Příklad: 3.1. PARCIÁLNÍ DERIVACE

1. $xy - \ln x + \sin y$

$\frac{\partial}{\partial x} f(x,y) = y - \frac{1}{x}$ $\frac{\partial}{\partial y} f(x,y) = x + \cos y$

3. $\ln\left(\frac{x+y}{2}\right)$

$\frac{\partial}{\partial x} f(x,y) = \frac{1}{x+y}$ $\frac{\partial}{\partial y} f(x,y) = \frac{1}{x+y}$

2. $y^2 \cos x$

$\frac{\partial}{\partial x} f(x,y) = -y^2 \sin x$ $\frac{\partial}{\partial y} f(x,y) = 2y \cdot \cos$

4. $y^2 \sin^2 x$

$\frac{\partial}{\partial x} f(xy) = 2y^2 \cdot \sin x \cdot \cos x$

$\frac{\partial}{\partial y} f(xy) = 0 + 2y \sin^2 x = 2y \sin^2 x$

5. $y^2 e^{2x}$

$\frac{\partial}{\partial x} f(x,y) = 0 + y^2 \cdot e^{2x} \cdot 2 = 2y^2 \cdot e^{2x}$

$\frac{\partial}{\partial y} f(x,y) = 2y e^{2x}$

6. $-\cos \cdot (x+y)$

$\frac{\partial}{\partial x} f(x,y) = \sin(x+y)$ $\frac{\partial}{\partial y} f(x,y) = \sin(x+y)$

7. $e^{x^2} \cdot y^2 =$

$\frac{\partial}{\partial y} f(x,y) = 2y \cdot x \cdot e^{x^2} \cdot y^2$

$\frac{\partial}{\partial x} f(x,y) = 2x \cdot y^2 \cdot e^{x^2} \cdot y^2$

Příklad 3.2 → PARCIÁLNÍ DERIVACE 2. ŘÁDU

1. $xy - \ln x + \sin y$

$\frac{\partial}{\partial x} f(x,y) = y - \frac{1}{x}$ $\quad \frac{\partial}{\partial xx} f(x,y) = \frac{1}{x^2}$

$\frac{\partial}{\partial y} f(x,y) = x + \cos y$ $\quad \frac{\partial}{\partial yy} f(x,y) = 1 - \sin y = -\sin y$

$\frac{\partial}{\partial xy} f(x,y) = \frac{\partial}{\partial y}\left(\frac{1}{x^2}\right) = \underline{\underline{1}}$

2. $y^2 \cdot \cos x$

$\frac{\partial}{\partial x} f(x,y) = -y^2 \sin x$ $\quad \frac{\partial}{\partial xx} f(x,y) = -y^2 \cdot \cos x$

$\frac{\partial}{\partial y} f(x,y) = 2y \cdot \cos x$ $\quad \frac{\partial}{\partial yy} = 2\cos x$

$\frac{\partial}{\partial xy} f(x,y) = \frac{\partial}{\partial y}\left(-y^2 \cdot \cos x\right) = -2y \sin x$

3. $\ln\left(\frac{x+y}{2}\right)$

$\frac{\partial}{\partial x} f(x,y) = \ln\left(\frac{x+y}{2}\right)$ $\quad \frac{\partial}{\partial xx} f(x,y) = -\frac{1}{(x+y)^2}$

$\frac{\partial}{\partial y} f(x,y) = \ln\left(\frac{x+y}{2}\right)$ $\quad \frac{\partial}{\partial yy} f(x,y) = -\frac{1}{(x+y)^2}$

$\frac{\partial}{\partial xy} f(x,y) = \frac{\partial}{\partial y}\left(-\frac{1}{(x+y)^2}\right) = -\frac{1}{(x+y)^2}$

4. $y^2 \sin^2 x$

$\frac{\partial}{\partial x} f(x,y) = 2y^2 \cdot \sin x \cos x$ $\quad \frac{\partial}{\partial xx} f(x,y) = -2y^2 \cos^2 x \sin^2 x$

$\frac{\partial}{\partial y} f(x,y) = 2y \sin^2 x$ $\quad \frac{\partial}{\partial yy} f(x,y) = \cancel{2x}\ 2\sin^2 x$

$\frac{\partial}{\partial xy} f(x,y) = \frac{\partial}{\partial y}\left(2y^2 \sin x \cos x\right) = 4y \sin x \cos x$

5. $y^2 e^{2x}$

$\frac{\partial}{\partial x} f(x,y) = 2y^2 e^{2x}$ $\quad \frac{\partial}{\partial xx} f(x,y) = 4y^2 e^{2x}$

