03/C

1. $xy^2 - e^x + \cos y$

$\frac{\partial}{\partial x} f(xy) = y^2 - e^x$ $\frac{\partial}{\partial y} f(xy) = 2xy - \sin y$

2. $x^2 \cos y^2$

$\frac{\partial}{\partial x} f(xy) = 2x \cos y^2$ $\frac{\partial}{\partial y} (xy) = -2x^2 \sin y^2$

3. $\ln \frac{x}{y}$

$\frac{\partial}{\partial x} f(xy) = \frac{1}{x}$ $\frac{\partial}{\partial y} f(xy) = -\frac{1}{y}$

4. $y^2 e^{xy}$

$\frac{\partial}{\partial x} (f(xy) = y^3 e^{xy}$ $\frac{\partial}{\partial y} f(xy) = y e^{xy} (2y + e^{xy})$

5. $\frac{y-2}{x+1} = (y-2)(x+1)^{-1}$

$\frac{\partial}{\partial x} f(xy) = \frac{2-y}{(x+1)^2}$ $\frac{\partial}{\partial y} f(xy) = \frac{1}{x+1}$

6. $\sin(x^2 + y^2)$

$\frac{\partial}{\partial x} f(xy) = \cos(2x)(x^2+y^2)$ $\frac{\partial}{\partial y} f(xy) = \cos(2y)(x^2+y^2)$

7. $x^2 \ln y^2$

$\frac{\partial}{\partial x} f(xy) = 2x \ln y^2$ $\frac{\partial}{\partial y} f(xy) = \frac{2x^2}{y} = 2x^2 (y)^{-1}$

3.2

1. $xy^2 - e^x + \cos y$

$\frac{\partial^2}{\partial x^2} f(xy) \Rightarrow \frac{\partial}{\partial x}(y^2 - e^x) = -e^x$ $\quad \frac{\partial^2}{y^2} f(xy) \Rightarrow \frac{\partial}{\partial y}(2xy - \sin y) = 2x - \cos y$

$\frac{\partial^2}{\partial x \partial y} f(xy) \Rightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x} f(xy)\right) = \frac{\partial}{\partial y}(y^2 - e^x) = 2y$

$\frac{\partial^2}{\partial x \partial y} f(xy) \Rightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y} f(xy)\right) = \frac{\partial}{\partial x}(2xy - \sin y) = 2y$

2. $x^2 \cos y^2$

$\frac{\partial^2}{\partial x^2} f(xy) \Rightarrow \frac{\partial}{\partial x}(2x \cos y^2) = 2 \cos y^2$ $\quad \frac{\partial^2}{\partial y^2} f(xy) = \frac{\partial}{\partial y}(-2x^2 y \sin y^2) = \cancel{2x\cos y}$

$= -2x^2(\sin y^2 + 2y^2 \cos y^2)$

$\frac{\partial^2}{\partial x \partial y} f(xy) \Rightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x} f(xy)\right) = \frac{\partial}{\partial y}(2x \cos y^2) = -4xy \sin y^2$

$\frac{\partial}{\partial x}\left(\frac{\partial}{y} f(xy)\right) = \frac{\partial}{\partial x}(-2x^2 y \sin y^2) = -4x y \sin y^2$

3. $\ln \frac{x}{y}$

$\frac{\partial^2}{\partial x^2} f(xy) = \frac{\partial}{\partial x}\left(\frac{1}{x}\right) = \frac{\partial}{\partial x}(x^{-1}) = -(x)^{-2}$

$\frac{\partial^2}{\partial y^2} f(xy) = \frac{\partial}{y}\left(-\frac{1}{y}\right) = \frac{\partial}{\partial y}(-y^{-1}) = y^{-2}$

$\frac{\partial^2}{\partial x \partial y} f(xy) = \frac{\partial}{\partial y}(x^{-1}) = 0$ $\quad \frac{\partial}{\partial x}(-y^{-1}) = 0$

IE | EXTREM

4. $y^2 e^{xy}$

$$\frac{\partial^2}{\partial x^2} f(xy) = \frac{\partial}{\partial x}\left(y^3 e^{xy}\right) = y^4 e^{x}_{y}$$

$$\frac{\partial^2}{\partial y^2} f(xy) = \frac{\partial}{\partial y}\left(y e^{xy}\left(2y + e^{xy}\right)\right) = e^{xy}\left(2 + e^{xy}\right) ?$$

$$\frac{\partial^2}{\partial x \partial y} f(xy) = \frac{\partial}{\partial y}\left(y^4 e^{x}_{y}\right) = y^2 e^{xy}(xy+3) ?$$

