C. SKUPINA

3.1

1. $xy^2 - e^x + \cos y$

$\frac{d}{dx} = 1y^2 - e^x + 0 = y^2 - e^x$

$\frac{d}{dy} = x.2y - 0 - \sin y = 2xy - \sin y$

2. $x^2 \cos y^2$

$\frac{d}{dx} = 2x \cos y^2$

$\frac{d}{dy} = x^2.(-\sin y^2) = -x^2 \sin y^2$

3. $\ln \frac{x}{y}$ ..... $\ln x . \frac{1}{y}$ $\ln x . \ln y^{-1}$

$\frac{d}{dx} = \frac{1}{x}$

$\frac{d}{dy} = -\frac{1}{y}$

4. $y^2 e^{x}$

$\frac{d}{dx} = y^2 e^{xy} . y = y^3 e^{xy}$

$\frac{d}{dy} = ???$

5. $\frac{y-2}{x+1} \dots (y-2)\cdot\frac{1}{(x+1)} \dots (y-2)^{1}\cdot(x+1)^{-1}$

$\frac{d}{dx} = (y-2)\cdot(-1)\cdot(x+1)^{-2} = \frac{y-2}{(x+1)^2}$

$\frac{d}{dy} = \frac{1}{x+1}$

6. $\sin(x^2+y^2)$

$\frac{d}{dx} = 2x\cos(x^2+y^2)$

$\frac{d}{dy} = 2y(\cos(x^2+y^2))$

2. $x^2 \ln y^2$

$\frac{d}{dx} = 2x \ln y^2$

$\frac{d}{dy} = x^2 \cdot 2\frac{1}{y} = x^2 \cdot \frac{2}{y}$

3.2 1. $xy^2 - e^x + \cos y$

$\frac{d}{dx} = y^2 - e^x \qquad \frac{d^2}{dx^2} = -e^x$

$\frac{d}{dy} = 2xy - \cos(y) \qquad \frac{d^2}{dy^2} - 2x - \sin(y)$

$\frac{d^2}{dxdy} = 2y$

$\frac{d}{dydx} = 2y$

2. $x^2\cos y^2$

$\frac{d}{dx} = 2x\cos(y^2) \qquad \frac{d^2}{dx^2} - 2\cos(y^2) \qquad \frac{d^2}{dxy} = -4xy\sin(y^2)$

$\frac{d}{dy} = -2x^2y \quad \sin(y^2) \quad \frac{d^2}{dy^2} = -2x^2(\sin(y^2) + 2y\cos(y^2))$

$\frac{d^2}{dyx} = -4xy\sin(y^2)$

3. $\ln\frac{x}{y}$

$\frac{d}{dx} = \frac{1}{x} \qquad \frac{d^2}{dxx} = -\frac{1}{x^2} \qquad \frac{d^2}{dxy} = 0$

$\frac{d}{dy} = -\frac{1}{y} \qquad \frac{d^2}{dy^2} = \frac{1}{y^2} \qquad \frac{d^2}{dyx} = 0$

4) $y^2e^{xy} = dx$

$dx = y^2\cdot e^{xy}\cdot y = y^3e^{xy}$

$dxx = y^3\cdot e^x\cdot y = y^4\cdot e^{xy}$

$dxy = y^3\cdot e^{xy} = 3y^2\cdot e^{xy} + y^3\cdot e^{xy}\cdot x = e^x(3y^2 + y^3x) = y^2\cdot e^{xy}(3+yx)$

$dy = 2y\cdot e^{xy} + y^2e^{xy}\cdot x$

$dyy = 2\cdot e^{xy} + 2y\cdot e^{xy}\cdot x + 2ye^{xy}\cdot x + y^2\cdot e^{xy}\cdot x\cdot x =$

$2e^{xy} + 2ye^{xy}x + 2e^{xy}xy + y^2x^2e^{xy} = 2e^{xy} + 4ye^{xy}x$

$+ y^2x^2e^{xy} = e^{xy}(2 + 4yx + y^2x^2)$

5. $\frac{y-2}{x+1}$

$dx = -\frac{y-2}{(x+1)^2}$

$dxx = \frac{y-2}{(x+1)^4}\cdot 2(x+1) = \frac{2(y-2)}{(x+1)^3}$

$dxy = -\frac{y-2}{(x+1)^2} = -\frac{1}{(x+1)^2}\cdot(y-2)' = \frac{1}{(x+1)^2}\cdot 1$

$dy = \frac{y-2}{x+1} = (y-2)\cdot\left(\frac{1}{x+1}\right) = 1\cdot\frac{1}{(x+1)}$

$dyy = 0$

6. $\sin(x^2+y^2)$

$dx = \cos(x^2+y^2)\cdot 2x$

$dxx = -\sin(x^2+y^2)\cdot 2x \qquad 2x + \cos(x^2+y^2)\cdot 2 = 2\cos(x^2+y^2) - 4x^2\sin(x^2+y^2)$

$dxy = \cos(x^2+y^2)\cdot 2x = 2x(-\sin(x^2+y^2))\cdot 2y = -4xy\sin(x^2+y^2)$

$dy = \cos(x^2+y^2)\cdot 2y$

$dyy = -\sin(x^2+y^2)\cdot 2y\cdot 2y + \cos(x^2+y^2)\cdot 2 = 2\cos(x^2+y^2) - 4y^2\cdot\sin(x^2+y^2)$

7. $x^2\ln y^2$

$\frac{d}{dx} = 2x\ln y^2 \qquad \frac{d}{dy} = x^2\cdot\frac{1}{y^2}\cdot 2y = \frac{2x^2}{y} \qquad \frac{d^2}{dy^2} = -\frac{2x^2}{y^2}$

