1. $2xy + \ln y - \cos x$

$\frac{d}{dx} = 2y + \sin x$

$\frac{d}{dy} = 2x + \frac{1}{y}$

2. $\sin(x-y)$

$\frac{d}{dx} = \cos(x-y)$

$\frac{d}{dy} = -\cos(x-y)$

3. $\frac{1}{xy} + \ln x = (xy)^{-1} + \ln x$

$\frac{d}{dx} = -y^{-1}x^{-2} + \frac{1}{x} = \frac{1}{x^2 y} + \frac{1}{x}$?

$\frac{d}{dy} = -\frac{1}{xy^2}$

4. $\ln(x^2 - y^2)$

$\frac{d}{dx} = \frac{1}{x^2-y^2} \cdot 2x = \frac{2x}{x^2-y^2}$

$\frac{d}{dy} = \frac{2y}{x^2-y^2}$

5. $\frac{x^2}{y^2} = x^2 \cdot y^{-2}$

$\frac{d}{dx} = \frac{2x}{y^2}$

$\frac{d}{dy} = -2x \cdot y^{-3} = -\frac{2x}{y^3}$

6. $\cos xy$

$\frac{d}{dx} = -\sin xy \cdot y = -y\sin(xy)$

$\frac{d}{dy} = -x\sin(xy)$

7. $e^{(1-x^2)y}$

$\frac{d}{dx} = e^{(1-x^2)y} \cdot (-2xy) = -2xy e^{(1-x^2)y}$

$\frac{d}{dy} = e^{(1-x^2)y} \cdot (1-x^2) = (1-x^2) e^{(1-x^2)y}$?

$\cdot(-2xy) = -2xy\,e^{(1-x^2)y}$

$\frac{d}{dy} = e^{(1-x^2)y} \cdot (1-x^2) = (1-x^2)\,e^{(1-x^2)y}$?

3.2

1. $2xy + \ln x - \cos x$

$\frac{2y + \sin x}{dx^2} = \cos x$

$\frac{2x + y^{-1}}{dy^2} = -y^{-2} = -\frac{1}{y^2}$

~~$\frac{2y + \sin x}{dx\,dy} = 2$~~

$\frac{d^2}{dx\,dy} = \frac{2y + \sin x}{dy} = 2$

$\frac{d^2}{dy\,dx} = \frac{2x + y^{-1}}{dx} = 2$

2. $\sin(x-y)$

$\frac{d^2}{dx^2} = \frac{\cos(x-y)}{dx} = -\sin(x-y)$

$\frac{d^2}{dy^2} = \frac{-\cos(x-y)}{dy} = -\sin(x-y)$

$\frac{d^2}{dx\,dy} = \frac{\cos(x-y)}{dy} = \sin(x-y)$

$\frac{d^2}{dy\,dx} = \frac{-\cos(x-y)}{dx} = \sin(x-y)$

3. $\frac{1}{xy} + \ln x$

$\frac{d^2}{dx} = \frac{\frac{xy-1}{x^2y}}{dx} =$

5 $\frac{x^2}{y}$

$$\frac{d^2}{d^2x} = \frac{2xy^{-2}}{dx} = 2y^{-1} = \frac{2}{y^2}$$

$$\frac{d^2}{d2y} = \frac{-2x^2y^{-3}}{dy} = 6x^2y^{-4} = \frac{6x^2}{y^4}$$

$$\frac{d^2}{dxdy} = \frac{2xy^{-2}}{dy} = -4xy^{-3} = -\frac{4x}{y^3}$$

$$\frac{d^2}{dydx} = \frac{-2x^2y^{-3}}{dx} = -\frac{4x}{y^3}$$

6. $\cos xy$

$$\frac{d^2}{d^2x} = \frac{-y\sin(xy)}{dx} =$$

7. $e^{(1-x^2)y}$

$$\frac{d_e^2}{d_x^2} = \frac{-2xy \cdot e^{(1-x^2)y}}{dx} =$$

3.3

1. Uvažujme $f(x,y) = 2xy - 2x - 4y$

$$\frac{d}{dx} = 2y - 2$$

$$\frac{d}{dy} = 2x - 4$$

$$\begin{array}{l} 2y - 2 = 0 \iff y = 1 \\ 2x - 4 = 0 \implies x = 2 \end{array}$$

stĺ $[2,1]$

$$\frac{d^2}{dx^2} = \frac{2y-2}{dx} = 0$$

$$\frac{d^2}{dxdy} = \frac{2y-2}{dy} = 2$$

$$\frac{d^2}{dy^2} = \frac{2x-4}{dy} = 0$$

$$\frac{d^2}{dydx} = \frac{2x-4}{dx} = 2$$

$$A = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \quad D = 0 \cdot 0 - 2 \cdot 2 = -4$$