5. $\frac{y-2}{x+1}$

$$\frac{\partial^2}{\partial x^2} f(xy) = \frac{\partial}{\partial x}\left(\left(\frac{2-y}{(x+1)^2}\right)\right) = \frac{2(y-2)}{(x+1)^3}$$

$$\frac{\partial^2}{\partial y^2} f(xy) = \frac{\partial}{\partial y}\left(\frac{1}{x+1}\right) = 0$$

$$\frac{\partial^2}{\partial x \partial y} f(xy) = \frac{\partial}{\partial x}\left(\frac{1}{x+1}\right) = \cancel{\frac{\partial}{\partial x}\left(1(x+1)^{-1}\right)} = -\frac{1}{(x+1)^2}$$

$$\frac{\partial}{\partial y}\left(\frac{2-y}{(x+1)^2}\right) = \left(\frac{2-1}{(x+1)^2} = \right)\frac{1}{(x+1)^2}$$

6. $\sin(x^2+y^2)$

$$\frac{\partial^2}{\partial x^2} f(xy) = \frac{\partial}{\partial x}\left(2x\cos(x^2+y^2)\right) = 2\cos(x^2+y^2) -$$

$$- 4x^2 \sin(x^2+y^2)$$

$$\frac{\partial^2}{\partial y^2} f(xy) = \frac{\partial}{\partial y}\left(2y\cos(x^2+y^2)\right) = 2\cos(x^2+y^2) -$$

$$- 4y^2 \sin(x^2+y^2)$$

$$\frac{\partial^2}{\partial x \partial y} f(xy) = \frac{\partial}{\partial x}\left(2y\cos(x^2+y^2)\right) = -4xy\sin(x^2+y^2)$$

$$\frac{\partial}{\partial y}\left(2x\cos(x^2+y^2)\right) = -4xy\sin(x^2+y^2)$$

7. $x^2 \ln y^2$

$$\frac{\partial^2}{\partial x^2} f(xy) = \frac{\partial}{\partial x}\left(2x\ln y^2\right) = 2\ln y^2$$

$$\frac{\partial^2}{\partial y^2} f(xy) = \frac{\partial}{\partial y}\left(\frac{2x^2}{y}\right) = -\frac{2x^2}{y^2}$$

$$\frac{\partial^2}{\partial x \partial y} f(xy) = \frac{\partial}{\partial x}\left(\frac{2x^2}{y}\right) = \frac{4x}{y}$$

$$\frac{\partial}{\partial y}\left(2x\ln y^2\right) = \frac{4x}{y}$$

3.3.

1. $f(x,y) = x^2 + y^2 - xy - 2x + y$

STACIONÁRNÍ BODY:

a) $\frac{\partial}{\partial x} f(x,y) = 2x + 0 - 1y - 2 + 0 = 2x - y - 2$

b) $\frac{\partial}{\partial y} f(xy) = 0 + 2y - 1\cancel{x} + 0 + 1 = 2y - \cancel{x} + 1$

a) $\cancel{y = -2x + 2}$

$\cancel{0 = -2x + 2 - y}$

$y = +2x - 2$

$y = +2 - 2$

$y = 0$

$y = +2x - 2$

$\cancel{2y = -x + 1}$

$0 = 2y - x + 1$

$0 = 2(+2x - 2) - x + 1$

$0 = +4x - 4 - x + 1$

$0 = +3x - 3 \quad / + 3$

$+3 = -3x \quad / :(3)$

$1 = x$

$[1, 0]$

stacionární bod

$\frac{\partial^2}{\partial x^2} = \frac{\partial}{\partial x}(2x - y - 2) =$

$= 2$

$\frac{\partial^2}{\partial y^2} = \frac{\partial}{\partial y}(2y - x + 1) = +2$

$\frac{\partial}{\partial x \partial y} = \frac{\partial}{\partial x}(2y - x + 1) = -1$

$\frac{\partial}{\partial y}(2x - y - 2) = -1$

Hessova matice

Hessián

$$H \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \Rightarrow |H| = (2 \cdot 2) - (1 \cdot 1) = 4 - 1 = \underline{\underline{3}} > 0$$

$3 > 0 \Rightarrow$ extrém $[1;0]$

minimum $\rightarrow$ konvexní

minimum $[1;0]$

2. $f(xy) = y^2x + 3xy - 6y$

a) $\frac{\partial}{\partial x} f(xy) = y^2 + 3y + 0 = y^2 + 3y = y(y+3)$

$\frac{\partial}{\partial y} f(xy) = 2yx + 3x - 6$

$3x = 6 - 2yx$

a) $y^2 + 3y = 0$

$D = b^2 - 4ac$

$D = 9 - 0$

$D = 9$

$x_{1,2} \frac{-b \pm \sqrt{D}}{2(1)}$

$x_{1,2} = \frac{-3 \pm \sqrt{2}}{6}$ $\Rightarrow$ $x_{1,2} \frac{-3 \pm \sqrt{9}}{2} = \frac{-3+3}{2} // \frac{0}{2} = \underline{\underline{0}}$

