

Skupina D

PŘ.: 3.1

3.1

1)  
 $x: 2xy - \cos x$   
 $y: x^2 + e^y$

2)  
 $x: \ln(xy) + 7$   
 $y: \frac{x}{y}$

3)  
 $x: \frac{2e^{2x}}{y}$   
 $y: -\frac{e^{2x}}{y^2}$

4)  
 $x: \sin(x+y) + x \cos(x+y)$   
 $y: x \cos(x+y)$

5)  
 $x: \frac{1}{x-y}$   
 $y: -\frac{1}{x-y}$

6)  
 $x: \cos x \cos y$   
 $y: -\sin x \sin y$

7)  
 $x: -2xy \sin(x^2y)$   
 $y: -x^2 \sin(x^2y)$

PŘ.: 3.2

5.2

1)  $yx^2 + e^x - \sin x$

$$\frac{\partial}{\partial x} = 2xy + 0 - \cos(x) = 2xy - \cos(x)$$

$$\frac{\partial^2}{\partial x^2} = 2y + \sin(x)$$

$$\frac{\partial}{\partial y} = x^2 + e^x$$

$$\frac{\partial^2}{\partial y^2} = 0 + e^x$$

$$\frac{\partial^2}{\partial x \partial y} = 2x \quad \frac{\partial^2}{\partial y \partial x} = 2x + 0$$


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2)  $x \ln xy$

$$\frac{\partial}{\partial x} = \ln(xy) + 1 \Rightarrow \frac{\partial^2}{\partial x^2} = \frac{1}{x}$$

$$\frac{\partial}{\partial y} = \frac{x}{y} \Rightarrow \frac{\partial^2}{\partial y^2} = -\frac{x}{y^2}$$

$$\frac{\partial^2}{\partial x \partial y} = -\frac{x}{y^2}$$

$$\frac{\partial^2}{\partial y \partial x} = \frac{1}{y}$$


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3)  $\frac{e^{2x}}{y}$

$$\frac{\partial}{\partial x} = \frac{2e^{2x}}{y} \quad \frac{\partial}{\partial y} = -\frac{e^{2x}}{y^2}$$

$$\frac{\partial^2}{\partial x^2} = \frac{4e^{2x}}{y} \quad \frac{\partial^2}{\partial y^2} = \frac{2e^{2x}}{y^3}$$

$$\frac{\partial^2}{\partial x \partial y} = 2e^{2x} \quad \frac{\partial^2}{\partial y \partial x} = -\frac{2e^{2x}}{y^2}$$


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4)  $x \sin(x+y)$


$$\frac{\partial}{\partial x} = \sin(x+y) + \cos(x+y) \quad \frac{\partial}{\partial y} = x \cos(x+y)$$


$$\frac{\partial^2}{\partial x^2} = 2\cos(x+y) - x \sin(x+y)$$

$$\frac{\partial^2}{\partial y^2} = -x \sin(x+y)$$

$$\frac{\partial^2}{\partial x \partial y} = \cos(x+y) - x \sin(x+y)$$

$$\frac{\partial^2}{\partial y \partial x} = -\sin(x+y)$$


 КОМПАНИА ПРО ХОСПОДАРСКЕ СТУКЫ СЕ СНС  
 ТОРГОВО-ПРОМЫШЛЕННАЯ ПАЛАТА ПО СТРАНАМ СНГ  
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 ROSATOM

5)  $\ln(x-y)$

$$\frac{df}{dx}(x,y) = g_1'(x) = \frac{1}{x-y} = (x-y)^{-1}$$

$$y'' = -1(x-y)^{-2} = -\frac{1}{(x-y)^2}$$

$$\frac{df}{dy}(x,y) = g_2'(y) = \frac{1}{x-y}$$

$$y'' = -\frac{1}{(x-y)^2}$$

6)  $\sin x \cos y$

$$\frac{df}{dx}(x,y) = g_1'(x) = \cos y \cdot \cos x$$

$$y'' = -\sin x \cdot \cos y$$

$$\frac{df}{dy}(x,y) = g_2'(y) = \sin x \cdot (-\sin y) = -\sin x \sin y$$

$$y'' = \cos x \cos y - \sin x \cdot (-\cos y) = \cos y \cdot \sin x$$

7)  $\cos x^2 y$

$$y' = \frac{df}{dx}(x,y) = -\sin x^2 y \cdot 2xy \Rightarrow y'' = -\cos x^2 y \cdot 2xy + 2xy + (-\sin x^2 y) \cdot 2y$$

$$y' = \frac{df}{dy}(x,y) = -\sin x^2 y \cdot x^2 \Rightarrow y'' = -\cos x^2 y \cdot x^2 \cdot x^2 = x^4 (-\cos x^2 y)$$