$\frac{d^2}{dx^2} = 2\ln y^2 \qquad \frac{d^2}{dxy} = 2x\cdot\frac{1}{y^2}\cdot 2y = 4x\cdot\frac{y}{y^2} = \frac{4x}{y}$

3.3 1. $f(x,y) = x^2 + y^2 - xy - 2x + y$

$\frac{d}{dx} = 2x + 0 - y - 2 + 0 \Rightarrow 2x - y - 2 \Rightarrow y = 2x - 2$

$y \nearrow$

$\frac{\partial}{\partial y} = 0 + 2y - x - 0 + 1 \Rightarrow 2y - x + 1 \Rightarrow x = 2y + 1$

$x = 1$

$\frac{\partial^2}{\partial x^2} = 2$ $\quad \frac{\partial^2}{\partial xy} = -1$

$\frac{\partial^2}{\partial y^2} = 2$ $\quad \frac{\partial^2}{\partial yx} = -1$

$H \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} = 4 - 1 = \underline{3}$ $\rightarrow$ EXTRÉM

$\frac{\partial}{\partial x^2} > 0 \Rightarrow$ MINIMUM

3) $f(x,y) = 4(x-y) - x^2 - y^2 = 4x - 4y - x^2 - y^2$

$\frac{\partial}{\partial x} = 4x - 2x = 0 \quad 2x = 4 \quad x = 2$

$\frac{\partial}{\partial y} = -4 - 2y = 0 \quad 2y = -4 \quad y = -2$

$M[2; -2]$

2) $f(x,y) = y^2 x + 3xy - 6y$

$\frac{\partial}{\partial x} = y^2 + 3y$

$\frac{\partial}{\partial y} = 2yx + 3x - 6$ $\quad [x, y] = [-2, -3]$

$\frac{\partial^2}{\partial x^2} = 0$ $\quad \frac{\partial^2}{\partial x \partial y} = 2y + 3$ $\quad \begin{pmatrix} 0 & 2y+3 \\ 2y+3 & 2x \end{pmatrix}$

$\frac{\partial^2}{\partial y^2} = 2x$ $\quad \frac{\partial^2}{\partial y \partial x} = 2y + 3$

$\begin{pmatrix} 0 & -3 \\ -3 & -4 \end{pmatrix} = 0 - 9 = -9 \Rightarrow$ NEMÁ EXTRÉM

$S[-2; -3]$?

$S[2, 0]$

4. $f(x,y) = (2x^2 - 3)(y + 1)$

$= 2x^2 y + 2x^2 - 3y - 3$

$\frac{\partial}{\partial x} = 4xy + 4x - 0 - 0 = 4x(y+1)$ $\quad 4x(y+1) = 0$

$y = -1$

$\frac{\partial}{\partial y} = 2x^2 + 0 - 3 - 0 = 2x^2 - 3$ $\quad 2x^2 - 3 = 0$

$2x^2 = 3$

$[x, y] = [\sqrt{\frac{3}{2}}; -1]$ $\quad x = \sqrt{\frac{3}{2}}$

$\frac{\partial^2}{\partial x^2} = 4$ $\quad \frac{\partial^2}{\partial y^2} = 0$ $\quad \frac{\partial}{\partial x \partial y} = 4x$ $\quad \frac{\partial^2}{\partial y \partial x} = 4x$

$\begin{pmatrix} 4 & 4x \\ 4x & 0 \end{pmatrix} = 0 - 16x^2 = -16 \cdot \sqrt{\frac{3}{2}} < 0$

$\hookrightarrow$ NEMÁ EXTRÉM

$S[\sqrt{\frac{3}{2}}; -1]$

5. $f(x,y) = x^4 + y^4 - x^2 - 2xy - y^2$

$\frac{\partial^2}{\partial x^2} = 4x - 2x - 2 = 0$

$2x = 2$

$x = 1$

$\frac{\partial^2}{\partial y^2} = 4y - 2 - 2y = 0$ $\quad m[1; 1]$

$2y = 2$

$y = 1$

$\frac{\partial^2}{\partial xy} = 0 + 0 + 0 - 1 - 0 = -1$

$\frac{\partial^2}{\partial y \partial x} = -1$

$H \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = 1 \cdot 1 - (-1)(-1) = 0$

6. $2x^2 - 6xy + 5y^2 - x + 3y + 2$

$\frac{\partial}{\partial x} = 4x - 6y + 0 - 1 + 0 + 0 - \cancel{4x - 6y - 1}\ 4x - 6y - 1$

$\frac{\partial}{\partial y} = 0 - 6x + 10y - 0 + 3 + 0 = 6x + 10y + 3$

$4x = 6y + 1$ $\qquad -\frac{6}{1}\left(\frac{6y+1}{4}\right) + 10y + 3 = 0$

$x = \frac{6y+1}{4}$ $\qquad y = -\frac{3}{2}$

$x = -2$ $\qquad [x; y]\ [-2; -\frac{3}{2}]$

$\frac{\partial^2}{\partial x^2} = 4$

EXTREM = MINIMUM

$\frac{\partial^2}{\partial y^2} = 10$ $\qquad \begin{pmatrix} 4 & -6 \\ -6 & 10 \end{pmatrix} - 4 > 0$

$\frac{\partial^2}{\partial xy} = -6$

$\frac{\partial}{\partial xy} = -6$

7. $(x, y) = xy(4 - x - y) = 4xy - x^2y - xy^2$

$\frac{\partial}{\partial x} = 4y - 2xy - y^2$ $\qquad 4y - 2xy - y^2 = 0$

$4x - x^2 - 2xy = 0$

$\frac{\partial}{\partial y} = 4x - x^2 - 2xy$