$\frac{-3-3}{2} // \frac{-6}{2} = \underline{\underline{-3}}$

$2yx + 3x - 6 = 0$

~~$2((y(y+3)x + 3x - 6 = 0$~~

? ~~Nejde spočítat x ani y.~~

~~$2xy + 3x - 6 = 0$~~ ?

~~$y(y+3) = 0$~~

x y

[0; -3] = stacionární body

$\frac{\partial^2}{x^2} = \frac{\partial}{\partial x}(y^2+3y) = 0$

$\frac{\partial^2}{\partial y^2} = \frac{\partial}{\partial y}(2yx + 3x - 6) = \underline{\underline{2x}}$

$\frac{\partial 2}{\partial x \partial y} = \frac{\partial}{\partial x}(2yx + 3x - 6) = \underline{\underline{2y + 3}}$

$\frac{\partial}{\partial y}(\cancel{y(y+3)} = 1(1\ (y^2 + 3y) = \underline{\underline{2y + 3}}$

$\begin{pmatrix} 0 & \overbrace{2y+3}^{2(-3)+3} \\ \underbrace{2y+3}_{2(-3)+3} & \underbrace{2x}_{0} \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & -3 \\ -3 & 0 \end{pmatrix} = 0 - (-3 \cdot (-3) = 0 - 9 = -9$

< 0

Maximum

konkávní

$\underline{\underline{M\ [0, -3]}}$

5. $f(x,y) = x^4 + y^4 - x^2 - 2xy - y^2$

$$\frac{\partial}{\partial x} f(x,y) = 4x^3 + 0 - 2x - 2y - 0 = 4x^3 - 2x - 2y = 0$$

$$\frac{\partial}{\partial y} f(x,y) = 0 + 4y^3 - 0 - 2x - 2y = 4y^3 - 2x - 2y = 0$$

$$4x^3 - 2x - 2y = 0 \rightarrow -2y = 2x - 4x^3$$

$$4y^3 - 2x - 2y = 0 \qquad y = \frac{2x - 4x^3}{-2}$$

$$4\left(-\frac{2x - 4x^3}{2}\right)^3 - 2x - 2\left(-\frac{2x - 4x^3}{2}\right) = 0$$

$$4\left(-\frac{8x^3 - 64x^9}{8}\right) - 2x + 2x + 4x^3 = 0 \quad /\cdot 2$$

$$-\cancel{8x^3} - 64x^9 - \cancel{4x} + \cancel{4x} + \cancel{8x^3} = 0$$

$$-64x^9 = 0$$

$$x = 0$$

$$\rightarrow y = \frac{2\cdot 0 - 4\cdot 0}{-2} = 0$$

[0,0] STACIONÁRNÍ BODY

$$\frac{\partial^2}{\partial x^2} = \frac{\partial}{\partial x}(4x^3 - 2x - 2y) = 12x^2 - 2$$

$$\frac{\partial^2}{\partial x \partial y} = \frac{\partial}{\partial y}(4x^3 - 2x - 2y) = -2$$

$$\frac{\partial^2}{\partial y^2} = \frac{\partial}{\partial y}(4y^3 - 2x - 2y) = 12y^2 - 2$$

$$\frac{\partial^2}{\partial y \partial x} = \frac{\partial}{\partial x}(4y^3 - 2x - 2y) = -2$$

$$\begin{pmatrix} 12x^2 - 2 & -2 \\ -2 & 12y^2 - 2 \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}$$

$$\rightarrow |H| = 4 - 4 = 0$$

nevíme jestli je extrém

3. $f(x,y) = 4(x-y) - x^2 - y^2 = 4x - 4y - x^2 - y^2$

$\frac{\partial}{\partial x} f(x,y) = 4 - 2x$

$\frac{\partial}{\partial y} f(x,y) = -4 - 2y$

$4 - 2x = 0 \rightarrow -2x = -4$

$x = 2$

$-4 - 2y = 0$

$\rightarrow -2y = 4$

$y = -2$

[2,-2] STACIONÁRNÍ BODY

$\frac{\partial^2}{\partial x^2} = \frac{\partial}{\partial x}(4-2x) = -2$

$\frac{\partial^2}{\partial x \partial y} = \frac{\partial}{\partial y}(4-2x) = 0$

