D

3.1 Parciální derivace prvního řádu

1. $yx^2 + e^y - \sin x$

$$\frac{\partial}{\partial x} f(x;y) = 0 \cdot x^2 + y \cdot 2x + 0 - \cos x = \underline{\underline{2xy - \cos x}}$$

$$\frac{\partial}{\partial y} f(x;y) = 1 \cdot x^2 + y \cdot 0 + e^y - 0 = \underline{\underline{x^2 + e^y}}$$

$$\cancel{\frac{\partial}{\partial x} f(x;y) = 1 \cdot \ln xy + x \cdot \overbrace{\frac{1}{xy} \cdot y}^{1} + \cancel{x \ln xy \cdot \overset{0}{0}} = \ln xy + 1}$$

$$\cancel{\frac{\partial}{\partial y} f(x;y) = \overbrace{0 \cdot \ln xy}^{0} \cdot y + x \cdot \overbrace{\frac{1}{xy} \cdot y}^{1} + x \ln xy \cdot 1 =}$$

$$\cancel{\frac{\partial}{\partial y} f(x;y) = 0 \cdot \ln xy \cdot y + x \cdot 0 \cdot y + x \ln xy \cdot 1 =}$$

$$\cancel{\frac{\partial}{\partial y} f(x;y) = \overbrace{0 \cdot \ln(xy)}^{0} + x \cdot \frac{1}{xy} \cdot \overbrace{(0 \cdot y + x \cdot 1)}^{x} = \frac{x}{y}}$$

2. $x \ln xy$

$$\frac{\partial}{\partial x} f(x;y) = 1 \cdot \ln(xy) + \cancel{x} \cdot \frac{1}{\cancel{xy}} \cdot \cancel{y} = \underline{\underline{\ln xy + 1}}$$

$$\frac{\partial}{\partial y} f(x;y) = \overbrace{0 \cdot \ln(xy)}^{0} + \cancel{x} \cdot \frac{1}{\cancel{x}y} \cdot x = \frac{x}{y}$$

3. $\frac{e^{2x}}{y} = e^{2x} \cdot y^{-1}$

$$\frac{\partial}{\partial x} f(x;y) = \frac{2 \cdot e^{2x} \cdot y - \overbrace{e^{2x} \cdot 0}^{0}}{y^2} = \frac{2e^{2x}}{y}$$

$$\frac{\partial}{\partial y} f(x;y) = \frac{\overbrace{0 \cdot y}^{0} - e^{2x} \cdot 1}{y^2} = -\frac{e^{2x}}{y^2}$$

4. $x \sin(x+y)$

$$\frac{\partial}{\partial x} f(x;y) = 1 \cdot \sin(x+y) + x \cdot \cos(x+y) \cdot (1+0) =$$
$$= \sin(x+y) + x\cos(x+y)$$

$$\frac{\partial}{\partial y} f(x;y) = \overbrace{0 \cdot \sin(x+y)}^{0} + x \cdot \cos(x+y) \cdot (0+1) =$$
$$= x\cos(x+y)$$

5. $\ln(x-y)$

$$\frac{\partial}{\partial x} f(x;y) = \frac{1}{x-y} \cdot (1-0) = \frac{1}{x-y}$$

$$\frac{\partial}{\partial y} f(x;y) = \frac{1}{x-y} \cdot (0-1) = -\frac{1}{x-y}$$

6. $\sin x \cos y$

$$\frac{\partial}{\partial x} f(x;y) = \cos x \cdot \cos y + \overbrace{\sin x \cdot 0}^{0} = \cos x \cos y$$

$$\frac{\partial}{\partial y} f(x;y) = \underbrace{0 \cdot \cos y}_{0} + \sin x \cdot (-\sin y) = -\sin y \cdot \sin x \quad (= -\sin(x)\sin(y))$$

