}{2^2-4^2} = \frac{12}{-12} = \underline{\underline{-1}}$

3. $\lim\limits_{x\to 3} \frac{x^3-x^2-5x-3}{x^2-9} = \frac{27-9-15-3}{9-9} = \frac{0}{0}$

	1	-1	-5	-3
1	1	0	-5	-8
-1	1	-2	-3	0
-1	1	-3	0	

$\pm 1 \quad \pm 3$

$x_1=-1 \quad (x+1)(x^2-2x-3)$

$x_2=-1 \quad (x+1)(x+1)(x-3)$

$x_3=3$

$\lim\limits_{x\to 3} \frac{(x+1)(x+1)\cancel{(x-3)}}{\cancel{(x-3)}(x+3)} = \frac{(x+1)(x+1)}{(x+3)} = \lim\limits_{x\to 3} \frac{(3+1)(3+1)}{(3+3)} = \frac{16}{6} = \underline{\underline{\frac{8}{3}}}$

4. $\lim\limits_{x\to 3} 3x^2-7 = 3\cdot 3^2-7 = \underline{\underline{20}}$

2.5.

1. $\lim\limits_{x\to -\infty} \frac{2x^2-x}{x} = \lim\limits_{x\to -\infty} \frac{x^{\cancel{2}}(2-\frac{1}{x})}{\cancel{x}} = \lim\limits_{x\to -\infty} -\infty(2-0) = \underline{\underline{-\infty}}$

2. $\lim\limits_{x\to -\infty} \frac{4x+2-3x^2-2x^7}{2x^3+x^5-3} = \lim\limits_{x\to -\infty} \frac{-2x^7-3x^2+4x+2}{x^5+2x^3-3} = \frac{x^7(-2-\frac{3}{x^5}+\frac{4}{x^6}+\frac{2}{x^7})}{x^5(1+\frac{2}{x^2}-\frac{3}{x^5})} = \frac{x^2(-2-\frac{3}{x^5}+\frac{4}{x^6}+\frac{2}{x^7})}{(1+\frac{2}{x^2}-\frac{3}{x^5})} =$

$\lim\limits_{x\to -\infty} \frac{-\infty(-2-0+0+0)}{1+0+0} = \underline{\underline{\infty}}$

3. $\lim\limits_{x\to \infty} \frac{3+x^3+x}{3x^5-2x^3-6x+2} = \lim\limits_{x\to\infty} \frac{x^3+x+3}{3x^5-2x^3-6x+2} = \frac{x^3(1+\frac{1}{x^2}+\frac{3}{x^3})}{x^5(3-\frac{2}{x^2}-\frac{6}{x^4}+\frac{2}{x^5})} = \frac{1+0+0}{\infty(3-0-0+0)} = \frac{1}{\infty} = \underline{\underline{0}}$

4. $\lim\limits_{x\to -\infty} \frac{7^x-5^x}{5^x} = \lim\limits_{x\to -\infty} \left(\frac{7}{5}\right)^x - \left(\frac{5}{5}\right)^x = \underline{\underline{-1}}$

5. $\lim\limits_{x\to \infty} \frac{2+\frac{1}{4^x}}{2-\frac{1}{5^x}} = \lim\limits_{x\to\infty} \frac{2+0}{2-0} = \underline{\underline{1}}$

6. $\lim\limits_{x\to -\infty} \frac{6^x-2^x}{3^x} = \lim\limits_{x\to -\infty} \left(\frac{6}{3}\right)^x - \left(\frac{2}{3}\right)^x = -\infty - 0 = \underline{\underline{-\infty}}$

7. $\lim\limits_{x\to \infty} \frac{1+2x-x^3+4x^5}{x^4+2x^5-x^3} = \lim\limits_{x\to\infty} \frac{4x^5-x^3+2x+1}{2x^5+x^4-x^3} = \lim\limits_{x\to\infty} \frac{\cancel{x^5}(4-\frac{1}{x^2}+\frac{2}{x^4}+\frac{1}{x^5})}{\cancel{x^5}(2+\frac{1}{x}-\frac{1}{x^2})} = \frac{4-0+0+0}{2+0-0} = \underline{\underline{2}}$

8. $\lim\limits_{x\to \infty} \frac{4^x-3^x}{5^x} = \left(\frac{4}{5}\right)^x - \left(\frac{3}{5}\right)^x = \infty - \infty = \underline{\underline{0}}$

2.6.

1. $(x^4+x^{-4}+x^0-\tan(x)+e^x)' = 4x^3 - 4x^{-5} - \frac{1}{\cos^2(x)} + e^x$

2. $\left(\frac{e^x+x^2-4x}{\ln(x)}\right)' = \frac{(e^x+2x-4)\ln(x) - (e^x+x^2-4x)\cdot\frac{1}{x}}{\ln^2(x)} = \frac{\cancel{\ln(x)}\cdot(e^x+2x-4)}{\ln^{\cancel{2}}(x)} - \frac{(e^x+x^2-4x)\cdot\frac{1}{x}}{\ln^2(x)} =$

$= \underline{\underline{\frac{e^x+2x-4}{\ln(x)} - \frac{e^x+x^2-4x}{x\ln^2(x)}}}$

3. $((x+x^4)\ln(x)-4x\sin(x))'=((1+4x^3)\ln(x)+(x+x^4)\frac{1}{x})-(4\sin(x)+4x\cdot\cos(x))=$
$=\underline{\underline{(1+4x^3)\ln(x)+x^3+1-4\sin(x)-4x\cos(x)}}$