$\frac{\partial^2}{\partial y^2} = \frac{\partial}{\partial y}(-4-2y) = -2$

$\frac{\partial^2}{\partial y \partial x} = \frac{\partial}{\partial x}(-4-2y) = 0$

$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \rightarrow |H| = 4 - 0 = 4 > 0$

[2,-2] je extrém

a je maximum $(-2 < 0)$

4. $f(x,y) = (2x^2-3)(y+1) = 2x^2y + 2x^2 - 3y - 3$

$$\frac{\partial}{\partial x} f(x,y) = 4xy + 4x - 0 - 0 = 4xy + 4x$$

$$\frac{\partial}{\partial y} f(x,y) = 2x^2 + 0 - 3 - 0 = 2x^2 - 3$$

$$4xy + 4x = 0$$
$$2x^2 - 3 = 0$$

$$\rightarrow x^2 = \frac{3}{2}$$

$$x = \sqrt{\frac{3}{2}}$$

$$4y\sqrt{\frac{3}{2}} + 4\sqrt{\frac{3}{2}} = 0$$

$$y = -1$$

$\left[\sqrt{\frac{3}{2}}, -1\right]$ STACIONÁRNÍ BODY

$$\frac{\partial^2}{\partial x^2} = \frac{\partial}{\partial x}(4xy + 4x) = 4y + 4$$

$$\frac{\partial^2}{\partial x \partial y} = \frac{\partial}{\partial y}(4xy + 4x) = 4x + 0$$

$$\frac{\partial^2}{\partial y^2} = \frac{\partial}{\partial y}(2x^2 - 3) = 0$$

$$\frac{\partial^2}{\partial y \partial x} = \frac{\partial}{\partial x}(2x^2 - 3) = 4x + 0$$

$$\begin{pmatrix} 4y+4 & 4x \\ 4x & 0 \end{pmatrix} = \begin{pmatrix} 4(-1)+4 & 4\cdot\sqrt{\frac{3}{2}} \\ 4\cdot\sqrt{\frac{3}{2}} & 0 \end{pmatrix} = \begin{pmatrix} 0 & 4\sqrt{\frac{3}{2}} \\ 4\sqrt{\frac{3}{2}} & 0 \end{pmatrix} \rightarrow$$

$$\rightarrow |H| = 0 - 4\sqrt{\frac{3}{2}} \cdot 4\sqrt{\frac{3}{2}} = 0 - 24 = -24 < 0$$

$\left[\sqrt{\frac{3}{2}}, -1\right]$ není extrém

6. $f(x,y) = 2x^2 - 6xy + 5y^2 - x + 3y + 2$

$$\frac{\partial}{\partial x} f(x,y) = 4x - 6y + 0 - 1 + 0 + 0 = 4x - 6y - 1 = 0 \quad \rightarrow \quad x = \frac{6y+1}{4}$$

$$\frac{\partial}{\partial y} f(x,y) = 0 - 6x + 10y - 0 + 3 + 0 = \underline{-6x + 10y + 3 = 0}$$

$$\overset{3}{\cancel{-6}}\left(\frac{6y+1}{\cancel{4}_2}\right) + 10y + 3 = 0$$

$$-3\frac{(6y+1)}{2} + 10y + 3 = 0 \,/\, 2$$

$$-18y - 3 + 20y + 6 = 0$$

$$2y = -3$$

$$y = -\frac{3}{2}$$

$$-6x + \overset{5}{\cancel{10}} \cdot \left(-\frac{3}{\cancel{2}_1}\right) + 3 = 0$$

$$-6x - 15 + 3 = 0$$

$$-6x = 12$$

$$x = -2$$

$\left[-2, -\frac{3}{2}\right]$ STACIONÁRNÍ BODY

$$\frac{\partial^2}{\partial x^2} = \frac{\partial}{\partial x}(4x - 6y - 1) = 4 - 0 - 0 = 4$$

$$\frac{\partial^2}{\partial x \partial y} = \frac{\partial}{\partial y}(4x - 6y - 1) = 0 - 6 - 0 = -6$$

$$\frac{\partial^2}{\partial y^2} = \frac{\partial}{\partial y}(-6x + 10y + 3) = 0 + 10 + 0 = 10$$

$$\frac{\partial^2}{\partial y \partial x} = \frac{\partial}{\partial x}(-6x + 10y + 3) = -6 + 0 + 0 = -6$$

$$\begin{pmatrix} 4 & -6 \\ -6 & 10 \end{pmatrix} \rightarrow |H| = 40 - 36 = 4 > 0$$

$\left[-2, -\frac{3}{2}\right]$ je extrém a je to minimum