PŘ.: 3.3 (zřejmě ne zcela hotový, nevěděli jsme jak na to)

$$\textcircled{1} f(x, y) = x^3 - 3xy - y^3$$

$$\frac{\partial}{\partial x} f(x, y) = 3x^2 - 3y - y^3$$

$$= 6x - 3y - y^3$$

$$y = 6x$$

$$\frac{\partial}{\partial y} f(x, y) = 0 - 3x - 3y^2$$

$$= 0 - 3x - 6y$$

$$S[0; 0]$$

$$-3x \cdot 6x = 0$$

$$-18x = 0$$

$$y = 6x$$

$$y = 6 \cdot 0 = 0$$

Extremy

$$f'(x) = 3x^2 - 3$$

$$f''(x) = 6$$

$$f'(y) = -3 - 3y^2$$

$$f''(y) = -6$$

$$\frac{\partial^2}{\partial x^2} = -1$$

$$\frac{\partial^2}{\partial y^2} = +1$$

$$H[-1; 1]$$

$$\textcircled{2} f(x, y) = x^3 - 2xy^2 - 3x + 5y - 1$$

$$\frac{\partial}{\partial x} f'(x, y) = 3x^2 - 3$$

$$f''(x, y) = 6x$$

$$y = 6x$$

$$\frac{\partial}{\partial y} f(x, y) = 0 - 4y - 0 + 5 - 0$$

$$f(x, y) = 4y + 5$$

$$4y = -5 \Rightarrow y = -\frac{5}{4}$$

okrajy =

~~$$f(x, y) = x^3 - 2xy^2 - 3x + 5y - 1$$

$$f'(x) = 3x^2 - 3$$

$$f'(y) = -4y + 5$$~~

$$y = 3x^2 - 3$$

$$-4y = 3x^2 - 3 \quad | \cdot 4$$

$$-5 = 12x^2 - 3$$

$$12x^2 = -3 + 5$$

$$12x^2 = 2$$

$$S\left[\frac{3}{2}; -\frac{5}{4}\right]$$

$$\textcircled{3} f(x, y) = x^2 - 2y^2 - 3x + 5y - 1$$

$$\frac{\partial}{\partial x} f(x, y) = 2x - 0 - 3 + 0 - 1$$
$$y = 2x - 4 \quad y = 0$$

$$\frac{\partial}{\partial y} f(x, y) = 0 - 4y - 0 + 5$$
$$= -4y + 5$$

$$x = -4 \cdot (6 - 2x) + 5$$

$$x = -24 + 8x + 5$$

$$x = -19 + 8x$$

$$0 = -19 + 7x$$

$$7x = 19$$

$$x = \frac{19}{7}$$

$$\textcircled{4} f(x, y) = 3 \cdot (x^2 - y^2)^2 = 3 \cdot (x^4 - 2x^2y^2 + y^4)$$

$$\frac{\partial}{\partial x} f(x, y) = \text{Ans } 12x^3 - 12xy^2$$

$$= 12x^3 - 12xy^2$$

$$\textcircled{5} f(x, y) = x^2y - 2xy + x$$

$$\frac{\partial}{\partial x} f(x, y) = 2xy - 2y + 1$$

$$2y = 2x + 1 \quad | :2$$

$$y = x + \frac{1}{2}$$

$$y=0 \quad 0 = x + \frac{1}{2} \quad x = -\frac{1}{2} \quad y=0 \quad x=0 \quad y = \frac{1}{2}$$

$$\frac{\partial}{\partial y} f(x, y) = 0 + 2x + 0$$

$$= 0 + 2x$$

$$= 0 + 2 \cdot (x + \frac{1}{2})$$

$$= 0 + 2x + 1$$

$$2x = 1 \quad x = \frac{1}{2} \quad y = -\frac{1}{2}$$

$$-\frac{1}{2} = x + \frac{1}{2} \\ = 1$$

$$S [0; \frac{1}{2}] \quad ; \quad S [2; -\frac{1}{2}]$$

$$\textcircled{c} f(x,y) = x^2 + xy + y^2 + 9x + 6y$$

$$\frac{\partial}{\partial x} f(x,y) = 2x + y + 0 + 9 + 0$$

$$= 2x + y + 9$$

$$-y = 2x + 9$$

$$y = \underline{\underline{-2x - 9}}$$

$$\frac{\partial}{\partial y} f(x,y) = 0 + x + 2y + 0 + 6$$

$$= 0 + x + 2(-2x - 9) + 6$$

$$= 0 + x + 2(-2x - 9) + 6$$

$$= x + (-4x - 18) + 6$$

$$= x - 4x - 12$$

$$= -3x - 12$$

$$3x = -12 \quad | :3$$

$$x = \underline{\underline{-4}}$$

$$y = -2 \cdot (-4) - 9$$

$$y = 8 - 9$$

$$y = \underline{\underline{-1}}$$

$$S[-4, -1]$$

$$\text{min} = \frac{\partial^2}{\partial x^2} =$$

$$\textcircled{1} f(x, y) = x^3 - 3x^2 + y^3 - 3y + 1$$

$$\frac{\partial}{\partial x} f(x, y) = 3x^2 - 6x + 0 - 0 + 0$$

$$0 = 3x^2 - 6x$$

$$\frac{\partial}{\partial y} f(x, y) = 0 - 0 + 2y^2 - 3 + 0$$

$$0 = 2y^2 - 3$$

$$2y^2 = 3$$

$$y^2 = \frac{3}{2}$$

$$y = \pm \sqrt{\frac{3}{2}}$$

$$\frac{\partial}{\partial x} f''(x, y) = 6x - 6$$

$$6x = 6$$

$$x = 1$$

$$\frac{\partial}{\partial y} f''(x, y) = 4y$$

$$0 = 1 - 1 + y^2 - 3y + 1$$

$$0 = y^2 - 3y + 1$$

$$0 = y^2 + 3y - 1$$

$$0 = y^2 - 3y - 1$$

$$y = \frac{3 \pm \sqrt{9 - 4}}{2}$$

$$y = \frac{3 \pm \sqrt{5}}{2}$$