$\frac{\partial}{\partial y} f(x,y) = 2y e^{2x}$ $\quad \frac{\partial}{\partial yy} f(x,y) = 2e^{2x}$

$\frac{\partial}{} f(x,y) = \left(2y^2 e^{2x}\right) \cdot \frac{\partial}{\partial y} = 4y e^{2x}$

6. $-\cos x \cdot (x+y)$

$\frac{\partial}{\partial x} f(x;y) = \sin(x+y)$ $\quad \frac{\partial}{\partial x} f(x,y) = \cos(x+y)$

$\frac{\partial}{\partial y} f(x;y) = \sin(x+y)$ $\quad \frac{\partial}{\partial y} f(x,y) = \cos(x+y)$

$\frac{\partial}{\partial y} f(x;y) = \sin(x+y)$ $\quad \frac{\partial}{\partial y} f(x,y) = \cos(x+y)$

$\frac{\partial}{\partial xy} f(x;y) = \frac{\partial}{\partial y} \overset{\sin}{\cancel{\cos}}(x+y)$

7. $e^{x^2 y^2}$

$\frac{\partial}{\partial x} f(x;y) = 2xy^2 e^{x^2y^2}$ $\quad \frac{\partial}{\partial xx} f(x;y) = 2y^2 e^{x^2y^2} \cdot (2x^2y^2+1)$

$\frac{\partial}{\partial y} f(x;y) = 2x^2y e^{x^2y^2}$ $\quad \frac{\partial}{\partial yy} f(x;y) = 2x^2 e^{x^2y^2} \cdot (2x^2y^2+1)$

$\frac{\partial}{\partial xy} f(x;y) = \frac{\partial}{\partial y} \left[2y^2 \cdot e^{x^2y^2} \cdot (2x^2y^2+1)\right] = 4xye^{x^2y^2} \cdot (x^2y^2+1)$

EXTRÉMY FCE

3.3

1. $f(x,y) = 1+6y+y^2-xy-x^2$

$f'_x(1+6y-y^2-xy-x^2) = 0+0-0-y-2x = -y-2x$

$f'_y(1+6y-y^2-xy-x^2) = 0+6-2y-x = 6-2y-x$

$-y-2x = 0$

$-y = 2x$

$y = -2x$

$6-2\cdot(-2x)-x) = 0$

$6+4x-x = 0$

$6+3x = 0$

$3x = -6$

$\underline{\underline{x = -2}}$

$y = -2\cdot(-2)$

$\underline{\underline{y = 4}}$

$[-2,4]$

stacionární body

$\frac{\partial}{\partial x^2}(1+6y+y^2-xy-x^2) = \frac{\partial}{\partial x}(-y-2x) = -2$

$\frac{\partial}{\partial y^2}(1+6y+y^2-xy-x^2) = \frac{\partial}{\partial y}(6-2y-x) = -2$

$\frac{\partial}{\partial x \partial y}(1+6y+y^2-xy-x^2) = \frac{\partial}{\partial x}(6-2y-x) = -1$

$\frac{\partial}{\partial y \partial x}(1+6y+y^2-xy-x^2) = \frac{\partial}{\partial y}(-y-2x) = -1$

Hessián

$H = \begin{pmatrix} -2 & -1 \\ -1 & -2 \end{pmatrix} = (-2)\cdot(-2) - (-1)\cdot(-1) = 4-1 = 3 > 0$

je extrém

$\frac{\partial}{\partial x^2} = -2 \rightarrow \ominus\ \cap = \text{maximum}$

$M[-2,4]$

2. $f(x,y) = x^3+y^3-3xy$

$3x^2-3y = 0$

$3(x^2)^2-3x = 0$

$\frac{\partial}{\partial x \partial y}(1+6y+y$ ... $\frac{\partial}{\partial x}($

$\frac{\partial}{\partial y \partial x}(1+6y+y^2-xy-x^2)=\frac{\partial}{\partial y}(-y-2x)=-1$

$M[-2,4]$

2. $f(x,y)=x^3+y^3-3xy$

$f'_x(x^3+y^3-3xy)=3x^2-3y$

$f'_y(x^3+y^3-3xy)=3y^2-3x$

$3x^2-3y=0$
$-3y=-3x^2$
$3y=3x^2$
$y=x^2$

$3(x^2)^2-3x=0$
$3x^4-3x=0$
$x^4-x=0$
$x(x^3-1)=0$

$\frac{\partial}{\partial x^2}(x^3+y^3-3xy)=\frac{\partial}{\partial x}(3x^2-3y)=6x$

$\frac{\partial}{\partial y^2}(x^3+y^3-3xy)=\frac{\partial}{\partial y}(3y^2-3x)=6y$