4. $\cos x^2 y$

$$\frac{\partial}{\partial x} f(x,y) = -\sin(x^2y)\cdot 2xy = -2xy \sin x^2 y$$

$$2x \cdot y + \underbrace{x^2 \cdot 0}_{0}$$

$$\frac{\partial}{\partial y} f(x,y) = -\sin(x^2y)\cdot x^2 = -x^2 \sin(x^2y)$$

3.2 Parciální derivace druhého řádu

1. $yx^2 + e^y - \sin x$

$$\frac{\partial^2}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} f(x,y)\right) = \frac{\partial}{\partial x}(2xy - \cos x) = 2y - (-\sin x) = \sin x + 2y$$

$$\frac{\partial^2}{\partial x \partial y} f(x,y) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y} f(x,y)\right) = \frac{\partial}{\partial y}\cdot(2xy - \underbrace{\cos x}_{0}) = 2x$$

$$\frac{\partial^2}{\partial y^2} f(x,y) = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y} f(x,y)\right) = \frac{\partial}{\partial y}(x^2 + e^y) = 0 + e^y = e^y$$

$$\frac{\partial^2}{\partial y \partial x} f(x,y) = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x} f(x,y)\right) = \frac{\partial}{\partial x}\cdot(x^2 + e^y) = 2x + 0 = 2x$$

2. $x \ln xy$

$$\frac{\partial^2}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}(\ln xy + 1) = \frac{1}{xy}\cdot y + 0 = \frac{1}{x}$$

$$\frac{\partial^2}{\partial x \partial y} f(x,y) = \frac{\partial}{\partial y}(\ln xy + 1) = \frac{1}{xy}\cdot x + 0 = \frac{1}{y}$$

$$\frac{\partial^2}{\partial y^2} f(x,y) = \frac{\partial}{\partial y}\left(\frac{x}{y}\right) = \frac{\overset{0}{0 \cdot y} - x \cdot 1}{y^2} = \frac{-x}{y^2}$$

$$\frac{\partial^2}{\partial y \partial x} f(x,y) = \partial x\left(\frac{x}{y}\right) = \frac{1 y - x \cdot 0}{y^2} = \frac{1}{y}$$

$$x \cdot y^{-1} = 1 \cdot y^{-1} + x \cdot 0 = y^{-1} = \frac{1}{y}$$

3. $\frac{e^{2x}}{y}$

$$\frac{\partial^2}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}\left(\frac{2e^{2x}}{y}\right) \overset{\frac{\partial}{\partial x}}{=} (2e^{2x}\cdot y^{-1}) = 4e^{2x}\cdot y^{-1} + \underbrace{2e^{2x}\cdot 0}_{0} = \frac{4e^{2x}}{y}$$

$$2\cdot e^{2x} = 0\cdot e^{2x} + 2\cdot e^{2x}\cdot 2 = 4e^{2x}$$

$$\frac{\partial^2}{\partial x\partial y} f(x,y) = \frac{\partial}{\partial y}\left(\frac{2e^{2x}}{y}\right) = \frac{\partial}{\partial y}(2e^{2x}\cdot y^{-1}) = \underbrace{0\cdot y^{-1}}_{0} + 2e^{2x}\cdot(-y^{-2}) = \frac{-2e^{2x}}{y^2}$$

$$\frac{\partial^2}{\partial y^2} f(x,y) = \frac{\partial}{\partial y}\left(-\frac{e^{2x}}{y^2}\right) \overset{\frac{\partial}{\partial y}}{=} (-1e^{2x}\cdot y^{-2}) = 0\cdot y^{-2} + (-e^{2x})\cdot(-2y^{-3}) = \frac{2e^{2x}}{y^3}$$

$$\frac{\partial^2}{\partial y\partial x} f(x,y) = \partial x\left(-\frac{e^{2x}}{y^2}\right) = -\frac{e^{2x}\cdot 2\cdot y^2 - (-e^{2x})\cdot 0}{y^4} = -\frac{2e^{2x}}{y^2}$$

4. $x\sin(x+y)$

$$\frac{\partial^2}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}(\sin(x+y) + x\cos(x+y)) = \cos(x+y)\cdot(1+0) + 1\cdot\cos(x+y) + x\cdot(-\sin(x+y))\cdot$$
$$\cdot(1+0) = \cos(x+y) + \cos(x+y) + x\cdot(-\sin(x+y)) = 2\cos(x+y) + x\cdot(-\sin(x+y))$$
$$(= 2\cos(x+y) - x\sin(x+y))$$

$$\frac{\partial^2}{\partial x\partial y} f(x,y) = \frac{\partial}{\partial y}(\sin(x+y) + x\cos(x+y)) = \cos(x+y)\cdot(0+1) + \underbrace{0\cdot\cos(x+y)}_{0} +$$
$$+ x\cdot(-\sin(x+y))\cdot(0+1) = \cos(x+y) + -x\sin(x+y)$$