4. $(\ln(\cos(x))+\ln(\ln(x)))'=\frac{1}{\cos(x)}\cdot(-\sin(x)+\frac{1}{\ln(x)}\cdot\frac{1}{x}=-\frac{\sin(x)}{\cos(x)}+\frac{1}{x\ln(x)}=\underline{\underline{-\tan(x)+\frac{1}{x\ln(x)}}}$

5. $(3\ln(x)\tan(x)+\sin(x)\cos(x))'=\left(3\cdot\frac{1}{x}\tan(x)+3\ln(x)\frac{1}{\cos^2x}\right)+(\cos(x)\cdot\cos(x)+\sin(x)\cdot(-\sin(x))=$
$=\underline{\underline{\frac{3\tan(x)}{x}+\frac{3\ln(x)}{\cos^2(x)}+\cos^2(x)-\sin^2(x)}}$

6. $(2x^6-x^4+3x^3+5x)'=\underline{\underline{12x^5-4x^3+9x^2+5}}$

7. $\left(\frac{x}{(1-x)^2}\right)'=\frac{1(1-x)^2-(x(2-2x)\cdot(-1))}{(1-x)^4}=\frac{(1-x)^2-(-2x+2x^2)}{(1-x)^4}=\frac{1-2x+x^2+2x-2x^2}{(1-x)^4}=$

$=\frac{1-x^2}{(1-x)^4}=\frac{(1-x)(1+x)}{(1-x)^4}=\underline{\underline{\frac{1+x}{(1-x)^3}}}$

8. $\left(\frac{-2}{\sin(2x+3)}\right)'=\frac{0(\sin(2x+3))-(-2\cdot\cos(2x+3)\cdot2)}{\sin^2(2x+3)}=\frac{4\cos(2x+3)}{\sin^2(2x+3)}$

2.7.

1. $(x^{-1}\ln(x))''=(-x^{-2}\ln(x)+x^{-1}\cdot\frac{1}{x})'=(-x^{-2}\ln(x)+x^{-2})'=2x^{-3}\ln(x)+(-x^{-2}\cdot\frac{1}{x})-2x^{-3}=$
$=2x^{-3}\ln(x)-x^{-2}\cdot\frac{1}{x}-2x^{-3}=2x^{-3}\ln(x)-x^{-2}\cdot x^{-1}-2x^{-3}=2x^{-3}\ln(x)-x^{-3}-2x^{-3}=$
$=x^{-3}(2\ln(x)-1-2)=\underline{\underline{\frac{2\ln(x)-3}{x^3}}}$

2. $(\cos(x^2)+\sin(2x))''=(-\sin(x^2)\cdot2x+\cos(2x)\cdot2)'=$
$=-\cos(x^2)\cdot2x\cdot2x-\sin(x^2)\cdot2-\sin(2x)\cdot2\cdot2+\cos(2x)\cdot0=$
$=-\cos(x^2)\cdot4x^2-\sin(x^2)\cdot2-\sin(2x)\cdot4=\underline{\underline{-4x^2\cos(x^2)-2\sin(x^2)-4\sin(2x)}}$

3. $(\sin(x)\ln(x))''=(\cos(x)\ln(x)+\sin(x)\cdot\frac{1}{x})'=-\sin(x)\ln(x)+\cos(x)\frac{1}{x}+\cos(x)\frac{1}{x}+\sin(x)(x^{-2})=$
$=-\sin(x)\ln(x)+\frac{2\cos(x)}{x}+\sin(x)\cdot(-x^{-2})=\underline{\underline{\frac{2\cos(x)}{x}-\sin(x)(\ln(x)+\frac{1}{x^2})}}$

4. $(2x^6-x^4+3x^3+4x^2-5)''=(12x^5-4x^3+9x^2+8x-5)'=60x^4-12x^2+18x+8=$
~~$=2(30x^4-12x^3+18x$~~
$=\underline{\underline{2(30x^4-6x^2-9x+4)}}$

2.8.

$$1.\ \lim_{x\to 3}\frac{x^3-x^2+5x-3}{x^2-9}=\lim_{x\to 3}\frac{3^3-3^2+5\cdot 3-3}{3^2-9}=\frac{27-9+15-3}{9-9}=\frac{30}{0}=\underline{\underline{30}}\quad \text{nelze užít}$$

$$2.\ \lim_{x\to 3}\frac{x^3-x^2-5x-3}{x^2-9}=\lim_{x\to 3}\frac{3^3-3^2-5\cdot 3-3}{3^2-9}=\frac{27-9-15-3}{9-9}=\frac{0}{0}\quad \checkmark$$

$$\lim_{x\to 3}\frac{(x^3-x^2-5x-3)'}{(x^2-9)'}=\lim_{x\to 3}\frac{3x^2-2x-5}{2x}=\frac{3\cdot 3^2-2\cdot 3-5}{2\cdot 3}=\frac{27-6-5}{6}=\frac{16}{6}=\underline{\underline{\frac{8}{3}}}$$

$$3.\ \lim_{x\to -1}\frac{3x^3-7x^2-2x+8}{4x^2+x-3}=\lim_{x\to -1}\frac{3\cdot(-1)^3-7(-1)^2-2\cdot(-1)+8}{4\cdot(-1)^2+(-1)-3}=\frac{-3-7+2+8}{4-1-3}=\frac{0}{0}\quad \checkmark$$

$$\lim_{x\to -1}\frac{(3x^3-7x^2-2x+8)'}{(4x^2+x-3)'}=\lim_{x\to -1}\frac{9x^2-14x-2}{8x+1}=\frac{9(-1)^2-14(-1)-2}{8(-1)+1}=\frac{9+14-2}{-8+1}=\frac{21}{-7}=-\underline{\underline{3}}$$