$[2,1]$ nie je stacionárny bod

2 $f(x,y) = 2x^3 + 3y^2 - 6xy$

$\frac{d}{dx} = 6x^2 - 6y$

$\frac{d}{dy} = 6y - 6x$

$6x^2 - 6y = 0$

$6y - 6x = 0$

$6x^2 - 6x = 0$

$\Delta = b^2 - 4ac$

$= 36$

$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{6 \pm 6}{12} \Rightarrow x_1 = 0$

$x_2 = 1$

$S_1: 6y - 6x = 0, \; x = 0$

$y = 0$

$[0, 0]$

$S_2: 6y - 6x = 0 \quad x = 1$

$6y - 6 = 0$

$y = 1$

$S_2 \; [1, 1]$

$\frac{d^2}{dx^2} = 12x \qquad \frac{d^2}{dxdy} = -6$

$\frac{d^2}{dy} = 6 \qquad \frac{d^2}{dydx} = -6$

S_1:

$H = \begin{pmatrix} 12x & -6 \\ -6 & 6 \end{pmatrix} = \begin{pmatrix} 0 & -6 \\ -6 & 6 \end{pmatrix}$

$D = 0 \cdot 6 - 36 = -36$ S_1 nije stac. bod

$S_2: \begin{pmatrix} 12x & -6 \\ -6 & 6 \end{pmatrix} = \begin{pmatrix} 12 & -6 \\ -6 & 6 \end{pmatrix} = 12 \cdot 6 - 36 = 72 - 36 = 36$ S_2 minimum ?

3. $f(x,y) = x^2 + 4xy + 6y^2 - 2x + 8y - 5$

$\frac{d}{dx} = 2x + 4y - 2$

$\frac{d}{dy} = 4x + 12y + 8$

$2x + 4y - 2 = 0 \quad \Rightarrow 2x - 12 - 2 = 0$

$4x + 12y + 8 = 0 \qquad x = 7$

$-4x - 8y + 4 = 0$

$4x + 12y + 8 = 0$

$4y + 12 = 0$

$y = -3$

$SB \; [7, -3]$

$\frac{d^2}{dx^2} = 2 \qquad \frac{d^2}{dxdy} = 4$

$\frac{d^2}{dy^2} = 12 \qquad \frac{d^2}{dydx} = 4$

$H = \begin{pmatrix} 2 & 4 \\ 4 & 12 \end{pmatrix} \quad D = 2 \cdot 12 - 4 \cdot 4 = 24 - 16 = 8$

SB je minimum

4. $f(x,y) = 5 + 6x - 4x^2 - 3y^2$

$\frac{d}{dx} = 6 - 8x$ $\quad x = \frac{6}{8} = \frac{3}{4}$ $\quad \frac{d^2}{d^2x} = -8$ $\quad \frac{d^2}{dx\,dy} = 0$

$\frac{d}{dy} = -6y$ $\quad y = 0$ $\quad \frac{d^2}{d^2y} = -6$ $\quad \frac{d^2}{dy\,dx} = 0$

SB $[\frac{3}{4}, 0]$

$H = \begin{pmatrix} -8 & 0 \\ 0 & -6 \end{pmatrix} = -8 \cdot -6 = 48$

SB je maximum

5. $f(x,y) = 8x^3 + y^3 - 6xy + 4$

$\frac{d}{dx} = 24x^2 - 6y$ $\quad 24x^2 - 6y = 0$

$\frac{d}{dy} = 3y - 6x$ $\quad 3y - 6x = 0 \Leftrightarrow y = 2x$

$24x^2 - 6 \cdot 2x = 0$

$24x^2 - 12x = 0$

$A = b^2 - 4ac$
$= 12^2 - 0$
$= 144$

$x_{1,2} = \frac{-b \pm \sqrt{A}}{2a} = \frac{6 \pm 12}{48} =$

$= \frac{18}{48}$

$= -\frac{6}{48}$ 2le

6. $f(x,y) = x(x-6) + y(y-9) + xy$

$= x^2 - 6x + y^2 - 9y + xy$

$\frac{d}{dx} = 2x - 6 + y$ $\quad 2x - 6 + y = 0 \Leftrightarrow y = +6 - 2x$

$\frac{d}{dy} = 2y - 9 + x$ $\quad 2y - 9 + x = 0$

$2(6 - 2x) - 9 + x = 0$

$12 - 4x - 9 + x = 0$

$-3x = -3$

$x = 1$

$y = 6 - 2$
$= 4$

SB $[1, 4]$

$\frac{d^2}{d^2x} = 2$ $\quad \frac{d^2}{dx\,dy} = 1$

$\frac{d^2}{d^2y} = 2$ $\quad \frac{d^2}{dy\,dx} = 1$

$H = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = 2 \cdot 2 - 1 \cdot 1 = 3$

SB je minimum