$$\frac{\partial^2}{\partial y^2} f(x,y) = \frac{\partial}{\partial y}(x\cos(x+y)) = \underbrace{0\cdot\cos(x+y)}_{0} + x\cdot(-\sin(x+y))\cdot(0+1) = -x\sin(x+y)$$

$$\frac{\partial^2}{\partial y\partial x} f(x,y) = \frac{\partial}{\partial x}(x\cos(x+y)) = 1\cdot\cos(x+y) + x\cdot(-\sin(x+y))\cdot(1+0) =$$
$$= \cos(x+y) - x\sin(x+y)$$

5. $\ln(x-y)$

$$\frac{\partial^2}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}\left(\frac{1}{x-y}\right) = \frac{\overbrace{0\cdot(x-y)}^{0} - 1\cdot(1-0)}{(x-y)^2} = \frac{-1}{(x-y)^2}$$

$$\frac{\partial^2}{\partial x\partial y} f(x,y) = \frac{\partial}{\partial y}\left(\frac{1}{x-y}\right) = \frac{\overbrace{0\cdot(x-y)}^{0} - 1\cdot(0-1)}{(x-y)^2} = \frac{1}{(x-y)^2}$$

$$\frac{\partial^2}{\partial y^2} f(x,y) = \frac{\partial}{\partial y}\left(-\frac{1}{x-y}\right) = \frac{\overbrace{0\cdot(x-y)}^{0} - (-1)\cdot(0-1)}{(x-y)^2} = \frac{-1}{(x-y)^2}$$

6. $\sin x \cos y$

$$\frac{\partial^2}{\partial x^2} f(x;y) = \frac{\partial}{\partial x}(\cos x \cos y) = -\sin x \cdot \cos y + \underbrace{\cos x \cdot 0}_{0} = \underline{\underline{-\sin x \cdot \cos y}}$$

$$\frac{\partial^2}{\partial x \partial y} f(x;y) = \frac{\partial}{\partial y}(\cos x \cos y) = 0 \cdot \cos y + \cos x \cdot (-\sin y) = \underline{\underline{-\sin y \cdot \cos x}}$$

$$\frac{\partial^2}{\partial y^2} f(x;y) = \frac{\partial}{\partial y}(-\sin y \cdot \sin x) = -\cos y \cdot \sin x + (-\sin y) \cdot 0 =$$

$$= \underline{\underline{-\cos y \sin x}}$$

$$\frac{\partial^2}{\partial x \partial y} f(x;y) = \frac{\partial}{\partial x}(-\sin y \cdot \sin x) = 0 \cdot \sin x + (-\sin y) \cdot \cos x =$$

$$= \underline{\underline{-\sin y \cos x}}$$

7. $\cos x^2 y$ - ve výsledcích chyba

$$\frac{\partial^2}{\partial x^2} f(x;y) = \frac{\partial}{\partial x}(-2xy \sin x^2 y) = -2y \cdot \sin x^2 y + (-2xy) \cdot \cos(x^2 y)(2xy) =$$

$$= -2y \sin(x^2 y) - 4x^2 y^2 \cos x^2 y =$$

$$= \underline{\underline{-2y\,(\sin(x^2 y) + 2x^2 y \cos(x^2 y))}}$$

$$\frac{\partial^2}{\partial x \partial y} f(x;y) = \frac{\partial}{\partial y}(-2xy \sin x^2 y) = -2x \cdot \sin(x^2 y) + (-2xy) \cdot \cos x^2 y \cdot x^2 =$$

$$= \underline{\underline{-2x\,(\sin(x^2 y) + x^2 y \cos(x^2 y))}}$$

$$\frac{\partial^2}{\partial y^2} f(x;y) = \frac{\partial}{\partial y}(-x^2 \sin(x^2 y)) = \overbrace{0 \cdot \sin(x^2 y)}^{0} + (-x^2) \cdot \cos(x^2 y) \cdot x^2 =$$

