5.1

1. $\int x\,dx = \int \frac{x^2}{2} + c$

2. $\int 5x^7\,dx = \int 5 \cdot \frac{x^8}{8} + c$

3. $\int \frac{4}{x^2}\,dx = 4\int \frac{1}{x^2} = 4\int x^{-2} = 4 \cdot \frac{x^{-1}}{-1} + c = 4 \cdot -\frac{1}{x} + c = -\frac{4}{x} + c$

4. $\int \frac{5}{x^6}\,dx = 5\int \frac{1}{x^6} = 5 \cdot \int x^{-6} = \cancel{5} \cdot \frac{x^{-5}}{-\cancel{5}} + c = \frac{x^{-5}}{-1} + c = -\frac{1}{x^5} + c$

5. $\int e^x\left(1 + \frac{e^{-x}}{\cos^2 x}\right)dx = \int e^x + \frac{e^x \cdot e^{-x}}{\cos^2 x}\,dx = \int e^x + \frac{1}{\cos^2 x}\,dx = e^x + \operatorname{tg} x + c$

6. $\int \frac{1}{3\cos^2 x}\,dx = \frac{1}{3}\int \frac{1}{\cos^2 x}\,dx = \frac{1}{3} \cdot \operatorname{tg} x + c$

7. $\int (1+\sqrt{x})^2\,dx = \int 1 + 2\sqrt{x} + x\,dx = \int 1 + 2 \cdot x^{\frac{1}{2}} + x\,dx = x + 2 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^2}{2} + c =$

$= x + 2 \cdot \frac{2}{3} \cdot x^{\frac{3}{2}} + \frac{x^2}{2} + c = x + \frac{4}{3} \cdot \sqrt[2]{x^3} + \frac{x^2}{2} + c = x + \frac{4}{3} \cdot \sqrt{x^2} \cdot \sqrt{x} + \frac{x^2}{2} + c = x + \frac{4}{3} \cdot x \cdot \sqrt{x} + \frac{x^2}{2} + c$

8. $\int \left(x + \frac{1}{x} + \sqrt{x} + \frac{1}{\sqrt{x}}\right)dx = \int \left(x + \frac{1}{x} + x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)dx = \frac{x^2}{2} + \ln|x| + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c =$

$= \frac{x^2}{2} + \ln|x| + \frac{2 \cdot \sqrt{x^3}}{3} + \frac{2}{1} \cdot \sqrt{2} + c = \frac{x^2}{2} + \ln|x| + \frac{2 \cdot \overset{x}{\sqrt{x^2}} \cdot \sqrt{x}}{3} + 2\sqrt{2} + c = \frac{x^2}{2} + \ln|x| + \frac{2x\sqrt{x}}{3} + 2\sqrt{x} + c$

$$\int\left(x+\frac{1}{x}+\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx=\int\left(x+\frac{1}{x}+x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)dx=\frac{x^2}{2}+\ln|x|+\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+c=$$

$$=\frac{x^2}{2}+\ln|x|+\frac{2\cdot\sqrt{x^3}}{3}+\frac{2}{1}\cdot\sqrt{2}+c=\frac{x^2}{2}+\ln|x|+\frac{2\cdot\sqrt{x^2}\cdot\sqrt{x}}{3}+2\sqrt{2}+c=\frac{x^2}{2}+\ln|x|+\frac{2x\sqrt{x}}{3}+2\sqrt{x}+c$$

9.

10. $\int\frac{5}{\sqrt{x}}dx=5\int\frac{1}{\sqrt{x}}dx=5\int x^{-\frac{1}{2}}dx=5\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+c=\cancel{5}\cdot\frac{2}{1}\cdot\sqrt{x}=10\sqrt{x}$

11. $\int\frac{(x+2)^3}{x^3}dx=\int\frac{x^3+6x^2+12x+8}{x^3}dx=\int\frac{x^3\left(1+\frac{6}{x}+\frac{12}{x^2}+\frac{8}{x^2}\right)}{x^3}dx=\int 1+\frac{6}{x}+\frac{12}{x^2}+\frac{8}{x^3}dx=$

$=\int 1+6\cdot\frac{1}{x}+12\cdot x^{-2}+8\cdot x^{-3}dx=x+6\ln|x|+12\cdot\frac{x^{-1}}{-1}+8\cdot\frac{x^{-2}}{-2}+c=x+6\ln|x|-\frac{12}{x}-\frac{4}{x^2}+c$