$\frac{\partial}{\partial x \partial y}(x^3+y^3-3xy)=\frac{\partial}{\partial x}(3y^2-3x)=-3$

$\frac{\partial}{\partial y \partial x}(x^3+y^3-3xy)=\frac{\partial}{\partial y}(3x^2-3y)=-3$

stacionární body $\begin{cases}[1,1] & x=1 \quad y=1^2 \quad y=1\\ [0,0] & x=0 \quad y=0^2 \quad y=0\end{cases}$

$H_{[0,0]}\begin{pmatrix}0 & -3\\ -3 & 0\end{pmatrix}=0-9=-9<0$ ← není extrém

$S[0,0]$

$H_{[1,1]}\begin{pmatrix}6 & -3\\ -3 & 6\end{pmatrix}=36-9=27>0$ ← je extrém

$\frac{\partial}{\partial x^2}=6=+$ $\underline{\cup}$ – minimum

$m[1,1]$

3. $f(x,y) = x^2 - y^2 + 2x - 2y$

$2x + 2 = 0$ $\quad$ $-2y - 2 = 0$

$x = -1$ $\quad$ $y = -1$

$f'_x(x,y) = 2x + 2$

$f'_y(x,y) = -2y - 2$

$[-1, -1]$

stacionární bod

$\frac{\partial}{\partial x^2}(x^2 - y^2 + 2x - 2y) = \frac{\partial}{\partial x}(2x + 2) = 2$

Hessian

$H = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} = -4 - 0 = -4 < 0$

$\uparrow$

není extrém

$\frac{\partial}{\partial y^2}(x^2 - y^2 + 2x - 2y) = \frac{\partial}{\partial y}(-2y - 2) = -2$

$\frac{\partial}{\partial x \partial y}(x^2 - y^2 + 2x - 2y) = \frac{\partial}{\partial x}(-2y - 2) = 0$

$S[-1, -1]$

$\frac{\partial}{\partial y \partial x}(x^2 - y^2 + 2x - 2y) = \frac{\partial}{\partial y}(2x + 2) = 0$

4. $f(x,y) = x(x-1) + y(y+1) - xy + 2$ $\quad$ $2x - 1 - y = 0$ $\quad$ $2y - 1 - \left(\frac{1+y}{2}\right) = 0$

$$\frac{\partial}{\partial y \partial x}\left(x^2 - y^2 + 2x - 2y\right) = \frac{\partial}{\partial y}(2x+2)$$

4. $f(x,y) = x(x-1) + y(y+1) - xy + 2$

$f'_x(x,y) = 2x - 1 - y$

$f'_y(x,y) = 2y - 1 - x$

$$\frac{\partial}{\partial x^2}\left(x(x-1)+y(y-1)-xy+2\right) = \frac{\partial}{\partial x}(2x-1-y) = 2$$

$$\frac{\partial}{\partial y^2}\left(x(x-1)+y(y-1)-xy+2\right) = \frac{\partial}{\partial y}(2y-1-x) = 2$$

$$\frac{\partial}{\partial x \partial y}\left(x(x-1)+y(y-1)-xy+2\right) = \frac{\partial}{\partial x}(2y-1-x) = -1$$

$$\frac{\partial}{\partial y \partial x}\left(x(x-1)+y(y-1)-xy+2\right) = \frac{\partial}{\partial y}(2x-1-y) = -1$$

$2x - 1 - y = 0$

$2x = 1 + y$

$x = \frac{1+y}{2}$

$x = \frac{1+1}{2}$

$x = 1$

$2y - 1 - \left(\frac{1+y}{2}\right) = 0$

$\frac{4y-2-1-y}{2} = 0$

$4y - 2 - 1 - y = 0$

$3y - 3 = 0$

$3y = 3$

$y = 1$

$[1,1]$

stacionární bod

Hessián

$$H = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} = 4 - 1 = 3 > 0$$

je extrém

$\frac{\partial}{\partial x^2} = 2 \rightarrow + \cup$ minimum

$m[1,1]$

3\. $f(x,y)$

6\. $f(x,y) = 4-(x-2)^2-(y+3)^2$

$f'_x(x,y) = -2x+4 = -(x^2-4x+4)'$

$f'_y(x,y) = -2y-6 = -(y^2+6x+9)'$

$-2x+4=0$

$2x=+4$

$x=2$

$-2y-6=0$

$2y=-6$

$y=-3$

$[2,-3]$

$$\frac{\partial}{\partial x^2}\left(4-(x-2)^2-(y+3)^2\right) = \frac{\partial}{\partial x}(-2x+4) = -2$$

$$\frac{\partial}{\partial y^2}\left(4-(x-2)^2-(y+3)^2\right) = \frac{\partial}{\partial y}(-2y-6) = -2$$

$$\frac{\partial}{\partial x \partial y}\left(4-(x-2)^2-(y+3)^2\right) = \frac{\partial}{\partial x}(-2y-6) = 0$$

$$\frac{\partial}{\partial y \partial y}\left(4-(x-2)^2-(y+3)^2\right) = \frac{\partial}{\partial y}(-2x+4) = 0$$

Hessián

$$H = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} = 4-0 = 4 > 0$$

je extrém

$$\frac{\partial}{\partial x^2} = -2 \rightarrow \ominus \sqcap \text{ maximum}$$

$\sqcap [2,-3]$

7\.