$$= \underline{\underline{-x^4 \cdot \cos(x^2 y)}}$$

$$\frac{\partial^2}{\partial y \partial x} f(x;y) = \frac{\partial}{\partial x}(-x^2 \sin(x^2 y)) = -2x \cdot \sin(x^2 y) + (-x^2) \cdot \cos(x^2 y) \cdot 2xy =$$

$$= \underline{\underline{-2x\,(\sin(x^2 y) + x^2 y \cos(x^2 y))}}$$

D 3.3.1

$f(x,y) = x^3 - 3xy - y^3$

$f'_x(x,y) = \frac{\partial}{\partial x}(x^3 - 3xy - y^3) = 3x^2 - 3y$ $\qquad$ $f'_y(x,y) = \frac{\partial}{\partial y}(x^3 - 3xy - y^3) = -3x - 3y^2$

$$\left.\begin{aligned} f'_x = 0 &= 3x^2 - 3y \to 3y = 3x^2,\ y = x^2 \\ f'_y = 0 &= -3x - 3y^2 \end{aligned}\right\} \begin{aligned} 0 &= -3x - 3(x^2)^2 \\ 0 &= -3x - 3x^4 \end{aligned}$$

$0 = x^4 + x = x(x^3+1)$ $\nearrow x = 0 \to y = 0$, $\searrow x = -1 \to y = 1$

$\to$ dva stacionárne body $S_1[0,0]$, $S_2[-1,1]$

$$\left.\begin{aligned} f''_{xx} &= \frac{\partial}{\partial x} f'_x = \frac{\partial}{\partial x}(3x^2 - 3y) = 6x \\ f''_{yy} &= \frac{\partial}{\partial y} f'_y = \frac{\partial}{\partial y}(-3x - 3y^2) = -6y \\ f''_{xy} &= \frac{\partial}{\partial y} f'_x = \frac{\partial}{\partial y}(3x^2 - 3y) = -3 \\ f''_{yx} &= \frac{\partial}{\partial x} f'_y = \frac{\partial}{\partial x}(-3x - 3y^2) = -3 \end{aligned}\right\}$$

Hessova matica

$$H = \begin{pmatrix} 6x & -3 \\ -3 & -6y \end{pmatrix}$$

- určenie vlastností - S_1: $H = \begin{pmatrix} 6\cdot 0 & -3 \\ -3 & -6\cdot 0 \end{pmatrix} = \begin{pmatrix} 0 & -3 \\ -3 & 0 \end{pmatrix}$

$|H| = 0\cdot 0 - (-3)(-3) = \boxed{-9}$ ← Hessián

Hessián $< 0 \to$ nie je extrém

- S_2: $H = \begin{pmatrix} 6\cdot(-1) & -3 \\ -3 & -6\cdot 1 \end{pmatrix} = \begin{pmatrix} -6 & -3 \\ -3 & -6 \end{pmatrix}$

$|H| = (-6)(-6) - (-3)(-3) = 36 - 9 = \boxed{27}$ Hessián

Hessián $> 0 \to$ je extrém

$f''_{xx}(S_2) < 0 \to$ maximum

$\to$ má maximum $M[-1,1]$ a stacionárny bod $S[0,0]$

2, $f(x,y) = x^2 - 2y^2 - 3x + 5y - 1$

$$\frac{\partial}{\partial x}\left(f(x,y)\right) = 2x - 3$$

$2x - 3 = 0 \Rightarrow x = \frac{3}{2}$

$-4y + 5 = 0 \Rightarrow y = \frac{5}{4}$

$$\frac{\partial}{\partial y}\left(f(x,y)\right) = -4y + 5$$

STAC.B. $\left[\frac{3}{2} ; \frac{5}{4}\right]$

$$\frac{\partial^2}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}(2x-3) = 2$$

$$\frac{\partial^2}{\partial y^2} f(x,y) = \frac{\partial}{\partial y}(-4y+5) = -4$$

$$\frac{\partial^2}{\partial x \partial y} f(x,y) = \frac{\partial}{\partial y}(2x-3) = 0$$

$$H = \begin{pmatrix} 2 & 0 \\ 0 & -4 \end{pmatrix} = \underline{\underline{-8}}$$

není extrém

$$\frac{\partial^2}{\partial y \partial x} f(x,y) = \frac{\partial}{\partial x}(-4y+5) = 0$$