12. $\int 8x^{\frac{3}{5}}dx=8\int x^{\frac{3}{5}}dx=8\cdot\frac{x^{\frac{8}{5}}+c}{\frac{8}{5}}=8\cdot\frac{5}{8}\cdot x^{\frac{8}{5}}+c=5\cdot x^{\frac{8}{5}}+c$

13. $\int\frac{3}{x}dx=3\int\frac{1}{x}dx=3\cdot\ln|x|+c$

$$\int \frac{3\cos x}{\sin^4 x}\,dx = \left| \begin{matrix} t = \sin x \\ dt = \cos x\,dx \end{matrix} \right| = 3\int t^{-4}\,dt = -t^{-3} = -\frac{1}{\sin^3 x}$$

$$\int \frac{-2}{\tan x \sin^2 x} = -2\int \frac{1}{\sin^2 x}\cdot\frac{1}{\tan x} = -2\int \frac{\cos x}{\sin^3 x}\,dt \left| \begin{matrix} t = \sin x \\ dt = \cos x\,dx \end{matrix} \right| =$$

$$= -2\int t^{-3}\,dt = t^{-2} = \frac{1}{\sin^2 x} + C$$

$$\frac{1}{\tan x} = \cancel{\cos}\ \cot g\, x = \frac{\cos x}{\sin x}$$

C

$(8-3x)^{6/5}$

5) $\int 33(8-3x)^{6/5}\,dx = 33\int \sqrt[5]{(8-3x)^6}\,dx = \left| \begin{array}{l} t = 8-3x \\ dt = -3\,dx \\ -\frac{dt}{3} = dx \end{array} \right| =$

$= -\frac{33}{3}\int t^{\frac{6}{5}}\,dt = -\frac{\cancel{33}^{11}}{\cancel{3}}\,\frac{t^{\frac{11}{5}}}{\frac{11}{5}}\,dt = -5(8-3x)^{\frac{11}{5}}$

$$\int -4exe^{-2x^2}dx = \left|\begin{array}{l} t=-2x^2 \\ dt=-4x\,dx \\ -\frac{dt}{4}=x\,dx \end{array}\right| = \int e^t dx = e^t = e^{-2x^2}+c$$

$$\int \frac{3\sqrt{\ln x}}{x}dx = \left|\begin{array}{l} t=\ln x \\ dt=\frac{1}{x}dx \end{array}\right| = 3\int t^{\frac{1}{2}}dx = 3\frac{t^{\frac{3}{2}}}{\frac{3}{2}} = 3(\ln x)^{\frac{3}{2}}$$

$$\int \frac{3\cos x}{\sin^4 x}dx = \left|\begin{array}{l} t=\sin x \\ dt=\cos x\,dx \end{array}\right| = 3\int t^{-4}dt = -t^{-3} = -\frac{1}{\sin^3 x}$$

$$\int \frac{1}{x^2-6x+9} = \int \frac{1}{(x-3)^2}\,dx = \left| \begin{array}{l} t = x-3 \\ dt = dx \end{array} \right. = \int t^{-2}\,dt = -t^{-1} = -\frac{1}{x-3}$$

$= \frac{-33}{3} \int t^{\frac{6}{5}}\, dt = -\frac{33}{3} \frac{t^{\frac{11}{5}}}{\frac{11}{5}}\, dt = -$

6, $\int \frac{(1+\ln x)^4}{x}\, dx = \left| \begin{matrix} t = 1+\ln x \\ dt = \frac{1}{x}\, dx \end{matrix} \right| = \int t^4\, dt = \frac{t^5}{5} = \frac{1}{5}(1+\ln x)^5 + c$

5. 3

$\cos \pi = -1 \qquad \cos 0 = 1$

1. $\int_0^{\pi} \sin x \, dx = \left[-\cos x\right]_0^{\pi} = (-(-1)) - (1) = \underline{\underline{2}}$

2. $\int_0^4 12\sqrt{x+\frac{1}{4}}\, dx = 12\int_0^4 (x+1)^{\frac{1}{2}} dx = 12\left[\frac{(x+\frac{1}{4})^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 = \left[12\cdot\frac{3}{2}\left(x+\frac{1}{4}\right)^{\frac{3}{2}}\right]_0^4 = \left[8\left(x+\frac{1}{4}\right)^{\frac{3}{2}}\right]_0^4 = \left[(4x+1)^{\frac{3}{2}}\right]_0^4$

$= 17^{\frac{3}{2}} - 1^{\frac{3}{2}} = \sqrt{17^3} - \sqrt{1^3} = \sqrt{17^2}\cdot\sqrt{17} - 1 = \underline{\underline{17\sqrt{17} - 1}}$