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$\frac{\partial}{\partial y \partial y}\left(4-(x-2)^2-(y+3)\right) \quad \partial y$

7. $f(x,y) = 2x^3 - xy^2 + 5x^2 + y^2$

$f'_x(x,y) = 6x^2 - y^2 + 10x$

$f'_y(x,y) = -2xy + 2y$

$-2xy + 2y = 0$

$-2xy = -2y$

$2xy = 2y$

$2x = 2$

$\boxed{x = 1}$

$6\cdot(1)^2 - y^2 + 10\cdot 1 = 0$

$-y^2 + 16 = 0$

$-y^2 = -16$

$y^2 = 16$

$\boxed{y = 4 \quad y = -4}$

$\frac{\partial}{\partial x^2}(2x^3 - xy^2 + 5x^2 + y^2) = \frac{\partial}{\partial x}(6x^2 - y^2 + 10x) = 12x + 10$

$6x^2 - y^2 + 10x = 0$

$-2xy + 2y = 0$

$\boxed{x = 0 \quad y = 0}$

$\frac{\partial}{\partial y}(2x^3 - xy^2 + 5x^2 + y^2) = \frac{\partial}{\partial y}(-2xy + 2y) = -2x + 2$

$\frac{\partial}{\partial x \partial y}(2x^3 - xy^2 + 5x + y^2) = \frac{\partial}{\partial x}(-2xy + 2y) = -2y$

$\frac{\partial}{\partial y \partial x}(2x^3 - xy^2 + 5x + y^2) = \frac{\partial}{\partial y}(6x^2 - y^2 + 10x) = -2y$

$$\underbrace{[1,4] \quad [1,-4] \quad [0,0] \quad [-\tfrac{5}{3},0]}_{\text{stacionární body}}$$

7.

$H_{[1,-4]}\begin{pmatrix} 22 & 8 \\ 8 & 0 \end{pmatrix} = 0-16 = -16 <$ není extrém

$H[1,4]\begin{pmatrix} 22 & -8 \\ -8 & 0 \end{pmatrix} = 0-((-8)(-8)) = 0-16 = -16 < 0$ - není extrém

$H[0,0]\begin{pmatrix} 10 & 0 \\ 0 & 2 \end{pmatrix} = 20-0 = 20 > 0$ - je extrém

$\frac{d}{dx^2} = 10 \rightarrow \oplus\ \underline{\cup}$ - minimum

$H[-\frac{5}{3},0]\begin{pmatrix} -10 & 0 \\ 0 & \frac{16}{3} \end{pmatrix} = -\frac{160}{3} < 0$ - není extrém

$m[0,0]\ ;\ S[-\frac{5}{3},0]\ ;\ S_{[1,4]}\ ;\ S[1,-4]$

$6x^2 - y^2 + 10x = 0$

$y^2 = 6x^2 + 10x$

$y = 6x + \sqrt{10x}$

$$\begin{array}{c|ccc} & -3 & -2 & +5 \\ \hline 1 & -3 & -5 & 0 \end{array} \quad (x-1)$$

$$\begin{array}{c|cc} & -3 & -5 \\ \hline -\frac{5}{3} & -3 & 0 \end{array} \quad (x+\frac{5}{3})$$

$-2x\cdot(6x+\sqrt{10x}) + 2(6x+\sqrt{10x}) = 0$

$-12x^2 - 2x\sqrt{10x} + 12x + 2\cdot\sqrt{10x} = 0 \,/^2$

$-12x^4 - 20x^2 + 12x^2 + 20x = 0$

$4x(-3x^3 - 2x + 5) = 0$

$-3x^3 - 2x + 5 = 0$

$($ $=$

$(x-1)\cdot(x+\frac{5}{3})\cdot(x-3) = 0$

$x = -\frac{5}{3}$

$y = 6\cdot(-\frac{5}{3}) - \sqrt{10\cdot(-\frac{5}{3})}$

$y =$ nejde

$y = 0$

$[-\frac{5}{3}, 0]$