3, $f(x,y) = xy - 2x + 3y - 6$

$$\frac{\partial}{\partial x} = y - 2$$

$y - 2 = 0 \quad y = 2$

$x + 3 = 0 \quad x = -3$

$$\frac{\partial}{\partial y} = x + 3$$

STAC. B $[-3;2]$

$$\frac{\partial^2}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}(y-2) = 0$$

$$\frac{\partial^2}{\partial y^2} f(x,y) = \frac{\partial}{\partial y}(x+3) = 0$$

$$\frac{\partial^2}{\partial x \partial y} f(x,y) = \frac{\partial}{\partial y}(y-2) = 1$$

$$\frac{\partial^2}{\partial y \partial x} f(x,y) = \frac{\partial}{\partial x}(x+3) = 1$$

$$H = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = -1$$

není extrém

4) $f(x,y) = 3\cdot(x^2+y^2)^2$ $\quad\quad 3\cdot(x^4+2x^2y^2+y^4) = 3x^4+6x^2y^2+3y^4$

$$\frac{\partial}{\partial x} f(x,y) = 12x^3 + 12xy^2$$

$$\frac{\partial}{\partial y} f(x,y) = 12x^2y + 12y^3$$

$$\begin{aligned} 12x^3 + 12xy^2 &= 0 \\ 12y^3 + 12x^2y &= 0 \\ \hline 12x\cdot(x^2+y^2) &= 0 \\ 12y(y^2+x^2) &= 0 \end{aligned} \quad \begin{aligned} x&=0 \\ y&=0 \end{aligned} \quad [0;0] \longrightarrow \text{STAC. BOD } [0;0]$$

$$\frac{\partial^2}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}(12x^3+12xy^2) = 36x^2+12y^2$$

$$\frac{\partial^2}{\partial y^2} f(x,y) = \frac{\partial}{\partial y}(12x^2y+12y^3) = 12x^2+36y^2$$

$$\frac{\partial^2}{\partial x \partial y} f(x,y) = \frac{\partial}{\partial y}(12x^3+12xy^2) = 24xy$$

$$\frac{\partial^2}{\partial y \partial x} f(x,y) = \frac{\partial}{\partial x}(12x^2y+12y^3) = 24xy$$

$$H = \begin{pmatrix} 36x^2+12y^2 & 24xy \\ 24xy & 12x^2+36y^2 \end{pmatrix} = \begin{pmatrix} 36\cdot0^2+12\cdot0^2 & 24\cdot0\cdot0 \\ 24\cdot0\cdot0 & 12\cdot0^2+36\cdot0^2 \end{pmatrix} = 0$$

nevime

D 3.3

6. $f(x,y) = x^2 + xy + y^2 + 9x + 6y$

$f'_x(x,y) = 2x + y + 9 \quad = 0 \rightarrow y = -2x - 9$

$f'_y(x,y) = x + 2y + 6 \quad = 0$

$x + 2(-2x - 9) + 6 = 0$

$x - 4x - 18 + 6 = 0$

$-3x - 12 = 0$

$x = -4 \rightarrow y = -2(-4) - 9 = 8 - 9 = -1$

→ stacionárny bod $S[-4,-1]$

$\left.\begin{array}{l} f''_{xx}(x,y) = 2 \\ f''_{xy}(x,y) = 1 \\ f''_{yx}(x,y) = 1 \\ f''_{yy}(x,y) = 2 \end{array}\right\}$ Hessova matica $H = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$|H| = 2 \cdot 2 - 1 \cdot 1 = 4 - 1 = \boxed{3}$ ← Hessián

Hessián > 0 → extrém

$f''_{xx}(S) = 2 > 0$ → minimum

→ má jedno minimum v bode $m[-4,-1]$

---

5. $f(x,y) = x^2 y - 2xy + x$

$f'_x(x,y) = 2yx - 2y + 1 \quad = 0$

$f'_y(x,y) = x^2 - 2x = x(x-2) \quad = 0 \rightarrow x = 0,\ x = 2$

$x = 0$: $2y \cdot 0 - 2y + 1 = 0$, $-2y + 1 = 0$, $y = \frac{1}{2}$

$x = 2$: $2y \cdot 2 - 2y + 1 = 0$, $4y - 2y + 1 = 0$, $2y + 1 = 0$, $y = -\frac{1}{2}$