3. $\int_1^2 \frac{6}{6x-1} dx = \left| \begin{matrix} t = 6x-1 \\ dt = 6dx \end{matrix} \right| = \int_1^2 \frac{1}{t} dt = \left[\ln|t|\right]_1^2 = \left[\ln|6x-1|\right]_1^2 = \ln|11| - \ln|5| =$

$= \underline{\underline{\ln\frac{11}{5}}}$

4. $\int_0^{\pi/2} 4\cdot\sin x \cos^3 x \, dx = \left| \begin{matrix} \cos x = t \\ -\sin x = dt \end{matrix} \right| = \int_0^{\pi/2} -4\cdot t^3 dt = \left[-4\cdot\frac{t^4}{4}\right]_0^{\frac{\pi}{2}} = \left[-t^4\right]_0^{\frac{\pi}{2}} =$

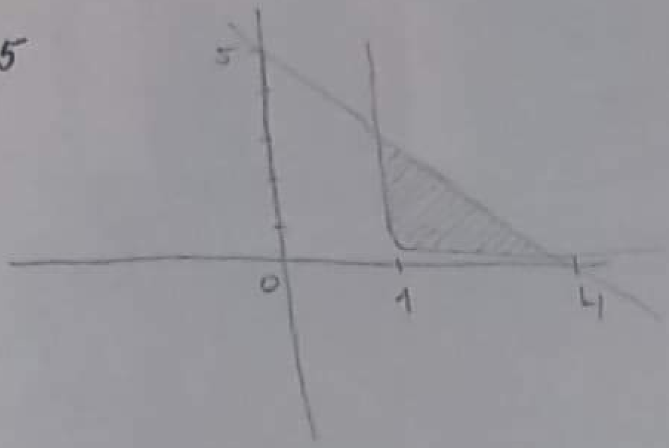
$= \left[-\cos^4 x\right]_0^{\frac{\pi}{2}} = 0 - (-1) = \underline{\underline{1}}$

5. $\int_{-1}^{3} (x^3 - 3x^2 + 1) dx = \left[\frac{x^4}{4} - x^3 + x\right]_{-1}^3 = \left[\frac{x^4 - 4x^3 + 4x}{4}\right]_{-1}^3 = \left(\frac{3^4 - 4\cdot(3)^3 + 4\cdot 3}{4}\right) - \left(\frac{(-1)^4 - 4\cdot(-1)^3 +}{4}\right)$

$= -\frac{15}{4} - \frac{1}{4} = -\frac{16}{4} = \underline{-4}$

Príklad: 5.4.

1. $yx = 4$; $x + y = 5$



$\frac{4}{x} = 5 - x$

$4 = 5x - x^2$

$x^2 - 5x + 4 = 0$ $\qquad x_{1,2} = \frac{5 \pm 3}{2} \nearrow 4, \searrow 1$

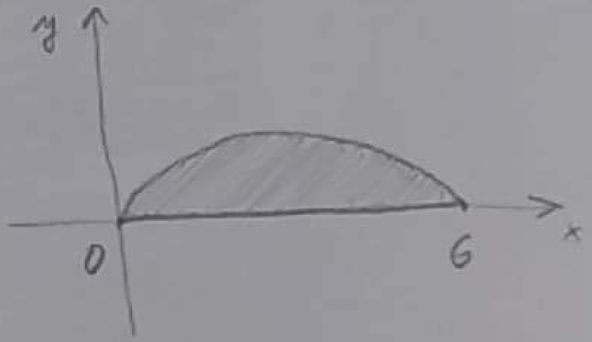
$D = 25 - 16 = 9$

$\int_1^4 \frac{4}{x} - (5 - x) + c = \int_1^4 4 \cdot \frac{1}{x} + x - 5 + c = \left[4 \ln|x| + \frac{x^2}{2} - 5x\right]_1^4 =$

$= \underline{\underline{\frac{15}{2} - 8 \ln 2}}$

Príklad: 5.4.

2. $y = 6x - x^2$; $y = 0$



$x \cdot (6 - x) = 0$

$x = 0$

$x = 6$

$\int_0^6 6x - x^2 dx = 6 \cdot \int_0^6 (x - x^2) dx = \left[6 \cdot \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\right]_0^6 = \left(6 \cdot \frac{36}{2} - \frac{216}{3}\right) - 0 =$

$= (6 \cdot 18) - 72 = \underline{\underline{36}}$