→ dva stacionárne body $S_1[0, \frac{1}{2}]$, $S_2[2; -\frac{1}{2}]$

$\left.\begin{array}{l} f''_{xx}(x,y) = 2y \\ f''_{xy}(x,y) = 2x - 2 \\ f''_{yx}(x,y) = 2x - 2 \\ f''_{yy}(x,y) = 0 \end{array}\right\}$ Hessova matica $H = \begin{pmatrix} 2y & 2x-2 \\ 2x-2 & 0 \end{pmatrix}$

– určenie vlastností – S_1: $H = \begin{pmatrix} 2 \cdot \frac{1}{2} & 2 \cdot 0 - 2 \\ 2 \cdot 0 - 2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ -2 & 0 \end{pmatrix}$

$|H| = 1 \cdot 0 - (-2)(-2) = \boxed{-4}$ ← Hessián

Hessián < 0 → nie je extrém

– S_2: $H = \begin{pmatrix} 2 \cdot (-\frac{1}{2}) & 2 \cdot 2 - 2 \\ 2 \cdot 2 - 2 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 2 & 0 \end{pmatrix}$

$|H| = (-1) \cdot 0 - (2) \cdot (2) = \boxed{-4}$ ← Hessián

Hessián < 0 → nie je extrém

→ má dva stacionárne body $S_1[0; \frac{1}{2}]$, $S_2[2; -\frac{1}{2}]$, nemá min/max

D 3.3

7. $f(x,y) = x^3 - 3x^2 + y^3 - 3y + 1$

$f'_x(x,y) = 3x^2 - 6x = 0 \rightarrow x(3x-6) = 0 \rightarrow$ ak $x = 0$, $x = 2$

$f'_y(x,y) = 3y^2 - 3 = 0 \rightarrow y^2 - 1 = 0$

$(y+1)(y-1) = 0 \rightarrow$ ak $y = -1$, $y = 1$

vždy platí kombinácia

$\rightarrow$ štyri stacionárne body $S_1[0,-1]$ $S_3[2,-1]$ $S_2[0,1]$ $S_4[2,1]$

$f''_{xx}(x,y) = 6x - 6$
$f''_{xy}(x,y) = 0$
$f''_{yx}(x,y) = 0$
$f''_{yy}(x,y) = 6y$

Hessova matica $H = \begin{pmatrix} 6x-6 & 0 \\ 0 & 6y \end{pmatrix}$

určenie vlastností – S_1: $H = \begin{pmatrix} 6\cdot 0-6 & 0 \\ 0 & 6\cdot(-1) \end{pmatrix} = \begin{pmatrix} -6 & 0 \\ 0 & -6 \end{pmatrix}$

$|H| = (-6)(-6) - (0)\cdot(0) = 36 - 0 = \boxed{36} \leftarrow$ Hessián

Hessián $> 0 \rightarrow$ extrém

$f''_{xx}(S_1) = -6 < 0 \rightarrow$ maximum

– S_2: $H = \begin{pmatrix} 6\cdot 0-6 & 0 \\ 0 & 6\cdot 1 \end{pmatrix} = \begin{pmatrix} -6 & 0 \\ 0 & 6 \end{pmatrix}$

$|H| = (-6)\cdot 6 - 0\cdot 0 = \boxed{-36} \leftarrow$ Hessián

Hessián $< 0 \rightarrow$ nie je extrém

– S_3: $H = \begin{pmatrix} 6\cdot 2-6 & 0 \\ 0 & 6\cdot(-1) \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & -6 \end{pmatrix}$

$|H| = 6\cdot(-6) - 0\cdot 0 = \boxed{-36} \leftarrow$ Hessián

Hessián $< 0 \rightarrow$ nie je extrém

– S_4: $H = \begin{pmatrix} 6\cdot 2-6 & 0 \\ 0 & 6\cdot 1 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$

$|H| = 6\cdot 6 - 0\cdot 0 = \boxed{36} \leftarrow$ Hessián

Hessián $> 0 \rightarrow$ extrém

$f''_{xx}(S_4) = 6 > 0 \rightarrow$ minimum

$\rightarrow$ S_1 = maximum; S_2, S_3 = stacionárne body; S_4 = minimum

$M[0,-1]$ $S_2[0,1]$ $S_3[2,-1]$ $m[2,1]$